Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.6-g+h-x-^m-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [143]. This is test number [37].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (143)	% 0. (0)
Mathematica	% 99.3 (142)	% 0.7 (1)
Maple	% 98.6 (141)	% 1.4 (2)
Maxima	% 8.39 (12)	% 91.61 (131)
Fricas	% 42.66 (61)	% 57.34 (82)
Sympy	% 6.99 (10)	% 93.01 (133)
Giac	% 32.87 (47)	% 67.13 (96)

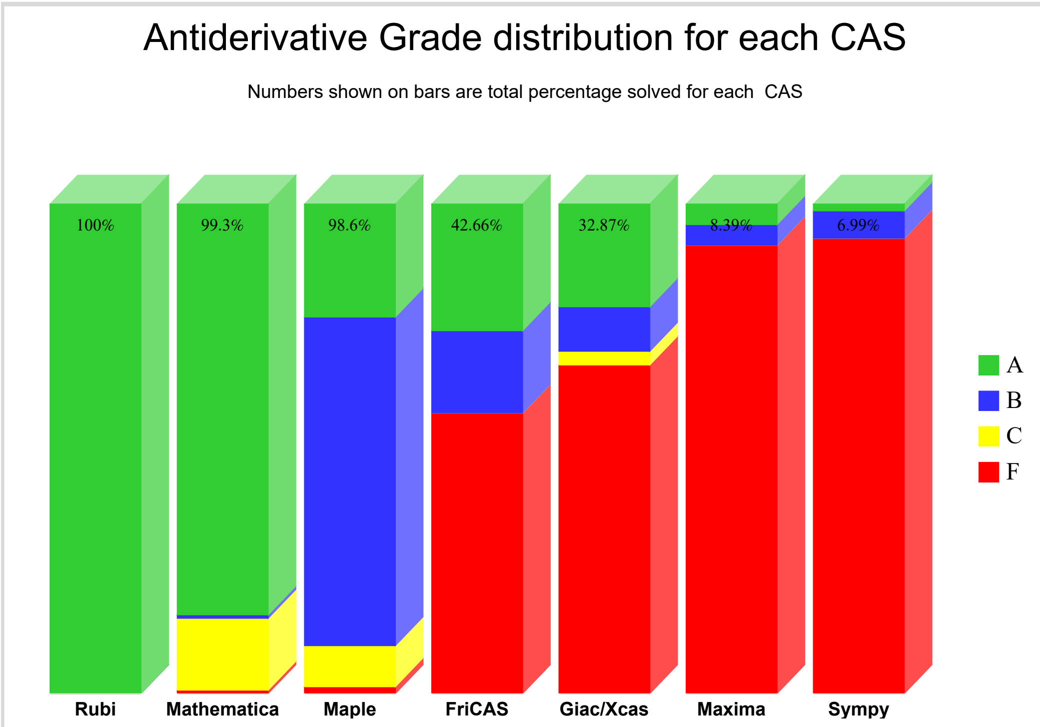
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

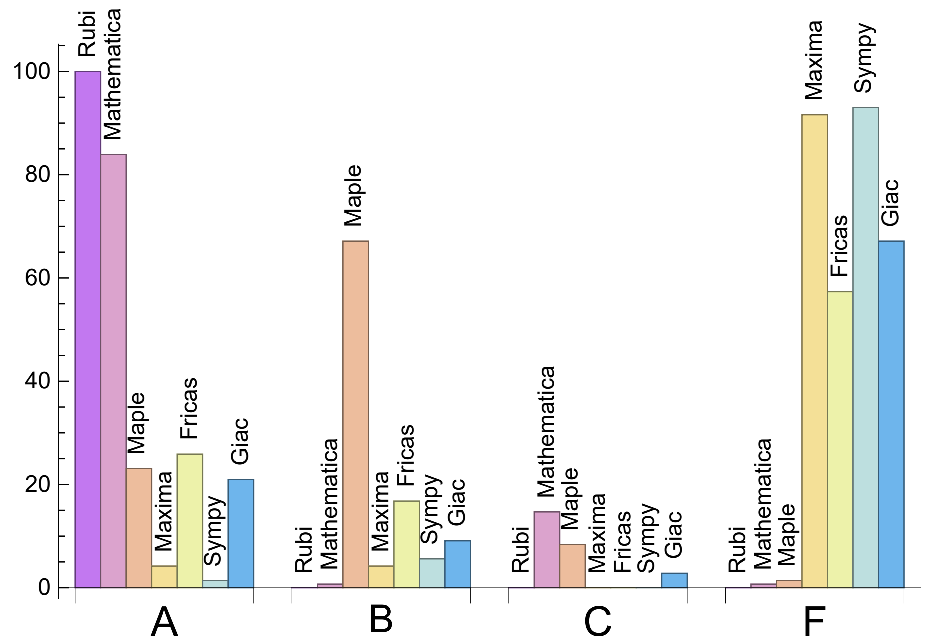
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	83.92	0.7	14.69	0.7
Maple	23.08	67.13	8.39	1.4
Maxima	4.2	4.2	0.	91.61
Fricas	25.87	16.78	0.	57.34
Sympy	1.4	5.59	0.	93.01
Giac	20.98	9.09	2.8	67.13

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.64	333.9	1.	302.	1.
Mathematica	1.21	352.03	1.17	290.5	0.97
Maple	0.23	52015.6	107.04	1172.	3.85
Maxima	1.5	568.	3.58	348.	2.56
Fricas	7.96	2115.46	9.83	821.	5.26
Sympy	8.69	599.4	3.67	506.	3.8
Giac	1.28	602.85	4.5	231.	1.91

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{25, 28, 142\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

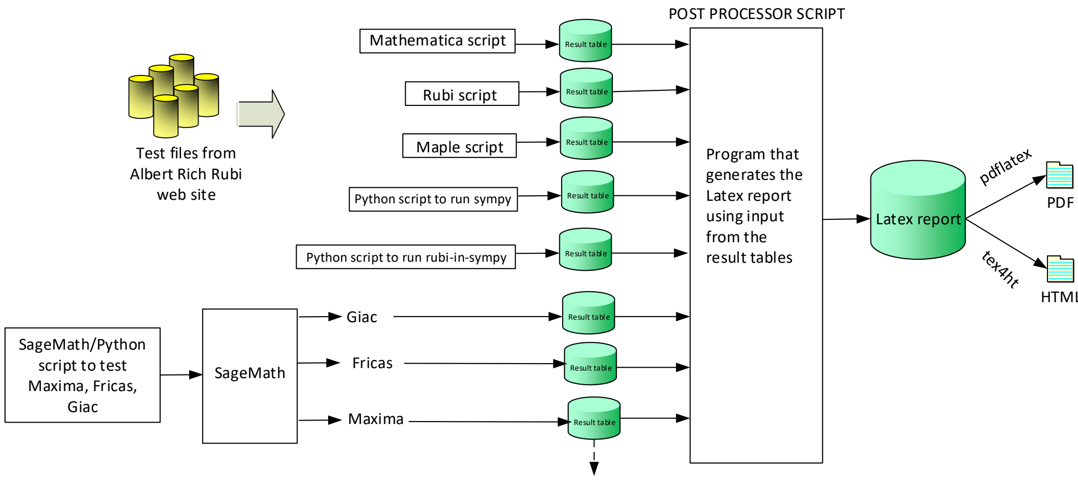
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 139, 140, 141 }

B grade: { 35 }

C grade: { 11, 12, 31, 32, 33, 34, 37, 38, 62, 63, 75, 76, 93, 126, 127, 128, 129, 130, 131, 132, 142 }

F grade: { 143}

2.1.3 Maple

A grade: { 1, 2, 3, 10, 22, 28, 31, 32, 33, 34, 36, 38, 39, 92, 94, 95, 96, 99, 100, 101, 126, 127, 128, 129, 130, 131, 132, 133, 134, 138, 139, 140, 141 }

B grade: { 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 35, 37, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 97, 98, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 135, 136, 137 }

C grade: { 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51 }

F grade: { 142, 143 }

2.1.4 Maxima

A grade: { 10, 92, 133, 134, 138, 139

B grade: { 25, 26, 27, 28, 29, 30 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 10, 13, 14, 33, 36, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141 }

B grade: { 11, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 37, 66, 67, 81, 82, 93, 97, 98, 116, 117, 135 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 12, 15, 16, 19, 20, 21, 22, 23, 35, 39, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

2.1.6 Sympy

A grade: { 33, 139 }

B grade: { 1, 2, 3, 13, 17, 18, 31, 138 }

C grade: { }

F grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 10, 13, 14, 17, 18, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 92, 126, 127, 129, 131, 133, 134, 137, 138, 139 }

B grade: { 5, 16, 31, 32, 33, 37, 38, 51, 128, 130, 132, 135, 136 }

C grade: { 11, 24, 34, 93 }

F grade: { 6, 7, 8, 9, 12, 15, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 35, 39, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 140, 141, 142, 143 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	86	133	0	452	332	117
normalized size	1	1.	0.91	1.41	0.	4.81	3.53	1.24
time (sec)	N/A	0.113	0.078	0.053	0.	1.88	2.537	1.212

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	204	373	0	1088	928	355
normalized size	1	1.	0.89	1.64	0.	4.77	4.07	1.56
time (sec)	N/A	0.33	0.222	0.052	0.	1.913	8.991	1.402

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	422	822	0	2164	1940	841
normalized size	1	1.	0.96	1.86	0.	4.91	4.4	1.91
time (sec)	N/A	0.621	0.493	0.053	0.	2.217	25.993	1.335

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	212	745	0	0	0	359
normalized size	1	1.	0.77	2.72	0.	0.	0.	1.31
time (sec)	N/A	0.282	0.404	0.199	0.	0.	0.	1.237

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	596	523	9311	0	0	0	1773
normalized size	1	1.	0.88	15.62	0.	0.	0.	2.97
time (sec)	N/A	1.772	2.17	0.204	0.	0.	0.	1.233

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	322	3358	0	0	0	0
normalized size	1	1.	0.97	10.15	0.	0.	0.	0.
time (sec)	N/A	0.601	0.805	0.45	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	249	714	0	0	0	0
normalized size	1	1.	1.	2.87	0.	0.	0.	0.
time (sec)	N/A	0.204	0.255	0.334	0.	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	380	440	2758	0	0	0	0
normalized size	1	1.	1.15	7.24	0.	0.	0.	0.
time (sec)	N/A	0.798	0.812	0.306	0.	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	797	796	674	6422	0	0	0	0
normalized size	1	1.	0.85	8.06	0.	0.	0.	0.
time (sec)	N/A	1.87	5.165	0.256	0.	0.	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	49	46	88	146	0	65
normalized size	1	1.	1.04	0.98	1.87	3.11	0.	1.38
time (sec)	N/A	0.034	0.008	0.05	1.536	1.526	0.	1.218

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	78	637	0	2379	0	336
normalized size	1	1.	0.67	5.44	0.	20.33	0.	2.87
time (sec)	N/A	0.169	0.034	0.161	0.	1.206	0.	1.298

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	484	136	6871419	0	0	0	0
normalized size	1	1.	0.28	14197.2	0.	0.	0.	0.
time (sec)	N/A	23.581	0.087	0.901	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	182	175	510	0	1280	1260	258
normalized size	1	0.99	0.95	2.77	0.	6.96	6.85	1.4
time (sec)	N/A	0.346	0.203	0.175	0.	1.799	25.275	1.149

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	535	1672	0	3933	0	996
normalized size	1	1.	0.99	3.08	0.	7.26	0.	1.84
time (sec)	N/A	1.102	0.626	0.167	0.	2.74	0.	1.155

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	398	267	1698	0	0	0	0
normalized size	1	0.98	0.66	4.18	0.	0.	0.	0.
time (sec)	N/A	0.478	0.54	0.314	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1075	1067	1376	51470	0	0	0	4355
normalized size	1	0.99	1.28	47.88	0.	0.	0.	4.05
time (sec)	N/A	4.177	7.641	0.312	0.	0.	0.	1.578

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	131	340	0	2399	709	296
normalized size	1	1.	0.94	2.43	0.	17.14	5.06	2.11
time (sec)	N/A	0.131	0.153	0.161	0.	1.931	3.775	1.208

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	131	340	0	2372	680	279
normalized size	1	1.	0.94	2.43	0.	16.94	4.86	1.99
time (sec)	N/A	0.104	0.028	0.167	0.	1.936	3.192	1.159

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	617	615	517	16209	0	0	0	0
normalized size	1	1.	0.84	26.27	0.	0.	0.	0.
time (sec)	N/A	8.998	2.152	0.309	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1092	1092	1627	59465	0	0	0	0
normalized size	1	1.	1.49	54.46	0.	0.	0.	0.
time (sec)	N/A	18.867	6.536	0.306	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	393	2269	0	0	0	0
normalized size	1	1.	0.94	5.45	0.	0.	0.	0.
time (sec)	N/A	2.702	4.235	0.431	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	780	780	254	784	0	0	0	0
normalized size	1	1.	0.33	1.01	0.	0.	0.	0.
time (sec)	N/A	5.162	0.438	0.355	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	283	1771	0	0	0	0
normalized size	1	1.	0.94	5.86	0.	0.	0.	0.
time (sec)	N/A	0.843	0.49	0.344	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	154	608	0	2943	0	11297
normalized size	1	1.	1.52	6.02	0.	29.14	0.	111.85
time (sec)	N/A	0.127	0.177	0.326	0.	2.497	0.	3.821

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	140	324	487	948	0	0
normalized size	1	1.	1.01	2.33	3.5	6.82	0.	0.
time (sec)	N/A	0.22	0.296	0.135	1.56	1.649	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	167	760	915	1029	0	0
normalized size	1	1.	1.01	4.58	5.51	6.2	0.	0.
time (sec)	N/A	0.221	0.38	0.106	1.584	1.451	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	185	1560	1723	1304	0	0
normalized size	1	1.	0.96	8.08	8.93	6.76	0.	0.
time (sec)	N/A	0.266	0.609	0.107	1.733	1.491	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	148	186	490	709	0	0
normalized size	1	1.	0.98	1.23	3.25	4.7	0.	0.
time (sec)	N/A	0.229	0.354	0.125	1.538	1.284	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	172	466	902	1131	0	0
normalized size	1	1.	0.99	2.68	5.18	6.5	0.	0.
time (sec)	N/A	0.255	0.576	0.11	1.565	1.477	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	190	878	1723	1422	0	0
normalized size	1	1.	0.96	4.46	8.75	7.22	0.	0.
time (sec)	N/A	0.303	0.722	0.101	1.762	1.457	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	79	14	0	136	36	78
normalized size	1	1.	5.27	0.93	0.	9.07	2.4	5.2
time (sec)	N/A	0.017	0.044	0.046	0.	1.323	3.443	1.158

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	101	40	0	327	0	146
normalized size	1	1.	2.3	0.91	0.	7.43	0.	3.32
time (sec)	N/A	0.052	0.054	0.046	0.	1.283	0.	1.129

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	90	20	0	103	68	127
normalized size	1	1.	3.75	0.83	0.	4.29	2.83	5.29
time (sec)	N/A	0.019	0.063	0.099	0.	1.262	3.175	1.259

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	114	45	0	1018	0	217
normalized size	1	1.	2.04	0.8	0.	18.18	0.	3.88
time (sec)	N/A	0.05	0.063	0.049	0.	1.474	0.	1.244

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	767	3606	0	0	0	0
normalized size	1	1.	3.08	14.48	0.	0.	0.	0.
time (sec)	N/A	0.91	1.654	0.325	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	46	48	0	181	0	109
normalized size	1	1.	0.96	1.	0.	3.77	0.	2.27
time (sec)	N/A	0.05	0.198	0.044	0.	3.785	0.	1.17

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	165	94	0	142	0	132
normalized size	1	1.	9.71	5.53	0.	8.35	0.	7.76
time (sec)	N/A	0.022	0.298	0.091	0.	1.795	0.	1.165

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	150	123	0	360	0	220
normalized size	1	1.	1.74	1.43	0.	4.19	0.	2.56
time (sec)	N/A	0.185	0.113	0.092	0.	1.645	0.	1.176

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	108	122	0	0	0	0
normalized size	1	1.	0.79	0.9	0.	0.	0.	0.
time (sec)	N/A	0.114	0.095	0.234	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	129	103	0	435	0	158
normalized size	1	1.	0.61	0.49	0.	2.05	0.	0.75
time (sec)	N/A	0.116	0.157	0.211	0.	1.966	0.	1.172

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	117	83	0	381	0	132
normalized size	1	1.	0.73	0.52	0.	2.37	0.	0.82
time (sec)	N/A	0.067	0.119	0.223	0.	1.904	0.	1.149

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	85	65	0	316	0	107
normalized size	1	1.	0.57	0.44	0.	2.14	0.	0.72
time (sec)	N/A	0.057	0.059	0.216	0.	1.873	0.	1.201

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	139	94	0	859	0	138
normalized size	1	1.	0.87	0.59	0.	5.37	0.	0.86
time (sec)	N/A	0.119	0.147	0.205	0.	1.853	0.	1.206

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	118	120	0	821	0	170
normalized size	1	1.	0.76	0.77	0.	5.26	0.	1.09
time (sec)	N/A	0.113	0.283	0.228	0.	1.72	0.	1.181

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	126	141	0	934	0	269
normalized size	1	1.	0.78	0.88	0.	5.8	0.	1.67
time (sec)	N/A	0.116	0.117	0.24	0.	1.723	0.	1.147

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	198	530	0	1250	0	497
normalized size	1	1.	0.62	1.67	0.	3.94	0.	1.57
time (sec)	N/A	0.333	0.282	0.214	0.	1.845	0.	1.185

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	147	381	0	930	0	362
normalized size	1	1.	0.65	1.68	0.	4.1	0.	1.59
time (sec)	N/A	0.127	0.137	0.213	0.	1.782	0.	1.192

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	134	257	0	683	0	250
normalized size	1	1.	0.68	1.3	0.	3.45	0.	1.26
time (sec)	N/A	0.101	0.128	0.202	0.	1.711	0.	1.211

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	149	214	0	1581	0	0
normalized size	1	1.	0.71	1.01	0.	7.49	0.	0.
time (sec)	N/A	0.221	0.202	0.244	0.	6.149	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	155	249	0	1562	0	284
normalized size	1	1.	0.77	1.23	0.	7.73	0.	1.41
time (sec)	N/A	0.194	0.195	0.216	0.	3.455	0.	1.212

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	161	358	0	1670	0	608
normalized size	1	1.	0.75	1.67	0.	7.77	0.	2.83
time (sec)	N/A	0.183	0.232	0.206	0.	4.573	0.	1.271

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	516	7739	0	0	0	0
normalized size	1	1.	1.14	17.12	0.	0.	0.	0.
time (sec)	N/A	1.967	2.596	0.319	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	422	5581	0	0	0	0
normalized size	1	1.	1.07	14.13	0.	0.	0.	0.
time (sec)	N/A	0.931	1.585	0.258	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	282	3249	0	0	0	0
normalized size	1	1.	0.95	10.9	0.	0.	0.	0.
time (sec)	N/A	0.376	0.414	0.256	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	314	3544	0	0	0	0
normalized size	1	1.	0.88	9.9	0.	0.	0.	0.
time (sec)	N/A	1.314	0.734	0.266	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	569	3703	0	0	0	0
normalized size	1	1.	1.49	9.69	0.	0.	0.	0.
time (sec)	N/A	1.417	3.202	0.311	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	507	507	642	3993	0	0	0	0
normalized size	1	1.	1.27	7.88	0.	0.	0.	0.
time (sec)	N/A	1.88	2.751	0.268	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	795	795	793	19148	0	0	0	0
normalized size	1	1.	1.	24.09	0.	0.	0.	0.
time (sec)	N/A	4.264	3.761	0.275	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	755	14709	0	0	0	0
normalized size	1	1.	1.37	26.6	0.	0.	0.	0.
time (sec)	N/A	2.431	2.168	0.26	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	482	603	8954	0	0	0	0
normalized size	1	1.	1.25	18.5	0.	0.	0.	0.
time (sec)	N/A	4.24	1.158	0.269	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	746	9728	0	0	0	0
normalized size	1	1.	1.5	19.61	0.	0.	0.	0.
time (sec)	N/A	2.569	1.627	0.262	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	604	604	885	9912	0	0	0	0
normalized size	1	1.	1.47	16.41	0.	0.	0.	0.
time (sec)	N/A	2.809	4.639	0.288	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	668	668	904	10298	0	0	0	0
normalized size	1	1.	1.35	15.42	0.	0.	0.	0.
time (sec)	N/A	3.465	3.547	0.312	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	378	2397	0	0	0	0
normalized size	1	1.	0.99	6.31	0.	0.	0.	0.
time (sec)	N/A	1.17	1.377	0.291	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	334	1796	0	0	0	0
normalized size	1	1.	0.97	5.22	0.	0.	0.	0.
time (sec)	N/A	0.541	0.732	0.269	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	275	1172	0	10211	0	0
normalized size	1	1.	0.94	3.99	0.	34.73	0.	0.
time (sec)	N/A	0.235	0.385	0.261	0.	8.071	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	247	589	0	10168	0	0
normalized size	1	1.	0.93	2.21	0.	38.23	0.	0.
time (sec)	N/A	0.151	0.336	0.263	0.	6.903	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	319	681	0	0	0	0
normalized size	1	1.	0.97	2.06	0.	0.	0.	0.
time (sec)	N/A	0.825	0.788	0.319	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	356	736	0	0	0	0
normalized size	1	1.	0.97	2.01	0.	0.	0.	0.
time (sec)	N/A	1.199	0.903	0.273	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	460	911	0	0	0	0
normalized size	1	1.	1.01	1.99	0.	0.	0.	0.
time (sec)	N/A	1.863	1.679	0.278	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	577	6124	0	0	0	0
normalized size	1	1.	1.16	12.27	0.	0.	0.	0.
time (sec)	N/A	2.107	2.91	0.276	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	509	4752	0	0	0	0
normalized size	1	1.	1.24	11.59	0.	0.	0.	0.
time (sec)	N/A	0.709	2.494	0.296	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	457	3000	0	0	0	0
normalized size	1	1.	1.11	7.3	0.	0.	0.	0.
time (sec)	N/A	0.826	0.912	0.29	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	320	1713	0	0	0	0
normalized size	1	1.	0.77	4.12	0.	0.	0.	0.
time (sec)	N/A	0.617	2.335	0.304	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	526	526	497	1945	0	0	0	0
normalized size	1	1.	0.94	3.7	0.	0.	0.	0.
time (sec)	N/A	2.183	4.021	0.273	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	618	618	557	2046	0	0	0	0
normalized size	1	1.	0.9	3.31	0.	0.	0.	0.
time (sec)	N/A	2.28	4.924	0.273	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	327	1817	0	0	0	0
normalized size	1	1.	0.83	4.64	0.	0.	0.	0.
time (sec)	N/A	0.952	1.03	0.266	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	302	1810	0	0	0	0
normalized size	1	1.	0.96	5.73	0.	0.	0.	0.
time (sec)	N/A	0.489	0.523	0.263	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	272	1667	0	0	0	0
normalized size	1	1.	0.96	5.91	0.	0.	0.	0.
time (sec)	N/A	0.295	0.333	0.258	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	253	1669	0	0	0	0
normalized size	1	1.	0.95	6.27	0.	0.	0.	0.
time (sec)	N/A	0.226	0.186	0.255	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	255	1764	0	2695	0	0
normalized size	1	1.	0.96	6.61	0.	10.09	0.	0.
time (sec)	N/A	0.778	0.313	0.253	0.	116.964	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	275	1819	0	2422	0	0
normalized size	1	1.	0.96	6.36	0.	8.47	0.	0.
time (sec)	N/A	0.706	0.456	0.293	0.	141.882	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	316	1953	0	0	0	0
normalized size	1	1.	0.9	5.53	0.	0.	0.	0.
time (sec)	N/A	0.879	0.549	0.299	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	447	4884	0	0	0	0
normalized size	1	1.	0.89	9.75	0.	0.	0.	0.
time (sec)	N/A	1.412	1.563	0.273	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	395	4900	0	0	0	0
normalized size	1	1.	0.95	11.75	0.	0.	0.	0.
time (sec)	N/A	1.015	1.082	0.269	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	330	4567	0	0	0	0
normalized size	1	1.	0.95	13.09	0.	0.	0.	0.
time (sec)	N/A	0.522	0.875	0.263	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	298	4574	0	0	0	0
normalized size	1	1.	0.95	14.52	0.	0.	0.	0.
time (sec)	N/A	0.517	0.842	0.263	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	755	4765	0	0	0	0
normalized size	1	1.	1.61	10.16	0.	0.	0.	0.
time (sec)	N/A	1.274	0.532	0.278	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	765	4799	0	0	0	0
normalized size	1	1.	1.65	10.37	0.	0.	0.	0.
time (sec)	N/A	1.201	0.669	0.268	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	303	5056	0	0	0	0
normalized size	1	1.	0.49	8.23	0.	0.	0.	0.
time (sec)	N/A	1.436	1.011	0.276	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	181	1346	0	0	0	0
normalized size	1	1.	0.96	7.12	0.	0.	0.	0.
time (sec)	N/A	0.285	0.621	0.2	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	102	112	197	0	99
normalized size	1	1.	1.	1.36	1.49	2.63	0.	1.32
time (sec)	N/A	0.058	0.039	0.055	1.718	1.802	0.	1.276

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	120	789	0	2356	0	369
normalized size	1	1.	0.92	6.07	0.	18.12	0.	2.84
time (sec)	N/A	0.16	0.172	0.127	0.	2.039	0.	1.418

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	300	516	0	0	0	0
normalized size	1	1.	0.81	1.4	0.	0.	0.	0.
time (sec)	N/A	0.809	1.923	0.28	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	325	410	0	0	0	0
normalized size	1	1.	1.13	1.43	0.	0.	0.	0.
time (sec)	N/A	0.633	1.202	0.265	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	250	399	0	0	0	0
normalized size	1	1.	0.94	1.5	0.	0.	0.	0.
time (sec)	N/A	0.216	0.456	0.279	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	211	354	0	5501	0	0
normalized size	1	1.	0.96	1.61	0.	25.	0.	0.
time (sec)	N/A	0.13	0.184	0.265	0.	5.229	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	209	358	0	5328	0	0
normalized size	1	1.	0.95	1.63	0.	24.22	0.	0.
time (sec)	N/A	0.118	0.097	0.26	0.	4.682	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	252	391	0	0	0	0
normalized size	1	1.	0.94	1.46	0.	0.	0.	0.
time (sec)	N/A	0.663	0.472	0.28	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	325	427	0	0	0	0
normalized size	1	1.	1.12	1.47	0.	0.	0.	0.
time (sec)	N/A	0.657	1.067	0.265	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	314	519	0	0	0	0
normalized size	1	1.	0.84	1.38	0.	0.	0.	0.
time (sec)	N/A	0.734	2.155	0.279	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	466	562	1648	0	0	0	0
normalized size	1	1.	1.21	3.54	0.	0.	0.	0.
time (sec)	N/A	1.347	1.64	0.283	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	414	1480	0	0	0	0
normalized size	1	1.	1.21	4.34	0.	0.	0.	0.
time (sec)	N/A	1.041	1.379	0.287	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	352	1427	0	0	0	0
normalized size	1	1.	1.19	4.8	0.	0.	0.	0.
time (sec)	N/A	0.454	0.468	0.309	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	356	1360	0	0	0	0
normalized size	1	1.	1.19	4.55	0.	0.	0.	0.
time (sec)	N/A	0.4	0.415	0.298	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	360	1376	0	0	0	0
normalized size	1	1.	1.16	4.44	0.	0.	0.	0.
time (sec)	N/A	0.411	0.447	0.292	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	436	1518	0	0	0	0
normalized size	1	1.	1.11	3.85	0.	0.	0.	0.
time (sec)	N/A	1.182	1.	0.26	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	488	1656	0	0	0	0
normalized size	1	1.	1.07	3.65	0.	0.	0.	0.
time (sec)	N/A	1.193	1.342	0.25	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	761	761	552	14815	0	0	0	0
normalized size	1	1.	0.73	19.47	0.	0.	0.	0.
time (sec)	N/A	3.135	2.394	0.312	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	549	496	10138	0	0	0	0
normalized size	1	1.	0.9	18.47	0.	0.	0.	0.
time (sec)	N/A	7.028	1.942	0.321	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	417	6019	0	0	0	0
normalized size	1	1.	0.97	13.97	0.	0.	0.	0.
time (sec)	N/A	0.65	0.764	0.32	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	521	454	6460	0	0	0	0
normalized size	1	1.	0.87	12.35	0.	0.	0.	0.
time (sec)	N/A	3.701	1.389	0.303	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	736	736	520	6765	0	0	0	0
normalized size	1	1.	0.71	9.19	0.	0.	0.	0.
time (sec)	N/A	3.476	1.839	0.313	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	550	3131	0	0	0	0
normalized size	1	1.	1.01	5.74	0.	0.	0.	0.
time (sec)	N/A	3.722	2.473	0.321	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	468	2321	0	0	0	0
normalized size	1	1.	1.01	5.01	0.	0.	0.	0.
time (sec)	N/A	3.436	1.211	0.342	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	407	1516	0	23019	0	0
normalized size	1	1.	1.01	3.77	0.	57.26	0.	0.
time (sec)	N/A	0.963	1.014	0.326	0.	48.067	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	376	761	0	22873	0	0
normalized size	1	1.	1.01	2.03	0.	61.16	0.	0.
time (sec)	N/A	0.314	0.805	0.319	0.	51.604	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	450	859	0	0	0	0
normalized size	1	1.	1.	1.9	0.	0.	0.	0.
time (sec)	N/A	2.632	2.61	0.374	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	533	983	0	0	0	0
normalized size	1	1.	0.98	1.81	0.	0.	0.	0.
time (sec)	N/A	4.592	1.598	0.344	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	679	669	1296	0	0	0	0
normalized size	1	1.	0.99	1.91	0.	0.	0.	0.
time (sec)	N/A	11.226	2.534	0.342	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	779	779	1066	14651	0	0	0	0
normalized size	1	1.	1.37	18.81	0.	0.	0.	0.
time (sec)	N/A	14.17	2.914	0.359	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	609	1097	11341	0	0	0	0
normalized size	1	1.	1.8	18.62	0.	0.	0.	0.
time (sec)	N/A	5.842	6.611	0.348	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	609	983	7163	0	0	0	0
normalized size	1	1.	1.61	11.76	0.	0.	0.	0.
time (sec)	N/A	5.643	6.333	0.352	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	666	700	4099	0	0	0	0
normalized size	1	1.	1.05	6.15	0.	0.	0.	0.
time (sec)	N/A	1.746	6.573	0.333	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	816	814	1121	4594	0	0	0	0
normalized size	1	1.	1.37	5.63	0.	0.	0.	0.
time (sec)	N/A	15.916	6.642	0.325	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	210	159	0	494	0	254
normalized size	1	1.	1.5	1.14	0.	3.53	0.	1.81
time (sec)	N/A	0.501	0.549	0.1	0.	2.197	0.	1.273

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	192	144	0	474	0	250
normalized size	1	1.	1.67	1.25	0.	4.12	0.	2.17
time (sec)	N/A	0.42	0.437	0.135	0.	2.2	0.	1.293

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	159	130	0	441	0	231
normalized size	1	1.	1.62	1.33	0.	4.5	0.	2.36
time (sec)	N/A	0.198	0.199	0.1	0.	1.857	0.	1.247

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	69	174	92	0	138	0	92
normalized size	1	1.01	2.56	1.35	0.	2.03	0.	1.35
time (sec)	N/A	0.062	0.18	0.098	0.	1.553	0.	1.216

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	150	121	0	365	0	223
normalized size	1	1.	1.58	1.27	0.	3.84	0.	2.35
time (sec)	N/A	0.111	0.109	0.1	0.	1.608	0.	1.256

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	200	152	0	473	0	269
normalized size	1	1.	1.54	1.17	0.	3.64	0.	2.07
time (sec)	N/A	0.417	0.453	0.102	0.	1.633	0.	1.307

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	225	169	0	518	0	363
normalized size	1	1.	1.49	1.12	0.	3.43	0.	2.4
time (sec)	N/A	0.454	0.456	0.111	0.	1.632	0.	1.279

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	87	147	209	398	0	115
normalized size	1	1.	0.58	0.99	1.4	2.67	0.	0.77
time (sec)	N/A	0.093	0.225	0.057	1.534	1.576	0.	1.161

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	72	96	140	263	0	95
normalized size	1	1.	0.7	0.93	1.36	2.55	0.	0.92
time (sec)	N/A	0.042	0.035	0.05	1.471	1.56	0.	1.156

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	37	163	0	153	0	85
normalized size	1	1.	1.32	5.82	0.	5.46	0.	3.04
time (sec)	N/A	0.048	0.11	0.058	0.	1.544	0.	1.173

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	114	245	0	370	0	215
normalized size	1	1.	1.36	2.92	0.	4.4	0.	2.56
time (sec)	N/A	0.078	0.244	0.064	0.	1.586	0.	1.227

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	131	306	0	674	0	313
normalized size	1	1.	0.94	2.2	0.	4.85	0.	2.25
time (sec)	N/A	0.117	0.38	0.071	0.	1.69	0.	1.216

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	16	15	31	31	15
normalized size	1	1.	0.87	1.07	1.	2.07	2.07	1.
time (sec)	N/A	0.004	0.007	0.046	0.988	1.494	0.449	1.155

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	13	14	12	31	10	15
normalized size	1	1.	0.81	0.88	0.75	1.94	0.62	0.94
time (sec)	N/A	0.006	0.003	0.045	1.015	1.661	10.071	1.183

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	20	0	31	0	0
normalized size	1	1.	0.87	1.33	0.	2.07	0.	0.
time (sec)	N/A	0.028	0.006	0.046	0.	1.478	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	18	0	31	0	0
normalized size	1	1.	0.87	1.2	0.	2.07	0.	0.
time (sec)	N/A	0.058	0.004	0.046	0.	1.458	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	268	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.582	0.803	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	488	488	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.36	0.549	3.165	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [63] had the largest ratio of [0.5556]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.	25	0.16
2	A	5	4	1.	27	0.148
3	A	5	4	1.	27	0.148
4	A	8	8	1.	27	0.296
5	A	9	9	1.	27	0.333
6	A	9	6	1.	30	0.2
7	A	5	3	1.	30	0.1
8	A	6	4	1.	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
9	A	7	5	1.	30	0.167
10	A	5	4	1.	23	0.174
11	A	5	4	1.	23	0.174
12	A	5	4	1.	30	0.133
13	A	6	5	0.99	28	0.179
14	A	6	5	1.	30	0.167
15	A	9	5	0.98	30	0.167
16	A	10	6	0.99	30	0.2
17	A	5	5	1.	34	0.147
18	A	5	5	1.	34	0.147
19	A	9	6	1.	32	0.188
20	A	10	7	1.	32	0.219
21	A	5	3	1.	32	0.094
22	A	5	3	1.	29	0.103
23	A	5	3	1.	29	0.103
24	A	6	6	1.	26	0.231
25	A	5	4	1.	30	0.133
26	A	7	6	1.	30	0.2
27	A	7	6	1.	30	0.2
28	A	5	3	1.	30	0.1
29	A	6	4	1.	30	0.133
30	A	7	5	1.	30	0.167
31	A	2	2	1.	26	0.077
32	A	5	5	1.	26	0.192
33	A	2	2	1.	24	0.083
34	A	5	5	1.	20	0.25
35	A	6	5	1.	36	0.139
36	A	2	2	1.	36	0.056
37	A	2	2	1.	32	0.062
38	A	13	9	1.	32	0.281
39	A	5	5	1.	38	0.132
40	A	6	6	1.	35	0.171

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	5	5	1.	33	0.152
42	A	5	5	1.	32	0.156
43	A	8	8	1.	35	0.229
44	A	8	8	1.	35	0.229
45	A	8	8	1.	35	0.229
46	A	6	6	1.	38	0.158
47	A	5	5	1.	36	0.139
48	A	5	5	1.	35	0.143
49	A	7	6	1.	38	0.158
50	A	7	6	1.	38	0.158
51	A	7	6	1.	38	0.158
52	A	9	6	1.	27	0.222
53	A	9	6	1.	25	0.24
54	A	8	5	1.	24	0.208
55	A	12	9	1.	27	0.333
56	A	18	12	1.	27	0.444
57	A	22	13	1.	27	0.482
58	A	10	7	1.	27	0.259
59	A	10	7	1.	25	0.28
60	A	9	6	1.	24	0.25
61	A	17	11	1.	27	0.407
62	A	21	14	1.	27	0.518
63	A	26	15	1.	27	0.556
64	A	10	6	1.	27	0.222
65	A	8	5	1.	27	0.185
66	A	5	3	1.	25	0.12
67	A	5	3	1.	24	0.125
68	A	10	7	1.	27	0.259
69	A	11	8	1.	27	0.296
70	A	15	9	1.	27	0.333
71	A	10	7	1.	27	0.259
72	A	6	4	1.	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
73	A	6	4	1.	25	0.16
74	A	6	4	1.	24	0.167
75	A	12	9	1.	27	0.333
76	A	14	11	1.	27	0.407
77	A	15	9	1.	28	0.321
78	A	9	6	1.	28	0.214
79	A	9	6	1.	26	0.231
80	A	8	5	1.	25	0.2
81	A	17	9	1.	28	0.321
82	A	16	8	1.	28	0.286
83	A	20	10	1.	28	0.357
84	A	17	10	1.	28	0.357
85	A	10	7	1.	28	0.25
86	A	10	7	1.	26	0.269
87	A	9	6	1.	25	0.24
88	A	19	11	1.	28	0.393
89	A	18	10	1.	28	0.357
90	A	26	13	1.	28	0.464
91	A	9	6	1.	24	0.25
92	A	8	6	1.	22	0.273
93	A	10	9	1.	17	0.529
94	A	13	7	1.	28	0.25
95	A	10	6	1.	28	0.214
96	A	8	5	1.	28	0.179
97	A	5	3	1.	26	0.115
98	A	5	3	1.	25	0.12
99	A	9	4	1.	28	0.143
100	A	10	5	1.	28	0.179
101	A	13	6	1.	28	0.214
102	A	13	9	1.	28	0.321
103	A	9	6	1.	28	0.214
104	A	6	4	1.	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
105	A	6	4	1.	26	0.154
106	A	6	4	1.	25	0.16
107	A	12	7	1.	28	0.25
108	A	12	7	1.	28	0.25
109	A	9	6	1.	30	0.2
110	A	9	6	1.	28	0.214
111	A	8	5	1.	27	0.185
112	A	17	9	1.	30	0.3
113	A	23	10	1.	30	0.333
114	A	12	6	1.	30	0.2
115	A	8	5	1.	30	0.167
116	A	5	3	1.	28	0.107
117	A	5	3	1.	27	0.111
118	A	9	4	1.	30	0.133
119	A	12	5	1.	30	0.167
120	A	16	7	1.	30	0.233
121	A	10	7	1.	30	0.233
122	A	6	4	1.	30	0.133
123	A	6	4	1.	28	0.143
124	A	6	4	1.	27	0.148
125	A	12	7	1.	30	0.233
126	A	24	14	1.	30	0.467
127	A	20	13	1.	30	0.433
128	A	16	12	1.	30	0.4
129	A	6	4	1.01	28	0.143
130	A	10	8	1.	27	0.296
131	A	17	11	1.	30	0.367
132	A	20	12	1.	30	0.4
133	A	8	6	1.	34	0.176
134	A	6	5	1.	30	0.167
135	A	3	3	1.	34	0.088
136	A	5	5	1.	34	0.147

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
137	A	7	7	1.	34	0.206
138	A	1	1	1.	17	0.059
139	A	1	1	1.	15	0.067
140	A	2	2	1.	23	0.087
141	A	3	3	1.	21	0.143
142	A	2	2	1.	40	0.05
143	A	2	2	1.	104	0.019

Chapter 3

Listing of integrals

3.1 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx$

Optimal. Leaf size=94

$$-\frac{\log\left(d+fx^2\right)(-aBf-Abf+Bcd)}{2f^2}-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}f^{3/2}}+\frac{x(Ac+bB)}{f}+\frac{Bcx^2}{2f}$$

```
[Out] ((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*ArcTan[(Sqrt
[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f - a*B*f)*Log[d + f*x^2
])/(2*f^2)
```

Rubi [A] time = 0.112756, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1629, 635, 205, 260}

$$-\frac{\log\left(d+fx^2\right)(-aBf-Abf+Bcd)}{2f^2}-\frac{\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}f^{3/2}}+\frac{x(Ac+bB)}{f}+\frac{Bcx^2}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] ((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*ArcTan[(Sqrt
[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f - a*B*f)*Log[d + f*x^2
```

```
])/(2*f^2)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx &= \int \left(\frac{bB+Ac}{f}+\frac{Bcx}{f}-\frac{bBd+Acd-aAf+(Bcd-Abf-aBf)x}{f\left(d+fx^2\right)}\right)dx \\
&= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{\int \frac{bBd+Acd-aAf+(Bcd-Abf-aBf)x}{d+fx^2}\,dx}{f} \\
&= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{(bBd+Acd-aAf)\int \frac{1}{d+fx^2}\,dx}{f}-\frac{(Bcd-Abf-aBf)\int \frac{x}{d+fx^2}\,dx}{f} \\
&= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{(bBd+Acd-aAf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{3/2}}-\frac{(Bcd-Abf-aBf)\log\left(d+f\right.}{2f^2}
\end{aligned}$$

Mathematica [A] time = 0.077951, size = 86, normalized size = 0.91

$$\frac{\log\left(d+fx^2\right)(aBf+Abf-Bcd)-\frac{2\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}}+fx(2Ac+2bB+Bcx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] (f*x*(2*b*B + 2*A*c + B*c*x) - (2*Sqrt[f]*(b*B*d + A*c*d - a*A*f)*ArcTan[(S
qrt[f]*x)/Sqrt[d]])/Sqrt[d] + (-(B*c*d) + A*b*f + a*B*f)*Log[d + f*x^2])/(2
*f^2)
```

Maple [A] time = 0.053, size = 133, normalized size = 1.4

$$\frac{Bcx^2}{2f}+\frac{Acx}{f}+\frac{Bbx}{f}+\frac{\ln\left(fx^2+d\right)Ab}{2f}+\frac{\ln\left(fx^2+d\right)aB}{2f}-\frac{\ln\left(fx^2+d\right)Bcd}{2f^2}+Aa\arctan\left(fx\frac{1}{\sqrt{df}}\right)\frac{1}{\sqrt{df}}-\frac{Acd}{f}\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x)
```

```
[Out] 1/2*B*c*x^2/f+1/f*A*c*x+1/f*B*b*x+1/2/f*ln(f*x^2+d)*A*b+1/2/f*ln(f*x^2+d)*a
*B-1/2/f^2*ln(f*x^2+d)*B*c*d+1/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*a*A-1/f/
(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*c*d-1/f/(d*f)^(1/2)*arctan(x*f/(d*f)^
(1/2))*B*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88013, size = 452, normalized size = 4.81

$$\left[\frac{Bcdfx^2 + 2\,(Bb + Ac)dfx - \left(Aaf - (Bb + Ac)d\right)\sqrt{-df}\log\left(\frac{fx^2 - 2\sqrt{-df}x - d}{fx^2 + d}\right) - \left(Bcd^2 - (Ba + Ab)df\right)\log\left(fx^2 + d\right)}{2\,df^2}, \frac{Bcdf}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x - (A*a*f - (B*b + A*c)*d)*sqrt(-d*f
)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) - (B*c*d^2 - (B*a + A*b)*d*
f)*log(f*x^2 + d))/(d*f^2), 1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x + 2*(A*a
*f - (B*b + A*c)*d)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - (B*c*d^2 - (B*a + A*b
)*d*f)*log(f*x^2 + d))/(d*f^2)]
```

Sympy [B] time = 2.53689, size = 332, normalized size = 3.53

$$\frac{Bcx^2}{2f} + \left(\frac{Abf + Baf - Bcd}{2f^2} - \frac{\sqrt{-df^5}\left(Aaf - Acd - Bbd\right)}{2df^4}\right)\log\left(x + \frac{-Abdf - Badf + Bcd^2 + 2df^2\left(\frac{Abf + Baf - Bcd}{2f^2} - \frac{\sqrt{-df^5}(}{}\right.}{Aaf^2 - Acdf - Bbdf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] B*c*x**2/(2*f) + ((A*b*f + B*a*f - B*c*d)/(2*f**2) - sqrt(-d*f**5)*(A*a*f -
 A*c*d - B*b*d)/(2*d*f**4))*log(x + (-A*b*d*f - B*a*d*f + B*c*d**2 + 2*d*f*
*2*((A*b*f + B*a*f - B*c*d)/(2*f**2) - sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d
)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f - B*b*d*f)) + ((A*b*f + B*a*f - B*c*d)/(
2*f**2) + sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4))*log(x + (-A*b*d
*f - B*a*d*f + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*c*d)/(2*f**2) + sqrt
(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f - B*b*d*
f)) + x*(A*c + B*b)/f
```

Giac [A] time = 1.21227, size = 117, normalized size = 1.24

$$-\frac{\left(Bbd + Acd - Aaf\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}f} - \frac{\left(Bcd - Baf - Abf\right)\log\left(fx^2 + d\right)}{2\,f^2} + \frac{Bcfx^2 + 2\,Bbfx + 2\,Acfx}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -(B*b*d + A*c*d - A*a*f)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) - 1/2*(B*c*d -
 B*a*f - A*b*f)*log(f*x^2 + d)/f^2 + 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x
)/f^2
```

3.2 $\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+fx^2}\,dx$

Optimal. Leaf size=228

$$-\frac{\log\left(d+fx^2\right)\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^3}+\frac{x^2\left(2Abcf-B\left(-2acf+b^2(-f)+c^2d\right)\right)}{2f^2}+x(-$$

```
[Out] ((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2*A*b*c*f -
 B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B + A*c)*x^3)/(3*f) +
(B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*Ar
cTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) - ((2*A*b*f*(c*d - a*f) - B*(c
^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)
```

Rubi [A] time = 0.329642, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1012, 635, 205, 260}

$$-\frac{\log\left(d+fx^2\right)\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^3}+\frac{x^2\left(2Abcf-B\left(-2acf+b^2(-f)+c^2d\right)\right)}{2f^2}+x(-$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] ((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2*A*b*c*f -
 B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B + A*c)*x^3)/(3*f) +
(B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*Ar
cTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) - ((2*A*b*f*(c*d - a*f) - B*(c
^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)
```

Rule 1012

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
 f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx = \int\left(\frac{Ab^2f-Ac(cd-2af)-bB(2cd-2af)}{f^2}+\frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)x}{f^2}+\frac{c(\cdots}{\cdots}\right.$$

$$=\frac{\left(Ab^2f-Ac(cd-2af)-bB(2cd-2af)\right)x}{f^2}+\frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)x^2}{2f^2}+\frac{c\cdots}{\cdots}$$

$$=\frac{\left(Ab^2f-Ac(cd-2af)-bB(2cd-2af)\right)x}{f^2}+\frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)x^2}{2f^2}+\frac{c\cdots}{\cdots}$$

$$=\frac{\left(Ab^2f-Ac(cd-2af)-bB(2cd-2af)\right)x}{f^2}+\frac{\left(2Abcf-B\left(c^2d-b^2f-2acf\right)\right)x^2}{2f^2}+\frac{c\cdots}{\cdots}$$

Mathematica [A] time = 0.222411, size = 204, normalized size = 0.89

$$\frac{6\log\left(d+fx^2\right)\left(B\left(a^2f^2-2acdf+b^2(-d)f+c^2d^2\right)+2Abf(af-cd)\right)+fx\left(4Ac\left(6af-3cd+cfx^2\right)+4bB\left(6af-6cd\cdots\right.\right.}{12f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] ((-(A*b^2*d*f) + 2*b*B*d*(c*d - a*f) + A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/
Sqrt[d]])/(Sqrt[d]*f^(5/2)) + (f*x*(12*A*b*c*f*x + 6*b^2*f*(2*A + B*x) + 3*
B*c*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*A*c*(-3*c*d + 6*a*f + c*f*x^2) + 4*b*B
```

```
*(-6*c*d + 6*a*f + 2*c*f*x^2)) + 6*(2*A*b*f*(-(c*d) + a*f) + B*(c^2*d^2 - b
^2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(12*f^3)
```

Maple [A] time = 0.052, size = 373, normalized size = 1.6

$$\frac{Bc^2x^4}{4f}+\frac{Ax^3c^2}{3f}+\frac{2Bx^3bc}{3f}+\frac{Abcx^2}{f}+\frac{Bx^2ac}{f}+\frac{Bx^2b^2}{2f}-\frac{Bc^2x^2d}{2f^2}+2\frac{aAcx}{f}+\frac{Ab^2x}{f}-\frac{Ac^2dx}{f^2}+2\frac{abBx}{f}-2\frac{Bbcdx}{f^2}+\frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x)
```

```
[Out] 1/4*B*c^2*x^4/f+1/3/f*A*x^3*c^2+2/3/f*B*x^3*b*c+1/f*A*x^2*b*c+1/f*B*x^2*a*c
+1/2/f*B*x^2*b^2-1/2/f^2*B*x^2*c^2*d+2/f*a*c*A*x+1/f*A*b^2*x-1/f^2*A*c^2*d*
x+2/f*a*b*B*x-2/f^2*B*b*c*d*x+1/f*ln(f*x^2+d)*A*a*b-1/f^2*ln(f*x^2+d)*A*b*c
*d+1/2/f*ln(f*x^2+d)*B*a^2-1/f^2*ln(f*x^2+d)*B*a*c*d-1/2/f^2*ln(f*x^2+d)*B*
b^2*d+1/2/f^3*ln(f*x^2+d)*B*c^2*d^2+1/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A
*a^2-2/f/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*a*c*d-1/f/(d*f)^(1/2)*arctan
(x*f/(d*f)^(1/2))*A*b^2*d+1/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*c^2*d
^2-2/f/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*a*b*d+2/f^2/(d*f)^(1/2)*arctan
(x*f/(d*f)^(1/2))*B*b*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9131, size = 1088, normalized size = 4.77

$$\left[\frac{3Bc^2df^2x^4+4\left(2Bbc+Ac^2\right)df^2x^3-6\left(Bc^2d^2f-\left(Bb^2+2\left(Ba+Ab\right)c\right)df^2\right)x^2-6\left(Aa^2f^2+\left(2Bbc+Ac^2\right)d^2-\left(2Bab\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f -
 (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2 - 6*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^
2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x
 - d)/(f*x^2 + d)) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2 + 2*A*a
*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f + (B*a^2 + 2*
A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3), 1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c
 + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f - (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2
 + 12*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)
*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b +
 A*b^2 + 2*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f
 + (B*a^2 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3)]
```

Sympy [B] time = 8.99137, size = 928, normalized size = 4.07

$$\frac{Bc^2x^4}{4f} + \left(\frac{2Aabf^2 - 2Abcdf + Ba^2f^2 - 2Bacdf - Bb^2df + Bc^2d^2}{2f^3} - \frac{\sqrt{-df^7}\left(Aa^2f^2 - 2Aacdf - Ab^2df + Ac^2d^2 - 2Ba\right.}{2df^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] B*c**2*x**4/(4*f) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*
f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a
*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))
*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f
 + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*A*a*b*f**2 - 2*A*b*c*d*f + B*
a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*f**7
)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B
*b*c*d**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c
**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f)) + ((2*A*a*b*f**2 - 2*A*b*c*d
*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) + sqrt(
-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*
f + 2*B*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B
*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*
A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*
d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*
```

```
c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*a*c
*d*f**2 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f))
 + x**3*(A*c**2 + 2*B*b*c)/(3*f) + x**2*(2*A*b*c*f + 2*B*a*c*f + B*b**2*f -
 B*c**2*d)/(2*f**2) + x*(2*A*a*c*f + A*b**2*f - A*c**2*d + 2*B*a*b*f - 2*B*
b*c*d)/f**2
```

Giac [A] time = 1.4019, size = 355, normalized size = 1.56

$$\frac{\left(2\,Bbcd^2 + Ac^2d^2 - 2\,Babdf - Ab^2df - 2\,Aacdf + Aa^2f^2\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}f^2} + \frac{\left(Bc^2d^2 - Bb^2df - 2\,Bacdf - 2\,Abcdf + Ba^2\right.}{2\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

```
[Out] (2*B*b*c*d^2 + A*c^2*d^2 - 2*B*a*b*d*f - A*b^2*d*f - 2*A*a*c*d*f + A*a^2*f^
2)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/2*(B*c^2*d^2 - B*b^2*d*f - 2*B
*a*c*d*f - 2*A*b*c*d*f + B*a^2*f^2 + 2*A*a*b*f^2)*log(f*x^2 + d)/f^3 + 1/12
*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 6*B*c^2*d*f^2*x^2 +
 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 24*B*b*c*d*f^2*x -
 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x)/f^4
```

3.3 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$

Optimal. Leaf size=441

$$-\frac{x^2\left(Abf\left(-6acf+b^2(-f)+3c^2d\right)-B\left(-3cf\left(b^2d-a^2f\right)+3ab^2f^2-3ac^2df+c^3d^2\right)\right)}{2f^3}+\frac{\log\left(d+fx^2\right)\left(Abf\left(-f\left(b^2d\right.\right.\right.}{}$$

```
[Out] -(((b^3*B*d*f + 3*A*b^2*f*(c*d - a*f) - 3*b*B*(c*d - a*f)^2 - A*c*(c^2*d^2
- 3*a*c*d*f + 3*a^2*f^2))*x)/f^3) - ((A*b*f*(3*c^2*d - b^2*f - 6*a*c*f) - B
*(c^3*d^2 - 3*a*c^2*d*f + 3*a*b^2*f^2 - 3*c*f*(b^2*d - a^2*f)))*x^2)/(2*f^3
) + ((b^3*B*f + 3*A*b^2*c*f - A*c^2*(c*d - 3*a*f) - 3*b*B*c*(c*d - 2*a*f))*
x^3)/(3*f^2) + (c*(3*A*b*c*f - B*(c^2*d - 3*b^2*f - 3*a*c*f))*x^4)/(4*f^2)
+ (c^2*(3*b*B + A*c)*x^5)/(5*f) + (B*c^3*x^6)/(6*f) + ((b^3*B*d^2*f + 3*A*b
^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(Sqrt[
f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + ((A*b*f*(3*c^2*d^2 - 6*a*c*d*f - f*(b^2
*d - 3*a^2*f)) - B*(c*d - a*f)*(c^2*d^2 - 2*a*c*d*f - f*(3*b^2*d - a^2*f)))
*Log[d + f*x^2])/(2*f^4)
```

Rubi [A] time = 0.621031, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1012, 635, 205, 260}

$$-\frac{x^2\left(Abf\left(-6acf+b^2(-f)+3c^2d\right)-B\left(-3cf\left(b^2d-a^2f\right)+3ab^2f^2-3ac^2df+c^3d^2\right)\right)}{2f^3}+\frac{\log\left(d+fx^2\right)\left(Abf\left(-f\left(b^2d\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] -(((b^3*B*d*f + 3*A*b^2*f*(c*d - a*f) - 3*b*B*(c*d - a*f)^2 - A*c*(c^2*d^2
- 3*a*c*d*f + 3*a^2*f^2))*x)/f^3) - ((A*b*f*(3*c^2*d - b^2*f - 6*a*c*f) - B
*(c^3*d^2 - 3*a*c^2*d*f + 3*a*b^2*f^2 - 3*c*f*(b^2*d - a^2*f)))*x^2)/(2*f^3
) + ((b^3*B*f + 3*A*b^2*c*f - A*c^2*(c*d - 3*a*f) - 3*b*B*c*(c*d - 2*a*f))*
x^3)/(3*f^2) + (c*(3*A*b*c*f - B*(c^2*d - 3*b^2*f - 3*a*c*f))*x^4)/(4*f^2)
+ (c^2*(3*b*B + A*c)*x^5)/(5*f) + (B*c^3*x^6)/(6*f) + ((b^3*B*d^2*f + 3*A*b
^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(Sqrt[
f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + ((A*b*f*(3*c^2*d^2 - 6*a*c*d*f - f*(b^2
*d - 3*a^2*f)) - B*(c*d - a*f)*(c^2*d^2 - 2*a*c*d*f - f*(3*b^2*d - a^2*f)))
```

```
*Log[d + f*x^2])/(2*f^4)
```

Rule 1012

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
 f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx = \int \left(-\frac{b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)}{f^3} - \frac{\left(Abf\right.}{}\right.$$

$$= -\frac{\left(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)\right)x}{f^3} - \frac{\left(Abf\right.}{}$$

$$= -\frac{\left(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)\right)x}{f^3} - \frac{\left(Abf\right.}{}$$

$$= -\frac{\left(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac\left(c^2d^2-3acdf+3a^2f^2\right)\right)x}{f^3} - \frac{\left(Abf\right.}{}$$

Mathematica [A] time = 0.492801, size = 422, normalized size = 0.96

$$\frac{fx\left(3b\left(4B\left(15a^2f^2+10acf\left(fx^2-3d\right)+c^2\left(15d^2-5dfx^2+3f^2x^4\right)\right)+15Acfx\left(4af-2cd+cfx^2\right)\right)+c\left(4A\left(45a^2f^2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] ((b^3*B*d^2*f + 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d -
a*f)^3)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + (f*x*(10*b^3*f*(-6
*B*d + 3*A*f*x + 2*B*f*x^2) + 15*b^2*f*(3*B*x*(-2*c*d + 2*a*f + c*f*x^2) +
4*A*(-3*c*d + 3*a*f + c*f*x^2)) + 3*b*(15*A*c*f*x*(-2*c*d + 4*a*f + c*f*x^2
) + 4*B*(15*a^2*f^2 + 10*a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^2 + 3
*f^2*x^4))) + c*(5*B*x*(18*a^2*f^2 + 9*a*c*f*(-2*d + f*x^2) + c^2*(6*d^2 -
3*d*f*x^2 + 2*f^2*x^4)) + 4*A*(45*a^2*f^2 + 15*a*c*f*(-3*d + f*x^2) + c^2*(
15*d^2 - 5*d*f*x^2 + 3*f^2*x^4)))) - 30*(A*b*f*(-3*c^2*d^2 + b^2*d*f + 6*a*
c*d*f - 3*a^2*f^2) + B*(c*d - a*f)*(c^2*d^2 - 3*b^2*d*f - 2*a*c*d*f + a^2*f
^2))*Log[d + f*x^2])/(60*f^4)
```

Maple [A] time = 0.053, size = 822, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x)
```

```
[Out] -3/2/f^2*ln(f*x^2+d)*B*a^2*c*d+6/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*
a*b*c*d^2+3/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*a*c^2*d^2-3/f/(d*f)^(
1/2)*arctan(x*f/(d*f)^(1/2))*B*a^2*b*d-3/f^3/(d*f)^(1/2)*arctan(x*f/(d*f)^(
1/2))*B*b*c^2*d^3-6/f^2*B*a*b*c*d*x+3/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2
))*A*b^2*c*d^2-3/f/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*a^2*c*d-3/f/(d*f)^
(1/2)*arctan(x*f/(d*f)^(1/2))*A*a*b^2*d+1/3/f*B*x^3*b^3+1/5/f*A*x^5*c^3+3/f
*a*b^2*A*x+3/2/f*B*x^2*a^2*c+3/f*b*a^2*B*x-1/f^2*b^3*B*d*x+3/2/f*B*x^2*a*b^
2+1/f^3*A*c^3*d^2*x+1/2/f^3*B*x^2*c^3*d^2+3/2/f*ln(f*x^2+d)*A*a^2*b+3/4/f*B
*x^4*b^2*c-1/4/f^2*B*x^4*c^3*d-1/3/f^2*A*x^3*c^3*d+3/4/f*A*x^4*b*c^2+3/4/f*
B*x^4*a*c^2+3/5/f*B*x^5*b*c^2+3/f*a^2*c*A*x+1/(d*f)^(1/2)*arctan(x*f/(d*f)^
(1/2))*A*a^3+1/2/f*ln(f*x^2+d)*B*a^3+1/2/f*A*x^2*b^3+1/f*A*x^3*b^2*c+1/f*A*
x^3*a*c^2+1/f^2/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*b^3*B*d^2-3/2/f^2*ln(f*
x^2+d)*B*a*b^2*d+3/2/f^3*ln(f*x^2+d)*B*a*c^2*d^2+3/2/f^3*ln(f*x^2+d)*B*b^2*
```

```
c*d^2-1/2/f^2*ln(f*x^2+d)*A*b^3*d-1/2/f^4*ln(f*x^2+d)*B*c^3*d^3-3/2/f^2*B*x
^2*b^2*c*d-3/f^2*ln(f*x^2+d)*A*a*b*c*d-1/f^3/(d*f)^(1/2)*arctan(x*f/(d*f)^(
1/2))*A*c^3*d^3-3/f^2*A*a*c^2*d*x-3/f^2*A*b^2*c*d*x+3/2/f^3*ln(f*x^2+d)*A*b
*c^2*d^2+1/6*B*c^3*x^6/f+3/f*A*x^2*a*b*c-3/2/f^2*A*x^2*b*c^2*d-3/2/f^2*B*x^
2*a*c^2*d-1/f^2*B*x^3*b*c^2*d+3/f^3*B*b*c^2*d^2*x+2/f*B*x^3*a*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.21747, size = 2164, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^
2*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*
d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^
3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B
*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 - 30*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (
B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A
*a^2*c)*d*f^2)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) + 6
0*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*
d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b
^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)
*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*log(f*x^2 + d))/(d*f^4), 1/60*(10*B*c
^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^2*f^2 - 3*(B*
b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*d^2*f^2 - (B*
b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^3*d^3*f - 3*(
B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*
b)*c)*d*f^3)*x^2 + 60*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a
```

```
*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2
)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) + 60*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3
+ 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2
*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a
*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*
log(f*x^2 + d))/(d*f^4)]
```

Sympy [B] time = 25.993, size = 1940, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d),x)
```

```
[Out] B*c**3*x**6/(6*f) + ((3*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 - A*b**3*d*f**2 +
3*A*b*c**2*d**2*f + B*a**3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3
*B*a*c**2*d**2*f + 3*B*b**2*c*d**2*f - B*c**3*d**3)/(2*f**4) - sqrt(-d*f**9
)*(A*a**3*f**3 - 3*A*a**2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*A*a*c**2*d**2*f
+ 3*A*b**2*c*d**2*f - A*c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B*a*b*c*d**2*f +
B*b**3*d**2*f - 3*B*b*c**2*d**3)/(2*d*f**8))*log(x + (-3*A*a**2*b*d*f**3 +
6*A*a*b*c*d**2*f**2 + A*b**3*d**2*f**2 - 3*A*b*c**2*d**3*f - B*a**3*d*f**3
+ 3*B*a**2*c*d**2*f**2 + 3*B*a*b**2*d**2*f**2 - 3*B*a*c**2*d**3*f - 3*B*b**
2*c*d**3*f + B*c**3*d**4 + 2*d*f**4*((3*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 -
A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a**3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*a
*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b**2*c*d**2*f - B*c**3*d**3)/(2*f**4
) - sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3*
A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*B
*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c**2*d**3)/(2*d*f**8)))/(A*a**3*f**4
- 3*A*a**2*c*d*f**3 - 3*A*a*b**2*d*f**3 + 3*A*a*c**2*d**2*f**2 + 3*A*b**2*c
*d**2*f**2 - A*c**3*d**3*f - 3*B*a**2*b*d*f**3 + 6*B*a*b*c*d**2*f**2 + B*b*
*3*d**2*f**2 - 3*B*b*c**2*d**3*f)) + ((3*A*a**2*b*f**3 - 6*A*a*b*c*d*f**2 -
 A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a**3*f**3 - 3*B*a**2*c*d*f**2 - 3*B*
a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b**2*c*d**2*f - B*c**3*d**3)/(2*f**
4) + sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**2*c*d*f**2 - 3*A*a*b**2*d*f**2 + 3
*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*c**3*d**3 - 3*B*a**2*b*d*f**2 + 6*
B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c**2*d**3)/(2*d*f**8))*log(x + (-3*A
*a**2*b*d*f**3 + 6*A*a*b*c*d**2*f**2 + A*b**3*d**2*f**2 - 3*A*b*c**2*d**3*f
 - B*a**3*d*f**3 + 3*B*a**2*c*d**2*f**2 + 3*B*a*b**2*d**2*f**2 - 3*B*a*c**2
*d**3*f - 3*B*b**2*c*d**3*f + B*c**3*d**4 + 2*d*f**4*((3*A*a**2*b*f**3 - 6*
A*a*b*c*d*f**2 - A*b**3*d*f**2 + 3*A*b*c**2*d**2*f + B*a**3*f**3 - 3*B*a**2
*c*d*f**2 - 3*B*a*b**2*d*f**2 + 3*B*a*c**2*d**2*f + 3*B*b**2*c*d**2*f - B*c
```

```
**3*d**3)/(2*f**4) + sqrt(-d*f**9)*(A*a**3*f**3 - 3*A*a**2*c*d*f**2 - 3*A*a
*b**2*d*f**2 + 3*A*a*c**2*d**2*f + 3*A*b**2*c*d**2*f - A*c**3*d**3 - 3*B*a*
*2*b*d*f**2 + 6*B*a*b*c*d**2*f + B*b**3*d**2*f - 3*B*b*c**2*d**3)/(2*d*f**8
)))/(A*a**3*f**4 - 3*A*a**2*c*d*f**3 - 3*A*a*b**2*d*f**3 + 3*A*a*c**2*d**2*
f**2 + 3*A*b**2*c*d**2*f**2 - A*c**3*d**3*f - 3*B*a**2*b*d*f**3 + 6*B*a*b*c
*d**2*f**2 + B*b**3*d**2*f**2 - 3*B*b*c**2*d**3*f)) + x**5*(A*c**3 + 3*B*b*
c**2)/(5*f) + x**4*(3*A*b*c**2*f + 3*B*a*c**2*f + 3*B*b**2*c*f - B*c**3*d)/
(4*f**2) + x**3*(3*A*a*c**2*f + 3*A*b**2*c*f - A*c**3*d + 6*B*a*b*c*f + B*b
**3*f - 3*B*b*c**2*d)/(3*f**2) + x**2*(6*A*a*b*c*f**2 + A*b**3*f**2 - 3*A*b
*c**2*d*f + 3*B*a**2*c*f**2 + 3*B*a*b**2*f**2 - 3*B*a*c**2*d*f - 3*B*b**2*c
*d*f + B*c**3*d**2)/(2*f**3) + x*(3*A*a**2*c*f**2 + 3*A*a*b**2*f**2 - 3*A*a
*c**2*d*f - 3*A*b**2*c*d*f + A*c**3*d**2 + 3*B*a**2*b*f**2 - 6*B*a*b*c*d*f
- B*b**3*d*f + 3*B*b*c**2*d**2)/f**3
```

Giac [A] time = 1.33529, size = 841, normalized size = 1.91

$$-\frac{\left(3\,Bbc^2d^3 + Ac^3d^3 - Bb^3d^2f - 6\,Babcd^2f - 3\,Ab^2cd^2f - 3\,Aac^2d^2f + 3\,Ba^2bdf^2 + 3\,Aab^2df^2 + 3\,Aa^2cdf^2 - Aa^3f^3\right)\mathrm{a}}{\sqrt{df}\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -(3*B*b*c^2*d^3 + A*c^3*d^3 - B*b^3*d^2*f - 6*B*a*b*c*d^2*f - 3*A*b^2*c*d^2
*f - 3*A*a*c^2*d^2*f + 3*B*a^2*b*d*f^2 + 3*A*a*b^2*d*f^2 + 3*A*a^2*c*d*f^2
- A*a^3*f^3)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^3) - 1/2*(B*c^3*d^3 - 3*B*b
^2*c*d^2*f - 3*B*a*c^2*d^2*f - 3*A*b*c^2*d^2*f + 3*B*a*b^2*d*f^2 + A*b^3*d*
f^2 + 3*B*a^2*c*d*f^2 + 6*A*a*b*c*d*f^2 - B*a^3*f^3 - 3*A*a^2*b*f^3)*log(f*
x^2 + d)/f^4 + 1/60*(10*B*c^3*f^5*x^6 + 36*B*b*c^2*f^5*x^5 + 12*A*c^3*f^5*x
^5 - 15*B*c^3*d*f^4*x^4 + 45*B*b^2*c*f^5*x^4 + 45*B*a*c^2*f^5*x^4 + 45*A*b*
c^2*f^5*x^4 - 60*B*b*c^2*d*f^4*x^3 - 20*A*c^3*d*f^4*x^3 + 20*B*b^3*f^5*x^3
+ 120*B*a*b*c*f^5*x^3 + 60*A*b^2*c*f^5*x^3 + 60*A*a*c^2*f^5*x^3 + 30*B*c^3*
d^2*f^3*x^2 - 90*B*b^2*c*d*f^4*x^2 - 90*B*a*c^2*d*f^4*x^2 - 90*A*b*c^2*d*f^
4*x^2 + 90*B*a*b^2*f^5*x^2 + 30*A*b^3*f^5*x^2 + 90*B*a^2*c*f^5*x^2 + 180*A*
a*b*c*f^5*x^2 + 180*B*b*c^2*d^2*f^3*x + 60*A*c^3*d^2*f^3*x - 60*B*b^3*d*f^4
*x - 360*B*a*b*c*d*f^4*x - 180*A*b^2*c*d*f^4*x - 180*A*a*c^2*d*f^4*x + 180*
B*a^2*b*f^5*x + 180*A*a*b^2*f^5*x + 180*A*a^2*c*f^5*x)/f^6
```

3.4 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+fx^2\right)}\,dx$

Optimal. Leaf size=274

$$\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\log\left(d+fx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}+\frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-$$

```
[Out] (Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2
*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((A*b^2*f + 2*A*c*(c*d - a*f) - b*
B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) + ((B*c*d + A*b*f - a*B*f)*Log[a
+ b*x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((B*c*d + A
*b*f - a*B*f)*Log[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

Rubi [A] time = 0.281907, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1023, 634, 618, 206, 628, 635, 205, 260}

$$\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-\frac{\log\left(d+fx^2\right)(-aBf+Abf+Bcd)}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}+\frac{\sqrt{f}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```



```
[Out] (Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2
*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((A*b^2*f + 2*A*c*(c*d - a*f) - b*
B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) + ((B*c*d + A*b*f - a*B*f)*Log[a
+ b*x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((B*c*d + A
*b*f - a*B*f)*Log[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```


Rule 1023

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
```

```
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+fx^2\right)}\,dx = \frac{\int \frac{-abBf+A\left(c^2d+b^2f-acf\right)+c(Bcd+Abf-aBf)x}{a+bx+cx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)} + \frac{\int \frac{f(bBd-Acd+aAf)-f(Bcd+Abf-aBf)x}{d+fx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)}$$

$$= \frac{(f(bBd-Acd+aAf))\int \frac{1}{d+fx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)} + \frac{(Bcd+Abf-aBf)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} - \frac{(f(Bcd+A\ldots}{c^2d^2-2ac\ldots}$$

$$= \frac{\sqrt{f}(bBd-Acd+aAf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} + \frac{(Bcd+Abf-aBf)\log\left(a+bx+cx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} - \frac{(Bcd\ldots}{2\left(c^2\ldots\right.}$$

$$= \frac{\sqrt{f}(bBd-Acd+aAf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)} - \frac{\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)\tanh^{-1}\ldots}{\sqrt{b^2-4ac}\left(c^2d^2-2acdf+f\left(b^2d+a^2f\ldots\right.\right.}$$

Mathematica [A] time = 0.404446, size = 212, normalized size = 0.77

$$\frac{\sqrt{d}\left(\sqrt{4ac-b^2}(-aBf+Abf+Bcd)\left(\log(a+x(b+cx))-\log\left(d+fx^2\right)\right)+2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(2Ac(cd-af)-bB(af+\ldots\right.\right.}{2\sqrt{d}\sqrt{4ac-b^2}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] (2*Sqrt[-b^2 + 4*a*c]*Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sq
rt[d]] + Sqrt[d]*(2*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(B*c*d + A*b*f - a*B*f
)*(-Log[d + f*x^2] + Log[a + x*(b + c*x)])))/(2*Sqrt[-b^2 + 4*a*c]*Sqrt[d]*
(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

Maple [B] time = 0.199, size = 745, normalized size = 2.7

$$\frac{\ln\left(cx^2+bx+a\right)Abf}{2\,a^2f^2-4\,acdf+2\,b^2df+2\,c^2d^2} - \frac{\ln\left(cx^2+bx+a\right)Baf}{2\,a^2f^2-4\,acdf+2\,b^2df+2\,c^2d^2} + \frac{c\ln\left(cx^2+bx+a\right)Bd}{2\,a^2f^2-4\,acdf+2\,b^2df+2\,c^2d^2} - 2\,\frac{\ldots}{\left(a^2f^2-\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x)
```

```
[Out] 1/2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln(c*x^2+b*x+a)*A*b*f-1/2/(a^2*f^2-
2*a*c*d*f+b^2*d*f+c^2*d^2)*ln(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-2*a*c*d*f+b^2
*d*f+c^2*d^2)*c*ln(c*x^2+b*x+a)*B*d-2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*a*c*f+1/(a^2*f^2-2*a
*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*A*b^2*f+2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))*A*c^2*d-1/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f-1/(a^2*f^2-2*a
*c*d*f+b^2*d*f+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))*B*b*c*d-1/2*f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln(f*x^2+d)*A*b+1/2*f/
(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*ln(f*x^2+d)*a*B-1/2/(a^2*f^2-2*a*c*d*f+
b^2*d*f+c^2*d^2)*ln(f*x^2+d)*B*c*d+f^2/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)/
(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*a*A-f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^
2)/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*A*c*d+f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c
^2*d^2)/(d*f)^(1/2)*arctan(x*f/(d*f)^(1/2))*B*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23739, size = 359, normalized size = 1.31

$$\frac{\left(Bcd - Baf + Abf\right)\log\left(cx^2 + bx + a\right)}{2\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)} - \frac{\left(Bcd - Baf + Abf\right)\log\left(fx^2 + d\right)}{2\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)} + \frac{\left(Bbdf - Acdf + Aaf^2\right)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\left(c^2d^2 + b^2df - 2\,acdf + a^2f^2\right)\sqrt{df}} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d - B*a*f + A*b*f)*log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f - 2*a*c
*d*f + a^2*f^2) - 1/2*(B*c*d - B*a*f + A*b*f)*log(f*x^2 + d)/(c^2*d^2 + b^2
*d*f - 2*a*c*d*f + a^2*f^2) + (B*b*d*f - A*c*d*f + A*a*f^2)*arctan(f*x/sqrt
(d*f))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(d*f)) - (B*b*c*d - 2
*A*c^2*d + B*a*b*f - A*b^2*f + 2*A*a*c*f)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(-b^2 + 4*a*c))
```

3.5 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+fx^2\right)}\,dx$

Optimal. Leaf size=596

$$-\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2+2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)+b\right.}{}$$

```
[Out] (A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*(A*b^2*f +
 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(a + b*x + c*x^2)) - (f^(3/2)*(A*b^2*d*f + 2*b*B*d*(c*d - a*f) - A*(
c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f +
f*(b^2*d + a^2*f))^2) - ((b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f) - 4*A*c^2*(
c*d - 3*a*f)*(c*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a^2*f^2) - 4*
A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*
d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])
/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - (f*(2*
A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[a + b*
x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) + (f*(2*A*b*f*(
c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(
2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

Rubi [A] time = 1.77168, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1018, 1074, 634, 618, 206, 628, 635, 205, 260}

$$-\frac{f^{3/2}\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2+2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}-\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] (A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*(A*b^2*f +
 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(a + b*x + c*x^2)) - (f^(3/2)*(A*b^2*d*f + 2*b*B*d*(c*d - a*f) - A*(
c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f +
f*(b^2*d + a^2*f))^2) - ((b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f) - 4*A*c^2*(
c*d - 3*a*f)*(c*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a^2*f^2) - 4*
```

```
A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*
d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])
/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - (f*(2*
A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[a + b*
x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) + (f*(2*A*b*f*(
c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(
2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1074

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d
*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c
*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x],
x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*
c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[
{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
```

```
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+fx^2\right)}\,dx = \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bB(cd+\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bB(cd+\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bB(cd+\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bB(cd+\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bB(cd+\right.}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)}$$

Mathematica [A] time = 2.16971, size = 523, normalized size = 0.88

$$-\frac{\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)-b^3Bf\left(a^2f^2+4acdf-5c^2d^2\right)+2bBc\left(3a^2cdf^2+3a^3f^3-7ac^2d^2f+c^3d^3\right)+2Ab^4f^2(af-cd)-4Ac^2(cd-3af)(cd-\right.}{\left(4ac-b^2\right)^{3/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] ((-2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))*(A*(b^3*f + b*c*(c*d - 3*a*f
) + b^2*c*f*x + 2*c^2*(c*d - a*f)*x) + B*(2*a^2*c*f - b*c^2*d*x - a*(2*c^2*
d + b^2*f + b*c*f*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*f^(3/2)*(-(A
*b^2*d*f) + A*(c*d - a*f)^2 + 2*b*B*d*(-(c*d) + a*f))*ArcTan[(Sqrt[f]*x)/Sq
rt[d]])/Sqrt[d] - (2*(b^5*B*d*f^2 - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + 2
*A*b^4*f^2*(-(c*d) + a*f) - b^3*B*f*(-5*c^2*d^2 + 4*a*c*d*f + a^2*f^2) - 4*
A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*
d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
/(-b^2 + 4*a*c)^(3/2) + f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f + f
*(-(b^2*d) + a^2*f)))*Log[d + f*x^2] + f*(2*A*b*f*(-(c*d) + a*f) + B*(-(c^2
*d^2) + 2*a*c*d*f + f*(b^2*d - a^2*f)))*Log[a + x*(b + c*x)])/(2*(c^2*d^2 -
```

```
2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

Maple [B] time = 0.204, size = 9311, normalized size = 15.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23283, size = 1773, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3
 - 2*A*a*b*f^3)*log(c*x^2 + b*x + a)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d
^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^
3 - 4*a^3*c*d*f^3 + a^4*f^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f
^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3)*log(f*x^2 + d)/(c^4*d^4 + 2*b
^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*
d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) - (2*B*b*c*d^2*f^2 - A
*c^2*d^2*f^2 - 2*B*a*b*d*f^3 + A*b^2*d*f^3 + 2*A*a*c*d*f^3 - A*a^2*f^4)*arc
tan(f*x/sqrt(d*f))/((c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^
2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3
 + a^4*f^4)*sqrt(d*f)) + (2*B*b*c^4*d^3 - 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f -
 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f + 20*A*a*c^4*d^2*f + B*b^5*d*f^2 -
4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*
d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^
3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4
*a*c))/((b^2*c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f +
 16*a^2*c^4*d^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^
2 - 24*a^3*c^3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*
d*f^3 + a^4*b^2*f^4 - 4*a^5*c*f^4)*sqrt(-b^2 + 4*a*c)) + (2*B*a*c^4*d^3 - A
*b*c^4*d^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f +
5*A*a*b*c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A
*a*b^3*c*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 -
A*a^2*b^3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 + (B*b*c^4*d^3 - 2*A*c^5*d^
3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a*c^4*d^2*f
 + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^2*c^2*d*f^
2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3*c^2*f^3)*
x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*
```

```
a*c))
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^{3/2}}+\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tan}{2\sqrt{d}f^{3/2}}$$

```
[Out] -((B*Sqrt[a + b*x + c*x^2])/f) - ((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[d]*f^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f
])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[d]*f^(3/2))
```

Rubi [A] time = 0.600582, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^{3/2}}+\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tan}{2\sqrt{d}f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((B*Sqrt[a + b*x + c*x^2])/f) - ((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[d]*f^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f
])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[d]*f^(3/2))
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{B\sqrt{a+bx+cx^2}}{f} + \frac{\int \frac{\frac{1}{2}(bBd+2aAf)+(Bcd+Abf+aBf)x+\frac{1}{2}(bB+2Ac)fx^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{-\frac{1}{2}(bB+2Ac)df-\frac{1}{2}f(bBd+2aAf)-f(Bcd+Abf+aBf)x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f^2} - \frac{(bB+2Ac)\int \frac{}{\sqrt{a\cdots}}}{2f}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(\left(B\sqrt{d}-A\sqrt{f}\right)\left(cd \cdots\right.\right.}{}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\left(\left(B\sqrt{d}-A\sqrt{f}\right)\left(cd-b\sqrt{d}\sqrt{\cdots}\right.\right.}{}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{cd-b\sqrt{d}\sqrt{\cdots}}}{}$$

Mathematica [A] time = 0.805055, size = 322, normalized size = 0.97

$$\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\cdots}{2\sqrt{d}f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*
Sqrt[c]*f) + (-2*B*Sqrt[d]*Sqrt[f]*Sqrt[a + x*(b + c*x)] + (B*Sqrt[d] + A*S
qrt[f])*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f
] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqr
t[a + x*(b + c*x)])] - (B*Sqrt[d] - A*Sqrt[f])*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
 + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[d]*f^
(3/2))
```

Maple [B] time = 0.45, size = 3358, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*A-1/2/f*((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*B-1/2/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2
)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*A+1/2*(d*f)^(1/2)/f^2*ln((1/2/f*(-2
*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f
*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/
2))*c^(1/2)*B+1/4/(d*f)^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/
2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b*A-1/4/f*ln((1/2/f*
(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^
(1/2))/c^(1/2)*b*B+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(
d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b
*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f
))*b*A-1/2*(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b
*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)
/f))*b*B-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(
d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b
*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f
))*a*A+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a
*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*B-1/2/(
d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a
*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c*d*A+1/2
/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+
1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d
))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c*d*B-1/2/(d*f)^(
1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2)*A-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*B-1/2/f*ln((1/2*
```

```
(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)
)*c^(1/2)*A-1/2*(d*f)^(1/2)/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1
/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*B-1/4/(d*f)^(1/2)*ln((1/2
*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*
c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2
))/c^(1/2)*b*A-1/4/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c
^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b*B+1/2/f/((b*(d*f)^(1/2)+a*f+c*d)/f
)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)
^(1/2)/f))*b*A+1/2*(d*f)^(1/2)/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*
(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(
d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*B
+1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f
+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)
+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*A+1/2/f/((b*(d*f)^(
1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^
(1/2))/(x-(d*f)^(1/2)/f))*a*B+1/2/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^
(1/2)/f))*c*d*A+1/2/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1
/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+
a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{A\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{Bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(A*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(B*x*sqrt(a
+ b*x + c*x**2)/(-d + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.7 $\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=249

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

```
[Out] -((B - (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
 - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((B + (A*Sqrt[f])/S
qrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.204315, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1033, 724, 206}

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -((B - (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
 - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((B + (A*Sqrt[f])/S
qrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
```

```
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \frac{1}{2}\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{\cdots}}$$

$$= \left(-B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{\cdots}\right.}{\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\cdots\right.}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+\cdots}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [A] time = 0.255337, size = 249, normalized size = 1.

$$\frac{-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (-(((B*Sqrt[d] - A*Sqrt[f])*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt
[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*
```

```
x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) - ((B*Sqrt[d] + A*Sqrt[f])*ArcT
anh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f])/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] time = 0.334, size = 714, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/
2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(
1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*A+1/2
/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*B+1/2/(d*f)^(1/2)/(
(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*
f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*A+1/2/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/
2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2
)/f))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{A}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx-\int \frac{Bx}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(A/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
 - Integral(B*x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2))
, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.8 $\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=381

$$-\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)+A\left(b^3f-bc(3af+cd)\right)+aB\left(2acf+b^2(-f)+2c^2d\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\left(B\sqrt{d}-\right.}{2}$$

```
[Out] (-2*(a*B*(2*c^2*d - b^2*f + 2*a*c*f) + A*(b^3*f - b*c*(c*d + 3*a*f)) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d*f -
 (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[f]*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[d]*(c*d - b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f]
 + a*f)^(3/2))
```

Rubi [A] time = 0.798083, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1018, 1033, 724, 206}

$$-\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\left(B\sqrt{d}-\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d*f -
 (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[f]*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[d]*(c*d - b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f]
```

```
 + a*f)^(3/2))
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-af)-2A\right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-af)-2A\right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-af)-2A\right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{2\left(Ab^3f-Abc(cd+3af)+aB\left(2c^2d-b^2f+2acf\right)+c\left(Ab^2f+bB(cd-af)-2A\right.\right.}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.811798, size = 440, normalized size = 1.15

$$\frac{2\left(\frac{B\left(2a^2cf+a\left(b^2(-f)-bcfx+2c^2d\right)+bc^2dx\right)+A\left(-bc(3af+cd)-2c^2x(af+cd)+b^2cfx+b^3f\right)}{\sqrt{a+x(b+cx)}}+\frac{\sqrt{f}\left(b^2-4ac\right)\left(A\sqrt{f}-B\sqrt{d}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}\right.}{2\sqrt{a+x(b+cx)}\sqrt{\ldots}}\right.}{4\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right.}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x) + B*(2
*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/Sqrt[a + x*(b + c*x)
] + ((b^2 - 4*a*c)*(-(B*Sqrt[d]) + A*Sqrt[f])*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt
[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))
/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((-b^2 + 4*a*c)*(B*Sqrt[d] + A*Sqrt
[f])*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqr
t[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])))/(
(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

Maple [B] time = 0.306, size = 2758, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*f/(-b*(d*f)^(1/2)+a*f+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*A-1
/2/(-b*(d*f)^(1/2)+a*f+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*
f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*B+2/(-b*(d*f)^(1/2
)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c^2*A-2*(d*f)^(1/2)/f
/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f
)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c^2*B-
1/(d*f)^(1/2)*f/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*x*b*c*A+1/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d)
)^(1/2)*x*b*c*B+1/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2
*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d
))^(1/2)*b*c*A-(d*f)^(1/2)/f/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)
^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+a*f+c*d))^(1/2)*b*c*B-1/2/(d*f)^(1/2)*f/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c
-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1
/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2*A+1/2/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*
c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2*B-1/2/(d*f)^(1/2)*f/(-b*(d*f)^(1/2)
+a*f+c*d)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+
c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*
f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*A+1/2/(-b*(d
*f)^(1/2)+a*f+c*d)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(
1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)
^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*B-1
/2/(d*f)^(1/2)*f/(b*(d*f)^(1/2)+a*f+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*A-1/2/(b*(d
*f)^(1/2)+a*f+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*B+2/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*
c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*c^2*A+2*(d*f)^(1/2)/f/(b*(d*f)^(1/2)+a*f+c*
d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
```

```
)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*c^2*B+1/(d*f)^(1/2)*f/(b*(d*f)^(1/2
)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b*c*A+1/(b*(d*f)^(1/2)+a*f+
c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b*c*B+1/(b*(d*f)^(1/2)+a*f+c*d)/(
4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)
+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b*c*A+(d*f)^(1/2)/f/(b*(d*f)^(1/2)+a*f+c*
d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b*c*B+1/2/(d*f)^(1/2)*f/(b*(d*f)^(1/2
)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2*A+1/2/(b*(d*f)^(1/2)+a*f+
c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2*B+1/2/(d*f)^(1/2)*f/(b*(d*f)^(1
/2)+a*f+c*d)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d
)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*A+1/2/(b*(d*f)^(1/2)+a*f+
c*d)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*(
(x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1
/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.9 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{5/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=797

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{2\sqrt{-\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(-\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}+\frac{\left(\sqrt{f}A+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}-2\,(3B$$

```
[Out] (-2*(a*B*(2*c^2*d - b^2*f + 2*a*c*f) + A*(b^3*f - b*c*(c*d + 3*a*f)) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/(3*(b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*B*d*f^2 + 24*a^2*B*c
^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 + 14*a*
c*d*f - 3*a^2*f^2) + A*b^3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2) + 2*b
^2*B*c*(2*c^3*d^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4*A*b*c^2
*(2*c^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*b^5*B*d*f
^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*c*d + 5*a*f) -
b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*A*b^2*c*f*(15*c^2*d^2 + 2
2*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3*d^3 + 11*a*c^2*d^2*f + 4*a^2*c*d*f
^2 - 5*a^3*f^3))*x))/(3*(b^2 - 4*a*c)^2*(c^2*d^2 + 2*a*c*d*f - f*(b^2*d - a
^2*f))^2*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*f^(3/2)*ArcTanh[
(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sq
rt[f] + a*f)^(5/2)) + ((B*Sqrt[d] + A*Sqrt[f])*f^(3/2)*ArcTanh[(b*Sqrt[d] +
 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
 + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(5/2))
```

Rubi [A] time = 1.87008, antiderivative size = 796, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1018, 1064, 1033, 724, 206}

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\tanh^{-1}\left(\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{2\sqrt{-\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(-\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}+\frac{\left(\sqrt{f}A+B\sqrt{d}\right)\tanh^{-1}\left(\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}-2\,(3B$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/(3*(b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*B*d*f^2 + 24*a^2*B*c
^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 + 14*a*
c*d*f - 3*a^2*f^2) + A*b^3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2) + 2*b
^2*B*c*(2*c^3*d^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4*A*b*c^2
*(2*c^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*b^5*B*d*f
^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*c*d + 5*a*f) -
b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*A*b^2*c*f*(15*c^2*d^2 + 2
2*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3*d^3 + 11*a*c^2*d^2*f + 4*a^2*c*d*f
^2 - 5*a^3*f^3))*x))/(3*(b^2 - 4*a*c)^2*(c^2*d^2 + 2*a*c*d*f - f*(b^2*d - a
^2*f))^2*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*f^(3/2)*ArcTanh[
(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sq
rt[f] + a*f)^(5/2)) + ((B*Sqrt[d] + A*Sqrt[f])*f^(3/2)*ArcTanh[(b*Sqrt[d] +
 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
 + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(5/2))
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1064

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p +
1)*(d + f*x^2)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2
*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) - B*(b*c*d + a
*b*f) + C*(b^2*d - 2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p +
1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)
```

```
*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + a
*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)
) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b - a*B)*(2*c^2*d + b^2*f - c
*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) - b*(
B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] &&
 ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{5/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(Ab^3f - Abc(cd+3af) + aB\left(2c^2d - b^2f + 2acf\right) + c\left(Ab^2f + bB(cd-af) - 2A \cdots\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f - Abc(cd+3af) + aB\left(2c^2d - b^2f + 2acf\right) + c\left(Ab^2f + bB(cd-af) - 2A \cdots\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f - Abc(cd+3af) + aB\left(2c^2d - b^2f + 2acf\right) + c\left(Ab^2f + bB(cd-af) - 2A \cdots\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f - Abc(cd+3af) + aB\left(2c^2d - b^2f + 2acf\right) + c\left(Ab^2f + bB(cd-af) - 2A \cdots\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$= -\frac{2\left(Ab^3f - Abc(cd+3af) + aB\left(2c^2d - b^2f + 2acf\right) + c\left(Ab^2f + bB(cd-af) - 2A \cdots\right.\right.}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(a+bx+cx^2\right)^{3/2}}$$

Mathematica [A] time = 5.16498, size = 674, normalized size = 0.85

$$2\left(-\frac{3f\left(-b^2\left(B\left(-a^2f^2+2acdf+c^2d^2\right)+2aAcf^2x\right)+b^3f(Bcdx-A(2af+cd))+bc(af+cd)(5aAf+aBfx+Acd-3Bcdx)+2c(af+cd)^2(Acx-aB)+b^4Bdf\right)}{\sqrt{a+x(b+cx)}\left(f\left(a^2f-b^2d\right)+2acdf+c^2d^2\right)} + \frac{B\left(2a^2cf+a\left(b \cdots\right.\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] (2*((4*c*(-(A*b^2*f) + b*B*(-(c*d) + a*f) + 2*A*c*(c*d + a*f))*(b + 2*c*x))
/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (3*f*(b^4*B*d*f + 2*c*(c*d + a*f)^
2*(-(a*B) + A*c*x) + b^3*f*(-(A*(c*d + 2*a*f)) + B*c*d*x) + b*c*(c*d + a*f)
*(A*c*d + 5*a*A*f - 3*B*c*d*x + a*B*f*x) - b^2*(B*(c^2*d^2 + 2*a*c*d*f - a^
2*f^2) + 2*a*A*c*f^2*x)))/((c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f))*Sqr
```

```
t[a + x*(b + c*x)]) + (A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*
d + a*f)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/(a
 + x*(b + c*x))^(3/2) + (3*(b^2 - 4*a*c)*f^(3/2)*(((-(B*Sqrt[d]) + A*Sqrt[f
])*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x
+ b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] - ((B*Sqrt[d] + A*Sqrt[
f])*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x)
 - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(4*Sqrt[d]*(-(b^2*d*
f) + (c*d + a*f)^2))))/(3*(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

Maple [B] time = 0.256, size = 6422, normalized size = 8.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(5/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.10 $\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx$

Optimal. Leaf size=47

$$\frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

```
[Out] -ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/2 + (3*ArcTanh[(1 - 3*x)/(2*Sqrt[-1
 + x + x^2])])/2
```

Rubi [A] time = 0.0342737, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1033, 724, 206, 204}

$$\frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/2 + (3*ArcTanh[(1 - 3*x)/(2*Sqrt[-1
 + x + x^2])])/2
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx &= \frac{1}{2}\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx+\frac{3}{2}\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx\\ &= -\left(3\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{-1+3x}{\sqrt{-1+x+x^2}}\right)\right)-\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,\frac{-3-x}{\sqrt{-1+x+x^2}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{-3-x}{2\sqrt{-1+x+x^2}}\right)+\frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0077584, size = 49, normalized size = 1.04

$$\frac{1}{2}\tan^{-1}\left(\frac{-x-3}{2\sqrt{x^2+x-1}}\right)-\frac{3}{2}\tanh^{-1}\left(\frac{3x-1}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] ArcTan[(-3 - x)/(2*Sqrt[-1 + x + x^2])]/2 - (3*ArcTanh[(-1 + 3*x)/(2*Sqrt[-
1 + x + x^2])])/2
```

Maple [A] time = 0.05, size = 46, normalized size = 1.

$$\frac{1}{2}\arctan\left(\frac{-3-x}{2}\frac{1}{\sqrt{(1+x)^2-2-x}}\right)-\frac{3}{2}\text{Artanh}\left(\frac{3\,x-1}{2}\frac{1}{\sqrt{(-1+x)^2-2+3\,x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x)
```

```
[Out] 1/2*arctan(1/2*(-3-x)/((1+x)^2-2-x)^(1/2))-3/2*arctanh(1/2*(3*x-1)/((-1+x)^
2-2+3*x)^(1/2))
```

Maxima [A] time = 1.53576, size = 88, normalized size = 1.87

$$-\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5|2x+2|}+\frac{6\sqrt{5}}{5|2x+2|}\right)-\frac{3}{2}\log\left(\frac{2\sqrt{x^2+x-1}}{|2x-2|}+\frac{2}{|2x-2|}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(2/5*sqrt(5)*x/abs(2*x + 2) + 6/5*sqrt(5)/abs(2*x + 2)) - 3/2*lo
g(2*sqrt(x^2 + x - 1)/abs(2*x - 2) + 2/abs(2*x - 2) + 3/2)
```

Fricas [A] time = 1.52567, size = 146, normalized size = 3.11

$$\arctan\left(-x+\sqrt{x^2+x-1}-1\right)-\frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}+2\right)+\frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(-x + sqrt(x^2 + x - 1) + 2) +
3/2*log(-x + sqrt(x^2 + x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{(x-1)(x+1)\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2-1)/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x - 1)*(x + 1)*sqrt(x**2 + x - 1)), x)
```

Giac [A] time = 1.21779, size = 65, normalized size = 1.38

$$\arctan\left(-x+\sqrt{x^2+x-1}-1\right)-\frac{3}{2}\,\log\left(\left|-x+\sqrt{x^2+x-1}+2\right|\right)+\frac{3}{2}\,\log\left(\left|-x+\sqrt{x^2+x-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(abs(-x + sqrt(x^2 + x - 1) + 2
)) + 3/2*log(abs(-x + sqrt(x^2 + x - 1)))
```

3.11 $\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx$

Optimal. Leaf size=117

$$\sqrt{\frac{1}{2}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

```
[Out] -(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[10*(2 + Sq
rt[5])]*Sqrt[-1 + x + x^2])]) + Sqrt[(-2 + Sqrt[5])/2]*ArcTanh[(5 - 2*Sqrt[
5] + Sqrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt[-1 + x + x^2])]
```

Rubi [A] time = 0.168886, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1036, 1030, 207, 203}

$$\sqrt{\frac{1}{2}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[10*(2 + Sq
rt[5])]*Sqrt[-1 + x + x^2])]) + Sqrt[(-2 + Sqrt[5])/2]*ArcTanh[(5 - 2*Sqrt[
5] + Sqrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt[-1 + x + x^2])]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x}{\left(1+x^2\right)\sqrt{-1+x+x^2}}\,dx = -\frac{\int \frac{-\sqrt{5}+\left(-5-2\sqrt{5}\right)x}{\left(1+x^2\right)\sqrt{-1+x+x^2}}\,dx}{2\sqrt{5}}+\frac{\int \frac{\sqrt{5}+\left(-5+2\sqrt{5}\right)x}{\left(1+x^2\right)\sqrt{-1+x+x^2}}\,dx}{2\sqrt{5}}$$

$$= -\left(\left(-5+2\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{10\left(2-\sqrt{5}\right)+x^2}\,dx, x, \frac{-5+2\sqrt{5}-\sqrt{5}x}{\sqrt{-1+x+x^2}}\right)\right)+\left(5+2\sqrt{5}\right)\text{Subst}$$

$$= -\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{5+2\sqrt{5}-\sqrt{5}x}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right)+\sqrt{\frac{1}{2}\left(-2+\sqrt{5}\right)}\tanh^{-1}\left(\frac{5-}{\sqrt{10\left(-2\right.}}\right.$$

Mathematica [C] time = 0.0338414, size = 78, normalized size = 0.67

$$-\frac{1}{2}i\left(\sqrt{2+i}\tanh^{-1}\left(\frac{\sqrt{2+i}(x-i)}{2\sqrt{x^2+x-1}}\right)-\sqrt{2-i}\tanh^{-1}\left(\frac{\sqrt{2-i}(x+i)}{2\sqrt{x^2+x-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] (-I/2)*(Sqrt[2 + I]*ArcTanh[(Sqrt[2 + I]*(-I + x))/(2*Sqrt[-1 + x + x^2])]
- Sqrt[2 - I]*ArcTanh[(Sqrt[2 - I]*(I + x))/(2*Sqrt[-1 + x + x^2])])
```

Maple [B] time = 0.161, size = 637, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x)
```

```
[Out] (10*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5*5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+
2-x)^2+10+5*5^(1/2))^(1/2)*5^(1/2)*(arctan(1/5*5^(1/2)*((-2+5^(1/2))*(-(-5^
(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+4*5^(1/2)+9))^(1/2)*(20+10*5^(1/2))^(1/2)*(5^
(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+2*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2
-5^(1/2)+2)*(-2+5^(1/2))*(-5^(1/2)-2+x)/(-5^(1/2)+2-x)/((-5^(1/2)-2+x)^4/(-
5^(1/2)+2-x)^4-18*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+1))*5^(1/2)+arctanh((10
*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-5*5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x
)^2+10+5*5^(1/2))^(1/2)/(20+10*5^(1/2))^(1/2))+2*arctan(1/5*5^(1/2)*((-2+5^
(1/2))*(-(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+4*5^(1/2)+9))^(1/2)*(20+10*5^(1/
2))^(1/2)*(5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+2*(-5^(1/2)-2+x)^2/(-5
^(1/2)+2-x)^2-5^(1/2)+2)*(-2+5^(1/2))*(-5^(1/2)-2+x)/(-5^(1/2)+2-x)/((-5^(1
/2)-2+x)^4/(-5^(1/2)+2-x)^4-18*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2+1)))/(-5*(
5^(1/2)*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)^2-2*(-5^(1/2)-2+x)^2/(-5^(1/2)+2-x)
^2-5^(1/2)-2)/(1+(-5^(1/2)-2+x)/(-5^(1/2)+2-x))^2)^(1/2)/(1+(-5^(1/2)-2+x)/
(-5^(1/2)+2-x))/(20+10*5^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+x-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)), x)
```

Fricas [B] time = 1.20553, size = 2379, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*5^(1/4)*sqrt(4*sqrt(5) + 10)*(2*sqrt(5) - 5)*log(2*x^2 - 2*sqrt(x^2 +
x - 1)*x + 1/5*(5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(sqrt(5
)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + x + sqrt(5)) - 1/20*5^(1/4)*sqrt
(4*sqrt(5) + 10)*(2*sqrt(5) - 5)*log(2*x^2 - 2*sqrt(x^2 + x - 1)*x - 1/5*(5
^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(sqrt(5)*(2*x + 1) - 5*x
))*sqrt(4*sqrt(5) + 10) + x + sqrt(5)) - 1/5*5^(3/4)*sqrt(4*sqrt(5) + 10)*a
rctan(2/55*sqrt(5)*(sqrt(5)*(2*x - 1) + 3*x + 4) + 1/275*sqrt(10*x^2 - 10*s
qrt(x^2 + x - 1)*x + (5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(
sqrt(5)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + 5*x + 5*sqrt(5))*((5^(3/4)
*(2*sqrt(5) + 3) + 2*5^(1/4)*(4*sqrt(5) - 5))*sqrt(4*sqrt(5) + 10) + 2*sqrt
(5)*(3*sqrt(5) + 10) - 20*sqrt(5) + 80) - 2/55*sqrt(x^2 + x - 1)*(sqrt(5)*(
2*sqrt(5) + 3) + 8*sqrt(5) - 10) + 1/55*sqrt(5)*(16*x + 3) + 1/275*(5^(3/4)
*(sqrt(5)*(3*x + 4) + 10*x - 5) - sqrt(x^2 + x - 1)*(5^(3/4)*(3*sqrt(5) + 1
0) - 10*5^(1/4)*(sqrt(5) - 4)) - 10*5^(1/4)*(sqrt(5)*(x - 6) - 4*x + 13))*s
qrt(4*sqrt(5) + 10) - 4/11*x + 2/11) - 1/5*5^(3/4)*sqrt(4*sqrt(5) + 10)*arc
tan(-2/55*sqrt(5)*(sqrt(5)*(2*x - 1) + 3*x + 4) + 1/275*sqrt(10*x^2 - 10*sq
rt(x^2 + x - 1)*x - (5^(1/4)*sqrt(x^2 + x - 1)*(2*sqrt(5) - 5) - 5^(1/4)*(s
qrt(5)*(2*x + 1) - 5*x))*sqrt(4*sqrt(5) + 10) + 5*x + 5*sqrt(5))*((5^(3/4)*
(2*sqrt(5) + 3) + 2*5^(1/4)*(4*sqrt(5) - 5))*sqrt(4*sqrt(5) + 10) - 2*sqrt(
5)*(3*sqrt(5) + 10) + 20*sqrt(5) - 80) + 2/55*sqrt(x^2 + x - 1)*(sqrt(5)*(2
*sqrt(5) + 3) + 8*sqrt(5) - 10) - 1/55*sqrt(5)*(16*x + 3) + 1/275*(5^(3/4)*
(sqrt(5)*(3*x + 4) + 10*x - 5) - sqrt(x^2 + x - 1)*(5^(3/4)*(3*sqrt(5) + 10
) - 10*5^(1/4)*(sqrt(5) - 4)) - 10*5^(1/4)*(sqrt(5)*(x - 6) - 4*x + 13))*sq
rt(4*sqrt(5) + 10) + 4/11*x - 2/11)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\left(x^2+1\right)\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+1)/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + x - 1)), x)
```

Giac [C] time = 1.29809, size = 336, normalized size = 2.87

$$\frac{1}{4}\sqrt{2\sqrt{5}-4}\left(-\frac{i}{\sqrt{5}-2}+1\right)\log\left(-4\sqrt{5\sqrt{5}+11}\left(\frac{2i}{5\sqrt{5}+11}-1\right)-(12i+4)x+(12i+4)\sqrt{x^2+x-1}-4i+12\right)-\frac{1}{4}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(5) - 4)*(-I/(sqrt(5) - 2) + 1)*log(-4*sqrt(5*sqrt(5) + 11)*
(2*I/(5*sqrt(5) + 11) - 1) - (12*I + 4)*x + (12*I + 4)*sqrt(x^2 + x - 1) -
4*I + 12) - 1/4*sqrt(2*sqrt(5) - 4)*(-I/(sqrt(5) - 2) + 1)*log(-4*sqrt(5*sq
rt(5) + 11)*(-2*I/(5*sqrt(5) + 11) + 1) - (12*I + 4)*x + (12*I + 4)*sqrt(x^
2 + x - 1) - 4*I + 12) - 1/4*sqrt(2*sqrt(5) - 4)*(I/(sqrt(5) - 2) + 1)*log(
-4*sqrt(5*sqrt(5) - 11)*(2*I/(5*sqrt(5) - 11) - 1) - (4*I + 12)*x + (4*I +
12)*sqrt(x^2 + x - 1) + 12*I - 4) + 1/4*sqrt(2*sqrt(5) - 4)*(I/(sqrt(5) - 2
) + 1)*log(-4*sqrt(5*sqrt(5) - 11)*(-2*I/(5*sqrt(5) - 11) + 1) - (4*I + 12)
*x + (4*I + 12)*sqrt(x^2 + x - 1) + 12*I - 4)
```

3.12 $\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=484

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\tan^{-1}\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}-x\left((a-c)\left(\sqrt{a^2-2\cdots}\right)\right)}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c\cdots\right)}}\right)}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

```
[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a
^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (b^2 +
(a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2
*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) -
a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*
(a^2 + b^2 - 2*a*c + c^2)^(1/4))) - (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2
 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b*Sqrt
[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c
+ c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c
+ Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]
*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))
```

Rubi [A] time = 23.581, antiderivative size = 484, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1036, 1030, 208, 205}

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\tan^{-1}\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}-x\left((a-c)\left(\sqrt{a^2-2\cdots}\right)\right)}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c\cdots\right)}}\right)}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a
^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (b^2 +
(a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2
*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) -
a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*
(a^2 + b^2 - 2*a*c + c^2)^(1/4))) - (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2
```

```
 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b*Sqrt
[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c
+ c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c
+ Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]
*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a-c+bx}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx = -\frac{\int \frac{-b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)-b\sqrt{a^2+b^2-2ac+c^2}x}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{a^2+b^2-2ac+c^2}} + \frac{\int \frac{-b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+b\sqrt{a^2+b^2-}}{\left(1+x^2\right)\sqrt{a+bx+cx^2}}}{2\sqrt{a^2+b^2-2ac+c^2}}$$

$$= \left(b\left(b^2+(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{}{-2b\sqrt{a^2+b^2-2ac+c^2}\left(b^2+\right.}\right.$$

$$= -\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}}$$

Mathematica [C] time = 0.0869691, size = 136, normalized size = 0.28

$$\frac{1}{2}i\left(\sqrt{a+ib-c}\tanh^{-1}\left(\frac{2a+b(x+i)+2icx}{2\sqrt{a+ib-c}\sqrt{a+x(b+cx)}}\right)-\sqrt{a-ib-c}\tanh^{-1}\left(\frac{2a+b(x-i)-2icx}{2\sqrt{a-ib-c}\sqrt{a+x(b+cx)}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (I/2)*(-(Sqrt[a - I*b - c]*ArcTanh[(2*a - (2*I)*c*x + b*(-I + x))/(2*Sqrt[a
 - I*b - c]*Sqrt[a + x*(b + c*x)])]) + Sqrt[a + I*b - c]*ArcTanh[(2*a + (2*
I)*c*x + b*(I + x))/(2*Sqrt[a + I*b - c]*Sqrt[a + x*(b + c*x)])])
```

Maple [B] time = 0.901, size = 6871419, normalized size = 14197.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx + a - c}{\sqrt{cx^2 + bx + a}(x^2 + 1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx - c}{(x^2 + 1)\sqrt{a + bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x**2+1)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((a + b*x - c)/((x**2 + 1)*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.13 $\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx$

Optimal. Leaf size=184

$$-\frac{\log\left(d+ex+fx^2\right)\left(Af(ce-bf)-B\left(af^2-bef-cdf+ce^2\right)\right)}{2f^3}-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(2af^2-bef-2cdf+ce^2\right)+B}{f^3\sqrt{e^2-4d}}$$

```
[Out] -(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((A*f*(c*e^2 - 2*c*d
*f - b*e*f + 2*a*f^2) + B*(f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f))
)*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^3*Sqrt[e^2 - 4*d*f]) - ((A*f*(
c*e - b*f) - B*(c*e^2 - c*d*f - b*e*f + a*f^2))*Log[d + e*x + f*x^2])/(2*f^
3)
```

Rubi [A] time = 0.345593, antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1628, 634, 618, 206, 628}

$$-\frac{\log\left(d+ex+fx^2\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)}{2f^3}-\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(2af^2-bef-2cdf+ce^2\right)+}{f^3\sqrt{e^2-4d}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] -(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((B*f*(b*e^2 - 2*b*d
*f - a*e*f) - B*c*(e^3 - 3*d*e*f) + A*f*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2)
)*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^3*Sqrt[e^2 - 4*d*f]) - ((B*f*(
b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))*Log[d + e*x + f*x^2])/(2*f^
3)
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx &= \int \left(-\frac{Bce-bBf-Acf}{f^2}+\frac{Bcx}{f}+\frac{-Af(cd-af)+Bd(ce-bf)-\left(Bf(be-af)+Af(ce-\right.}{f^2\left(d+ex+fx^2\right)}\right.\\
&= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}+\frac{\int \frac{-Af(cd-af)+Bd(ce-bf)-\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)x}{d+ex+fx^2}\,dx}{f^2}\\
&= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}+\frac{\left(-Bf(be-af)-Af(ce-bf)+Bc\left(e^2-df\right)\right)\int \frac{e+2fx}{d+ex+f}}{2f^3}\\
&= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}-\frac{\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\log\left(d+ex\right.}{2f^3}\\
&= -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}-\frac{\left(Bf\left(be^2-2bdf-aef\right)-Bc\left(e^3-3def\right)+Af\left(ce^2-2\right.\right.}{f^3\sqrt{e^2-4df}}
\end{aligned}$$

Mathematica [A] time = 0.202927, size = 175, normalized size = 0.95

$$\frac{-\frac{2\tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)\left(Af\left(-2af^2+bef+2cdf-ce^2\right)+Bf\left(aef+2bdf-be^2\right)+Bc\left(e^3-3def\right)\right)}{\sqrt{4df-e^2}}+\log(d+x(e+fx))\left(Bf(af-be)+Af(bf-ce)+Bc\right.}{2f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] (2*f*(-(B*c*e) + b*B*f + A*c*f)*x + B*c*f^2*x^2 - (2*(B*f*(-(b*e^2) + 2*b*d
*f + a*e*f) + B*c*(e^3 - 3*d*e*f) + A*f*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f
^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*f*(-(b
*e) + a*f) + A*f*(-(c*e) + b*f) + B*c*(e^2 - d*f))*Log[d + x*(e + f*x)])/(2
*f^3)
```

Maple [B] time = 0.175, size = 510, normalized size = 2.8

$$\frac{Bcx^2}{2f}+\frac{Acx}{f}+\frac{Bbx}{f}-\frac{Bcex}{f^2}+\frac{\ln\left(fx^2+ex+d\right)Ab}{2f}-\frac{\ln\left(fx^2+ex+d\right)Ace}{2f^2}+\frac{\ln\left(fx^2+ex+d\right)Ba}{2f}-\frac{\ln\left(fx^2+ex+d\right.}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2*B*c*x^2/f+1/f*A*c*x+1/f*B*b*x-1/f^2*B*c*e*x+1/2/f*ln(f*x^2+e*x+d)*A*b-1
/2/f^2*ln(f*x^2+e*x+d)*A*c*e+1/2/f*ln(f*x^2+e*x+d)*B*a-1/2/f^2*ln(f*x^2+e*x
+d)*B*b*e-1/2/f^2*ln(f*x^2+e*x+d)*B*c*d+1/2/f^3*ln(f*x^2+e*x+d)*B*c*e^2+2/(
4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*a-2/f/(4*d*f-e^2)^(1
/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d-2/f/(4*d*f-e^2)^(1/2)*arctan(
(2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*d+3/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)
/(4*d*f-e^2)^(1/2))*B*c*d*e-1/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e
^2)^(1/2))*e*A*b+1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2)
)*A*c*e^2-1/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*B*a+1
/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*b*B*e^2-1/f^3/(4
*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79901, size = 1280, normalized size = 6.96

$$\left[\frac{\left(Bce^2f^2 - 4\,Bcdf^3\right)x^2 - \left(Bce^3 - 2\,Aaf^3 + (2\,(Bb + Ac)d + (Ba + Ab)e)f^2 - \left(3\,Bcde + (Bb + Ac)e^2\right)f\right)\sqrt{e^2 - 4\,df}\,\log\left(\frac{2\,f}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^2 - (B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*
c)*d + (B*a + A*b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*sqrt(e^2 - 4*d
*f)*log((2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))
/(f*x^2 + e*x + d)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e + (B*
b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4*(B*b
 + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f)*log
(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4), 1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^
2 + 2*(B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f^2 - (3*B*c
*d*e + (B*b + A*c)*e^2)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2
*f*x + e)/(e^2 - 4*d*f)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e
+ (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4
*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f
)*log(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4)]
```

Sympy [B] time = 25.2751, size = 1260, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] B*c*x**2/(2*f) + (-sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*
f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f +
B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*
f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x + (-A*a*e*f**2 + 2*A*b*d*f**2 - A*c
*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c*d**2*f + B*c*d*e**2 - 4*d*f**3*(-
sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f +
 B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(
4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e
**2)/(2*f**3)) + e**2*f**2*(-sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2
+ 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*
c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**
2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)))/(-2*A*a*f**3 + A*b*e*f**2 + 2*
A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*
e*f + B*c*e**3)) + (sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d
*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f +
 B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e
*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x + (-A*a*e*f**2 + 2*A*b*d*f**2 - A*
c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c*d**2*f + B*c*d*e**2 - 4*d*f**3*(
sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f +
 B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(
4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e
**2)/(2*f**3)) + e**2*f**2*(sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2 +
 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c
*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2
 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)))/(-2*A*a*f**3 + A*b*e*f**2 + 2*A
*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e
*f + B*c*e**3)) + x*(A*c*f + B*b*f - B*c*e)/f**2
```

Giac [A] time = 1.14854, size = 258, normalized size = 1.4

$$\frac{Bcfx^2 + 2\,Bbfx + 2\,Acfx - 2\,Bcxe}{2\,f^2} - \frac{\left(Bcdf - Baf^2 - Abf^2 + Bbfe + Acfe - Bce^2\right)\log\left(fx^2 + xe + d\right)}{2\,f^3} - \frac{\left(2\,Bbdf^2 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x - 2*B*c*x*e)/f^2 - 1/2*(B*c*d*f - B*
a*f^2 - A*b*f^2 + B*b*f*e + A*c*f*e - B*c*e^2)*log(f*x^2 + x*e + d)/f^3 - (
```

```
2*B*b*d*f^2 + 2*A*c*d*f^2 - 2*A*a*f^3 - 3*B*c*d*f*e + B*a*f^2*e + A*b*f^2*e
 - B*b*f*e^2 - A*c*f*e^2 + B*c*e^3)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/(
sqrt(4*d*f - e^2)*f^3)
```

3.14 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$

Optimal. Leaf size=542

$$\frac{\log\left(d+ex+fx^2\right)\left(B\left(-f^2\left(-a^2f^2+2abef+b^2\left(-\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3de^2f\right.\right.\right.}{2f^5}$$

```
[Out] ((B*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + A*f*(b^2*f^2 - 2*c*f*
(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4 - ((A*c*f*(c*e - 2*b*f) - B*(b^2*f^2
 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x^2)/(2*f^3) - (c*(B*c*e - 2*b*B*f
 - A*c*f)*x^3)/(3*f^2) + (B*c^2*x^4)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*f +
2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(
e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2)
 + f^2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) + 2*c*f*(a
*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*ArcTanh[(e + 2*f*x)
/Sqrt[e^2 - 4*d*f]])/(f^5*Sqrt[e^2 - 4*d*f]) + ((A*f*(c*e - b*f)*(f*(b*e -
2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b
*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e
*f))))*Log[d + e*x + f*x^2])/(2*f^5)
```

Rubi [A] time = 1.1025, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1011, 634, 618, 206, 628}

$$\frac{\log\left(d+ex+fx^2\right)\left(B\left(-f^2\left(-a^2f^2+2abef+b^2\left(-\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3de^2f\right.\right.\right.}{2f^5}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```



```
[Out] ((B*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + A*f*(b^2*f^2 - 2*c*f*
(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4 - ((A*c*f*(c*e - 2*b*f) - B*(b^2*f^2
 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x^2)/(2*f^3) - (c*(B*c*e - 2*b*B*f
 - A*c*f)*x^3)/(3*f^2) + (B*c^2*x^4)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*f +
2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(
e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2)
 + f^2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) + 2*c*f*(a
*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*ArcTanh[(e + 2*f*x)
```

```
/Sqrt[e^2 - 4*d*f]])/(f^5*Sqrt[e^2 - 4*d*f]) + ((A*f*(c*e - b*f)*(f*(b*e -
2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b
*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e
*f))))*Log[d + e*x + f*x^2])/(2*f^5)
```

Rule 1011

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x +
c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && IGtQ[p, 0] && In
tegerQ[q]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = \int \left(\frac{B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2-df\right)\right.}{f^4}\right.$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2-df\right)\right)\right)x}{f^4}$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2-df\right)\right)\right)x}{f^4}$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2-df\right)\right)\right)x}{f^4}$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+Af\left(b^2f^2-2cf(be-af)+c^2\left(e^2-df\right)\right)\right)x}{f^4}$$

Mathematica [A] time = 0.626175, size = 535, normalized size = 0.99

$$\frac{6\log(d+x(e+fx))\left(B\left(f^2\left(a^2f^2-2abef+b^2\left(e^2-df\right)\right)-2cf\left(af\left(df-e^2\right)+b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] (12*f*(-(B*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f))) + A*f*(b^2*f
^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x + 6*f^2*(A*c*f*(-(c*e) + 2*
b*f) + B*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x^2 + 4*c*f^3*
(-(B*c*e) + 2*b*B*f + A*c*f)*x^3 + 3*B*c^2*f^4*x^4 - (12*(-(A*f*(c^2*(e^4 -
 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 - 2*d*f))
+ 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))) + B*(c^2*(e^5 - 5*d*e^3*f
 + 5*d^2*e*f^2) + f^2*(a^2*e*f^2 + 2*a*b*f*(-e^2 + 2*d*f) + b^2*(e^3 - 3*d*
e*f)) - 2*c*f*(-(a*e*f*(e^2 - 3*d*f)) + b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*
ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + 6*(A*f*(-(c*e)
 + b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f +
d^2*f^2) + f^2*(-2*a*b*e*f + a^2*f^2 + b^2*(e^2 - d*f)) - 2*c*f*(a*f*(-e^2
+ d*f) + b*(e^3 - 2*d*e*f))))*Log[d + x*(e + f*x)])/(12*f^5)
```

Maple [B] time = 0.167, size = 1672, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

```
[Out] -4/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c^2*d*e^2-2/
f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e*A*a*b+1/4*B*c^2*x
^4/f+1/3/f*A*x^3*c^2+1/2/f*B*x^2*b^2+1/f*A*b^2*x-8/f^3/(4*d*f-e^2)^(1/2)*ar
ctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*c*d*e^2+6/f^2/(4*d*f-e^2)^(1/2)*arcta
n((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*a*c*d*e+6/f^2/(4*d*f-e^2)^(1/2)*arctan((2*
f*x+e)/(4*d*f-e^2)^(1/2))*A*b*c*d*e-1/f^2*A*c^2*d*x+2/f*a*b*B*x+1/f*B*x^2*a
*c-1/2/f^2*B*x^2*c^2*d+2/f*a*c*A*x+2/3/f*B*x^3*b*c-2/f^2*B*b*c*d*x+1/2/f^3*
ln(f*x^2+e*x+d)*B*c^2*d^2-1/2/f^4*ln(f*x^2+e*x+d)*A*c^2*e^3-1/2/f^2*ln(f*x^
2+e*x+d)*B*b^2*d+1/2/f^3*ln(f*x^2+e*x+d)*B*b^2*e^2+1/f*ln(f*x^2+e*x+d)*A*a*
b+1/2/f^5*ln(f*x^2+e*x+d)*B*c^2*e^4+1/f*A*x^2*b*c-1/2/f^2*ln(f*x^2+e*x+d)*A
*b^2*e-1/3/f^2*B*x^3*c^2*e-1/2/f^2*A*x^2*c^2*e+1/2/f^3*B*x^2*c^2*e^2+1/f^3*
c^2*A*e^2*x-1/f^2*b^2*e*B*x-1/f^4*c^2*e^3*B*x+5/f^4/(4*d*f-e^2)^(1/2)*arcta
n((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c^2*d*e^3-5/f^3/(4*d*f-e^2)^(1/2)*arctan((
2*f*x+e)/(4*d*f-e^2)^(1/2))*B*c^2*d^2*e-2/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f
*x+e)/(4*d*f-e^2)^(1/2))*e^3*B*a*c+2/f^4/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)
/(4*d*f-e^2)^(1/2))*e^4*B*b*c-4/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f
-e^2)^(1/2))*B*a*b*d+2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2)
)*A*a^2+1/2/f*ln(f*x^2+e*x+d)*B*a^2+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e
)/(4*d*f-e^2)^(1/2))*e^2*B*a*b+3/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*B*b^2*d*e+2/f^3*ln(f*x^2+e*x+d)*B*b*c*d*e+4/f^2/(4*d*f-e^2)
^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*c*d^2+2/f^2/(4*d*f-e^2)^(1/2
)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^2*A*a*c-2/f^3/(4*d*f-e^2)^(1/2)*arc
tan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^3*A*b*c-4/f/(4*d*f-e^2)^(1/2)*arctan((2*
f*x+e)/(4*d*f-e^2)^(1/2))*A*a*c*d+2/f^3*b*c*e^2*B*x+2/f^3*B*c^2*d*e*x-1/f^2
*ln(f*x^2+e*x+d)*B*a*b*e-1/f^2*ln(f*x^2+e*x+d)*B*a*c*d+1/f^3*ln(f*x^2+e*x+d
)*B*a*c*e^2-1/f^4*ln(f*x^2+e*x+d)*B*b*c*e^3-3/2/f^4*ln(f*x^2+e*x+d)*B*c^2*d
*e^2+1/f^4/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*e^4*A*c^2+
1/f^3*ln(f*x^2+e*x+d)*A*c^2*d*e-1/f^2*ln(f*x^2+e*x+d)*A*a*c*e-1/f^2*ln(f*x^
2+e*x+d)*A*b*c*d+1/f^3*ln(f*x^2+e*x+d)*A*b*c*e^2-2/f/(4*d*f-e^2)^(1/2)*arct
an((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b^2*d+2/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f
*x+e)/(4*d*f-e^2)^(1/2))*A*c^2*d^2+1/f^2/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)
/(4*d*f-e^2)^(1/2))*e^2*A*b^2-1/f/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f
-e^2)^(1/2))*e*B*a^2-1/f^3/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(
1/2))*e^3*B*b^2-1/f^5/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))
*e^5*B*c^2-1/f^2*B*x^2*b*c*e-2/f^2*b*c*A*e*x-2/f^2*c*a*e*B*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.73957, size = 3933, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(B*c^2*e^2*f^4 - 4*B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*
c + A*c^2)*d*f^5 - (4*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^
2*e^4*f^2 - 4*(B*b^2 + 2*(B*a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c +
 A*c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*
c + A*c^2)*e^3)*f^3)*x^2 - 6*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2
 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*
b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^
2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 -
(5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*sqrt(e^2 - 4*d*f)*log((2*f^2*x^2
 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))/(f*x^2 + e*x + d)
) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*(2*B*b*c + A
*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e
^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*
b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x + 6*(B*c^2*
e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*d^2 + 4*(2*B
*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B*c^2*d^3 + 8
*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 + (2*B*a*b + A
*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c^2)*d*e^3 +
(B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c + A*c^2)*e^5
)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6), 1/12*(3*(B*c^2*e^2*f^4 - 4*
B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*c + A*c^2)*d*f^5 - (4*B*c^2*
d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^2*e^4*f^2 - 4*(B*b^2 + 2*(B*
```

```
a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c + A*c^2)*d*e + (B*b^2 + 2*(B*
a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*c + A*c^2)*e^3)*f^3)*x^2 + 1
2*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*
A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e
+ (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^
2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c +
 A*c^2)*e^4)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2*f*x + e)/(
e^2 - 4*d*f)) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*
(2*B*b*c + A*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2
+ 2*A*a*c)*e^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 +
 2*(B*a + A*b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x
 + 6*(B*c^2*e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*
d^2 + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B
*c^2*d^3 + 8*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 +
(2*B*a*b + A*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c
^2)*d*e^3 + (B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c
+ A*c^2)*e^5)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15544, size = 996, normalized size = 1.84

$$\frac{3\,Bc^2f^3x^4 + 8\,Bbcf^3x^3 + 4\,Ac^2f^3x^3 - 4\,Bc^2f^2x^3e - 6\,Bc^2df^2x^2 + 6\,Bb^2f^3x^2 + 12\,Bacf^3x^2 + 12\,Abcf^3x^2 - 12\,Bbcf^2x^2e -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/12*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 4*B*c^2*f^2*x^3
*e - 6*B*c^2*d*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*
```

```
x^2 - 12*B*b*c*f^2*x^2*e - 6*A*c^2*f^2*x^2*e - 24*B*b*c*d*f^2*x - 12*A*c^2*
d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x + 6*B*c^2*f*x^2*
e^2 + 24*B*c^2*d*f*x*e - 12*B*b^2*f^2*x*e - 24*B*a*c*f^2*x*e - 24*A*b*c*f^2
*x*e + 24*B*b*c*f*x*e^2 + 12*A*c^2*f*x*e^2 - 12*B*c^2*x*e^3)/f^4 + 1/2*(B*c
^2*d^2*f^2 - B*b^2*d*f^3 - 2*B*a*c*d*f^3 - 2*A*b*c*d*f^3 + B*a^2*f^4 + 2*A*
a*b*f^4 + 4*B*b*c*d*f^2*e + 2*A*c^2*d*f^2*e - 2*B*a*b*f^3*e - A*b^2*f^3*e -
 2*A*a*c*f^3*e - 3*B*c^2*d*f*e^2 + B*b^2*f^2*e^2 + 2*B*a*c*f^2*e^2 + 2*A*b*
c*f^2*e^2 - 2*B*b*c*f*e^3 - A*c^2*f*e^3 + B*c^2*e^4)*log(f*x^2 + x*e + d)/f
^5 + (4*B*b*c*d^2*f^3 + 2*A*c^2*d^2*f^3 - 4*B*a*b*d*f^4 - 2*A*b^2*d*f^4 - 4
*A*a*c*d*f^4 + 2*A*a^2*f^5 - 5*B*c^2*d^2*f^2*e + 3*B*b^2*d*f^3*e + 6*B*a*c*
d*f^3*e + 6*A*b*c*d*f^3*e - B*a^2*f^4*e - 2*A*a*b*f^4*e - 8*B*b*c*d*f^2*e^2
 - 4*A*c^2*d*f^2*e^2 + 2*B*a*b*f^3*e^2 + A*b^2*f^3*e^2 + 2*A*a*c*f^3*e^2 +
5*B*c^2*d*f*e^3 - B*b^2*f^2*e^3 - 2*B*a*c*f^2*e^3 - 2*A*b*c*f^2*e^3 + 2*B*b
*c*f*e^4 + A*c^2*f*e^4 - B*c^2*e^5)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/(
sqrt(4*d*f - e^2)*f^5)
```

3.15 $\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$

Optimal. Leaf size=406

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(e^2-2df\right)\right)+c^2d^2\right)}+\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf-Ace+\ldots}{2\left(f\left(a^2f-abe+b^2d\right)-c\left(bde-a\left(e^2-2df\right)\right)\ldots}$$

```
[Out] -(((A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 + f*(b^2*d
- a*b*e + a^2*f) - c*(b*d*e - a*(e^2 - 2*d*f))))) + ((B*(c*d*e - 2*b*d*f +
a*e*f) - A*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^
2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 + f*(b^2*d - a*b*e + a^2*f) - c*(b
*d*e - a*(e^2 - 2*d*f)))) + ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + b*x +
c*x^2])/(2*(c^2*d^2 + f*(b^2*d - a*b*e + a^2*f) - c*(b*d*e - a*(e^2 - 2*d*f
)))) - ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[d + e*x + f*x^2])/(2*(c^2*d^2 +
 f*(b^2*d - a*b*e + a^2*f) - c*(b*d*e - a*(e^2 - 2*d*f))))
```

Rubi [A] time = 0.477553, antiderivative size = 398, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1022, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}+\frac{\log\left(a+bx+cx^2\right)(-aBf+Abf-Ace+\ldots}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(((A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 - b*c*d*e +
 f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))) + ((B*(c*d*e - 2*b*d*f +
a*e*f) - A*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^
2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2
*f) + a*c*(e^2 - 2*d*f))) + ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + b*x +
c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*
f))) - ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[d + e*x + f*x^2])/(2*(c^2*d^2 -
 b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Rule 1022

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*
(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 - b*c*d*e +
a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2]}, Dist[1/q, Int[Simp[g*c
^2*d - g*b*c*e + a*h*c*e + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d - g*c*e +
 g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[-(h*c*
d*e) + g*c*e^2 + b*h*d*f - g*c*d*f - g*b*e*f + a*g*f^2 - f*(h*c*d - g*c*e +
 g*b*f - a*h*f)*x, x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a,
b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\left(d+ex+fx^2\right)}\,dx = \frac{\int \frac{aB(ce-bf)+A\left(c^2d+b^2f-c(be+af)\right)+c(Bcd-Ace+Abf-aBf)x}{a+bx+cx^2}\,dx}{c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)} + \frac{\int \frac{-Af(be-af)+Ac\left(e^2-df\right)-B(cde-\cdots}{d+ex+\cdots}}{c^2d^2-bcde+f\left(b^2d-abe\cdots\right.}$$

$$= \frac{(Bcd-Ace+Abf-aBf)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} - \frac{(Bcd-Ace+Abf\cdots}{2\left(c^2d^2-bcde+f\left(b^2d-\cdots\right.\right.}$$

$$= \frac{(Bcd-Ace+Abf-aBf)\log\left(a+bx+cx^2\right)}{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} - \frac{(Bcd-Ace+Abf-\cdots}{2\left(c^2d^2-bcde+f\left(b^2d-\cdots\right.\right.}$$

$$= -\frac{\left(Ab^2f+2c(Acd+aBe-aAf)-b(Bcd+Ace+aBf)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} + \frac{\left(B(c\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.540324, size = 267, normalized size = 0.66

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)}{\sqrt{4ac-b^2}} - \frac{2\tan^{-1}\left(\frac{e+2fx}{\sqrt{4df-e^2}}\right)\left(A\left(-2af^2+bef+2cdf-ce^2\right)+B(aef-2bdf+cde)\right)}{\sqrt{4df-e^2}} + \log(a+x(b+\cdots}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - (2*(B*(c*d*e - 2*
b*d*f + a*e*f) + A*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2*f*
x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*c*d - A*c*e + A*b*f - a*B*f
)*Log[a + x*(b + c*x)] + (-(B*c*d) + A*c*e - A*b*f + a*B*f)*Log[d + x*(e +
f*x)])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f
)))
```

Maple [B] time = 0.314, size = 1698, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(c*x^2+b*
x+a)*A*b*f-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*
c*ln(c*x^2+b*x+a)*A*e-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*
e+c^2*d^2)*ln(c*x^2+b*x+a)*B*a*f+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2
*d*f-b*c*d*e+c^2*d^2)*c*ln(c*x^2+b*x+a)*B*d-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*
c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^
2)^(1/2))*A*a*c*f+1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*
d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b^2*f-1/(a^2*f
^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2))*A*b*c*e+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e
^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))*A*c^2*d-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*b*f+2/(a^2*f^2-
a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2))*B*a*c*e-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+
b^2*d*f-b*c*d*e+c^2*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/
2))*B*b*c*d-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)
*f*ln(f*x^2+e*x+d)*A*b+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d
*e+c^2*d^2)*ln(f*x^2+e*x+d)*A*c*e+1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^
2*d*f-b*c*d*e+c^2*d^2)*f*ln(f*x^2+e*x+d)*B*a-1/2/(a^2*f^2-a*b*e*f-2*a*c*d*f
+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*ln(f*x^2+e*x+d)*B*c*d+2/(a^2*f^2-a*b*e*f-
2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+
e)/(4*d*f-e^2)^(1/2))*A*a*f^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-
b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*b*
e*f-2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^
2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*A*c*d*f+1/(a^2*f^2-a*b*e*f-2*a
*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/
(4*d*f-e^2)^(1/2))*A*c*e^2-1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c
*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*a*e*f
+2/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^
(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2))*B*b*d*f-1/(a^2*f^2-a*b*e*f-2*a*c*
d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*
d*f-e^2)^(1/2))*B*c*d*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.16 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=1075

result too large to display

```
[Out] -((A*c*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e +
2*a*f)) + c*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B
*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(a + b*x +
 c*x^2))) - ((b^5*(B*d - A*e)*f^2 - 2*b^4*f*(B*c*d*e - A*(c*e^2 - c*d*f + a
*f^2)) - 4*c^2*(A*(c^3*d^3 - 3*a^3*f^3 - a^2*c*f*(e^2 - 7*d*f) + a*c^2*d*(3
*e^2 - 5*d*f)) - a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f))) - 4*b^2*c
*(B*c^2*d^2*e + A*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B
*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*
c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) - b^3*(A*c*e*(c*e^2 - 2*
c*d*f - 4*a*f^2) + B*(4*a*c*d*f^2 + a^2*f^3 - c^2*d*(e^2 + 5*d*f))))*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 + f*(b^2*d
- a*b*e + a^2*f) - c*(b*d*e - a*(e^2 - 2*d*f)))^2) + ((B*(c^2*d*e*(e^2 - 3*
d*f) - 2*c*d*f*(b*e^2 - 2*b*d*f - a*e*f) + f^2*(b^2*d*e - 4*a*b*d*f + a^2*e
*f)) - A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 -
b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*ArcTan
h[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 + f*(b^2*d -
a*b*e + a^2*f) - c*(b*d*e - a*(e^2 - 2*d*f)))^2) + ((A*(c*e - b*f)*(f*(b*e
- 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f)
- c^2*d*(e^2 - d*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 + f*(b^2*d - a*b*e
+ a^2*f) - c*(b*d*e - a*(e^2 - 2*d*f)))^2) - ((A*(c*e - b*f)*(f*(b*e - 2*a*
f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*
d*(e^2 - d*f)))*Log[d + e*x + f*x^2])/(2*(c^2*d^2 + f*(b^2*d - a*b*e + a^2*
f) - c*(b*d*e - a*(e^2 - 2*d*f)))^2)
```

Rubi [A] time = 4.17687, antiderivative size = 1067, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1016, 1072, 634, 618, 206, 628}

$$-\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aBe-aA\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((A*c*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e +
2*a*f)) + c*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B
*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(a + b*x +
 c*x^2))) - ((b^5*(B*d - A*e)*f^2 - 2*b^4*f*(B*c*d*e - a*A*f^2 - A*c*(e^2 -
 d*f)) - 4*c^2*(A*(c^3*d^3 - 3*a^3*f^3 - a^2*c*f*(e^2 - 7*d*f) + a*c^2*d*(3
*e^2 - 5*d*f)) - a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f))) - 4*b^2*(
B*c^3*d^2*e + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B
*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*
c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) - b^3*(A*c*e*(c*e^2 - 2*
c*d*f - 4*a*f^2) + B*(4*a*c*d*f^2 + a^2*f^3 - c^2*d*(e^2 + 5*d*f))))*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - b*c*d*e +
 f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((B*(c^2*d*e*(e^2 - 3*
d*f) - 2*c*d*f*(b*e^2 - 2*b*d*f - a*e*f) + f^2*(b^2*d*e - 4*a*b*d*f + a^2*e
*f)) - A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 -
b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*ArcTan
h[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f
*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((A*(c*e - b*f)*(f*(b*e
- 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f)
- c^2*d*(e^2 - d*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*
d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) - ((A*(c*e - b*f)*(f*(b*e - 2*a*
f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*
d*(e^2 - d*f)))*Log[d + e*x + f*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*
b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx = -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(Ab^2f+2c(\cdots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(a\cdots\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(Ab^2f+2c(\cdots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(a\cdots\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(Ab^2f+2c(\cdots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(a\cdots\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(Ab^2f+2c(\cdots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(a\cdots\right.}$$

$$= -\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(Ab^2f+2c(\cdots\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(a\cdots\right.}$$

Mathematica [A] time = 7.64068, size = 1376, normalized size = 1.28

$$\frac{-Afb^3+Aceb^2+aBfb^2-Acfxb^2-Ac^2db-aBceb+3aAcfb+Bc^2dxb+Ac^2exb+aBcfxb+2aBc^2d-2aAc^2e-2a^2B\cdots}{\left(b^2-4ac\right)\left(dfb^2-cdeb-aefb+c^2d^2+ace^2+a^2f^2-2acdf\right)\left(cx^2+bx+a\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] (-(A*b*c^2*d) + 2*a*B*c^2*d + A*b^2*c*e - a*b*B*c*e - 2*a*A*c^2*e - A*b^3*f
 + a*b^2*B*f + 3*a*A*b*c*f - 2*a^2*B*c*f + b*B*c^2*d*x - 2*A*c^3*d*x + A*b*
c^2*e*x - 2*a*B*c^2*e*x - A*b^2*c*f*x + a*b*B*c*f*x + 2*a*A*c^2*f*x)/((b^2
- 4*a*c)*(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2
*f^2)*(a + b*x + c*x^2)) + ((2*b*B*c^4*d^3 - 4*A*c^5*d^3 - 4*b^2*B*c^3*d^2*
e + 6*A*b*c^4*d^2*e + 4*a*B*c^4*d^2*e + b^3*B*c^2*d*e^2 + 2*a*b*B*c^3*d*e^2
 - 12*a*A*c^4*d*e^2 - A*b^3*c^2*e^3 + 6*a*A*b*c^3*e^3 - 4*a^2*B*c^3*e^3 + 5
*b^3*B*c^2*d^2*f - 8*A*b^2*c^3*d^2*f - 14*a*b*B*c^3*d^2*f + 20*a*A*c^4*d^2*
f - 2*b^4*B*c*d*e*f + 2*A*b^3*c^2*d*e*f + 4*a*A*b*c^3*d*e*f + 8*a^2*B*c^3*d
*e*f + 2*A*b^4*c*e^2*f - 12*a*A*b^2*c^2*e^2*f + 6*a^2*b*B*c^2*e^2*f + 4*a^2
*A*c^3*e^2*f + b^5*B*d*f^2 - 2*A*b^4*c*d*f^2 - 4*a*b^3*B*c*d*f^2 + 12*a*A*b
^2*c^2*d*f^2 + 6*a^2*b*B*c^2*d*f^2 - 28*a^2*A*c^3*d*f^2 - A*b^5*e*f^2 + 4*a
*A*b^3*c*e*f^2 + 6*a^2*A*b*c^2*e*f^2 - 12*a^3*B*c^2*e*f^2 + 2*a*A*b^4*f^3 -
```

```
 a^2*b^3*B*f^3 - 12*a^2*A*b^2*c*f^3 + 6*a^3*b*B*c*f^3 + 12*a^3*A*c^2*f^3)*A
rcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((b^2 - 4*a*c)*Sqrt[-b^2 + 4*a*c]*(c
^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)^2) +
((-(B*c^2*d*e^3) + A*c^2*e^4 + 3*B*c^2*d^2*e*f + 2*b*B*c*d*e^2*f - 4*A*c^2*
d*e^2*f - 2*A*b*c*e^3*f - 4*b*B*c*d^2*f^2 + 2*A*c^2*d^2*f^2 - b^2*B*d*e*f^2
 + 6*A*b*c*d*e*f^2 - 2*a*B*c*d*e*f^2 + A*b^2*e^2*f^2 + 2*a*A*c*e^2*f^2 - 2*
A*b^2*d*f^3 + 4*a*b*B*d*f^3 - 4*a*A*c*d*f^3 - 2*a*A*b*e*f^3 - a^2*B*e*f^3 +
 2*a^2*A*f^4)*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/(Sqrt[-e^2 + 4*d*f]*(
c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)^2) +
 ((B*c^2*d*e^2 - A*c^2*e^3 - B*c^2*d^2*f - 2*b*B*c*d*e*f + 2*A*c^2*d*e*f +
2*A*b*c*e^2*f + b^2*B*d*f^2 - 2*A*b*c*d*f^2 + 2*a*B*c*d*f^2 - A*b^2*e*f^2 -
 2*a*A*c*e*f^2 + 2*a*A*b*f^3 - a^2*B*f^3)*Log[a + b*x + c*x^2])/(2*(c^2*d^2
 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)^2) + ((-(B*
c^2*d*e^2) + A*c^2*e^3 + B*c^2*d^2*f + 2*b*B*c*d*e*f - 2*A*c^2*d*e*f - 2*A*
b*c*e^2*f - b^2*B*d*f^2 + 2*A*b*c*d*f^2 - 2*a*B*c*d*f^2 + A*b^2*e*f^2 + 2*a
*A*c*e*f^2 - 2*a*A*b*f^3 + a^2*B*f^3)*Log[d + e*x + f*x^2])/(2*(c^2*d^2 - b
*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)^2)
```

Maple [B] time = 0.312, size = 51470, normalized size = 47.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.57819, size = 4355, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3
 - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e + A*b^2*f^2*e + 2*A*a*c*f^2*
e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*log(c*x^2 + b*x + a)/(c^4*d^4
+ 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2
*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e -
2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e -
 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4*
a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a
^2*b*c*f*e^3 + a^2*c^2*e^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^
2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3 + 2*B*b*c*d*f*e - 2*A*c^2*d*f*e
 + A*b^2*f^2*e + 2*A*a*c*f^2*e - B*c^2*d*e^2 - 2*A*b*c*f*e^2 + A*c^2*e^3)*l
```

og(f*x^2 + x*e + d)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^
2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3
 + a^4*f^4 - 2*b*c^3*d^3*e - 2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*
d*f^2*e + 2*a^2*b*c*d*f^2*e - 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2
*e^2 + 4*a*b^2*c*d*f*e^2 - 4*a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^
2*e^2 - 2*a*b*c^2*d*e^3 - 2*a^2*b*c*f*e^3 + a^2*c^2*e^4) + (2*B*b*c^4*d^3 -
 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f - 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f +
 20*A*a*c^4*d^2*f + B*b^5*d*f^2 - 4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B
*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^
3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3
 - 4*B*b^2*c^3*d^2*e + 4*B*a*c^4*d^2*e + 6*A*b*c^4*d^2*e - 2*B*b^4*c*d*f*e
+ 2*A*b^3*c^2*d*f*e + 8*B*a^2*c^3*d*f*e + 4*A*a*b*c^3*d*f*e - A*b^5*f^2*e +
 4*A*a*b^3*c*f^2*e - 12*B*a^3*c^2*f^2*e + 6*A*a^2*b*c^2*f^2*e + B*b^3*c^2*d
*e^2 + 2*B*a*b*c^3*d*e^2 - 12*A*a*c^4*d*e^2 + 2*A*b^4*c*f*e^2 + 6*B*a^2*b*c
^2*f*e^2 - 12*A*a*b^2*c^2*f*e^2 + 4*A*a^2*c^3*f*e^2 - A*b^3*c^2*e^3 - 4*B*a
^2*c^3*e^3 + 6*A*a*b*c^3*e^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*
c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f + 16*a^2*c^4*d
^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^2 - 24*a^3*c^
3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*d*f^3 + a^4*b
^2*f^4 - 4*a^5*c*f^4 - 2*b^3*c^3*d^3*e + 8*a*b*c^4*d^3*e - 2*b^5*c*d^2*f*e
+ 10*a*b^3*c^2*d^2*f*e - 8*a^2*b*c^3*d^2*f*e - 2*a*b^5*d*f^2*e + 10*a^2*b^3
*c*d*f^2*e - 8*a^3*b*c^2*d*f^2*e - 2*a^3*b^3*f^3*e + 8*a^4*b*c*f^3*e + b^4*
c^2*d^2*e^2 - 2*a*b^2*c^3*d^2*e^2 - 8*a^2*c^4*d^2*e^2 + 4*a*b^4*c*d*f*e^2 -
 20*a^2*b^2*c^2*d*f*e^2 + 16*a^3*c^3*d*f*e^2 + a^2*b^4*f^2*e^2 - 2*a^3*b^2*
c*f^2*e^2 - 8*a^4*c^2*f^2*e^2 - 2*a*b^3*c^2*d*e^3 + 8*a^2*b*c^3*d*e^3 - 2*a
^2*b^3*c*f*e^3 + 8*a^3*b*c^2*f*e^3 + a^2*b^2*c^2*e^4 - 4*a^3*c^3*e^4)*sqrt(
-b^2 + 4*a*c)) - (4*B*b*c*d^2*f^2 - 2*A*c^2*d^2*f^2 - 4*B*a*b*d*f^3 + 2*A*b
^2*d*f^3 + 4*A*a*c*d*f^3 - 2*A*a^2*f^4 - 3*B*c^2*d^2*f*e + B*b^2*d*f^2*e +
2*B*a*c*d*f^2*e - 6*A*b*c*d*f^2*e + B*a^2*f^3*e + 2*A*a*b*f^3*e - 2*B*b*c*d
*f*e^2 + 4*A*c^2*d*f*e^2 - A*b^2*f^2*e^2 - 2*A*a*c*f^2*e^2 + B*c^2*d*e^3 +
2*A*b*c*f*e^3 - A*c^2*e^4)*arctan((2*f*x + e)/sqrt(4*d*f - e^2))/((c^4*d^4
+ 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2
*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4 - 2*b*c^3*d^3*e -
2*b^3*c*d^2*f*e + 2*a*b*c^2*d^2*f*e - 2*a*b^3*d*f^2*e + 2*a^2*b*c*d*f^2*e -
 2*a^3*b*f^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 + 4*a*b^2*c*d*f*e^2 - 4*
a^2*c^2*d*f*e^2 + a^2*b^2*f^2*e^2 + 2*a^3*c*f^2*e^2 - 2*a*b*c^2*d*e^3 - 2*a
^2*b*c*f*e^3 + a^2*c^2*e^4)*sqrt(4*d*f - e^2)) + (2*B*a*c^4*d^3 - A*b*c^4*d
^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f + 5*A*a*b*
c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A*a*b^3*c
*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 - A*a^2*b^
3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 - 3*B*a*b*c^3*d^2*e + 2*A*b^2*c^3*d
^2*e - 2*A*a*c^4*d^2*e - 2*B*a*b^3*c*d*f*e + 2*A*b^4*c*d*f*e + 2*B*a^2*b*c^
2*d*f*e - 6*A*a*b^2*c^2*d*f*e + 4*A*a^2*c^3*d*f*e - B*a^2*b^3*f^2*e + A*a*b
^4*f^2*e + B*a^3*b*c*f^2*e - 2*A*a^2*b^2*c*f^2*e - 2*A*a^3*c^2*f^2*e + B*a*
b^2*c^2*d*e^2 - A*b^3*c^2*d*e^2 + 2*B*a^2*c^3*d*e^2 + A*a*b*c^3*d*e^2 + 2*B

```
*a^2*b^2*c*f*e^2 - 2*A*a*b^3*c*f*e^2 - 2*B*a^3*c^2*f*e^2 + 5*A*a^2*b*c^2*f*
e^2 - B*a^2*b*c^2*e^3 + A*a*b^2*c^2*e^3 - 2*A*a^2*c^3*e^3 + (B*b*c^4*d^3 -
2*A*c^5*d^3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a
*c^4*d^2*f + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^
2*c^2*d*f^2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3
*c^2*f^3 - B*b^2*c^3*d^2*e - 2*B*a*c^4*d^2*e + 3*A*b*c^4*d^2*e - 4*B*a*b^2*
c^2*d*f*e + 2*A*b^3*c^2*d*f*e + 4*B*a^2*c^3*d*f*e - 2*A*a*b*c^3*d*f*e - B*a
^2*b^2*c*f^2*e + A*a*b^3*c*f^2*e - 2*B*a^3*c^2*f^2*e - A*a^2*b*c^2*f^2*e +
3*B*a*b*c^3*d*e^2 - A*b^2*c^3*d*e^2 - 2*A*a*c^4*d*e^2 + 3*B*a^2*b*c^2*f*e^2
 - 2*A*a*b^2*c^2*f*e^2 + 2*A*a^2*c^3*f*e^2 - 2*B*a^2*c^3*e^3 + A*a*b*c^3*e^
3)*x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2 - b*c*d*e - a*b*f*e + a*c*e
^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

3.17 $\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2} dx$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)}-\frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d^2*(a + b*x + c*x^2)^2)
+ (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d^2*(a + b*x + c*x^2)) -
 (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^
(5/2)*d^2)
```

Rubi [A] time = 0.13148, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)}-\frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```


```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d^2*(a + b*x + c*x^2)^2)
+ (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d^2*(a + b*x + c*x^2)) -
 (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^
(5/2)*d^2)
```


Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = \frac{\int \frac{g+hx}{(a+bx+cx^2)^3}\,dx}{d^2}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2(b^2-4ac)d^2(a+bx+cx^2)^2} - \frac{(3(2cg-bh))\int \frac{1}{(a+bx+cx^2)^2}\,dx}{2(b^2-4ac)d^2}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2(b^2-4ac)d^2(a+bx+cx^2)^2} + \frac{3(2cg-bh)(b+2cx)}{2(b^2-4ac)^2d^2(a+bx+cx^2)} + \frac{(3c(2cg}{}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2(b^2-4ac)d^2(a+bx+cx^2)^2} + \frac{3(2cg-bh)(b+2cx)}{2(b^2-4ac)^2d^2(a+bx+cx^2)} - \frac{(6c(2cg}{}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2(b^2-4ac)d^2(a+bx+cx^2)^2} + \frac{3(2cg-bh)(b+2cx)}{2(b^2-4ac)^2d^2(a+bx+cx^2)} - \frac{6c(2cg}{}$$

Mathematica [A] time = 0.153385, size = 131, normalized size = 0.94

$$\frac{\frac{(b^2-4ac)(2ah-bg+bhx-2cgx)}{(a+x(b+cx))^2} - \frac{12c(bh-2cg)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d^2(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (
3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d
^2)
```

Maple [B] time = 0.161, size = 340, normalized size = 2.4

$$-\frac{bxh}{2d^2(4ac-b^2)(cx^2+bx+a)^2} + \frac{cxg}{d^2(4ac-b^2)(cx^2+bx+a)^2} - \frac{ah}{d^2(4ac-b^2)(cx^2+bx+a)^2} + \frac{bg}{2d^2(4ac-b^2)(cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x)
```

```
[Out] -1/2/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^
2*x*c*g-1/d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d^2/(4*a*c-b^2)/(c*x^2+b*
x+a)^2*b*g-3/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*x*c*b*h+6/d^2/(4*a*c-b^2)^2/(c
*x^2+b*x+a)*x*c^2*g-3/2/d^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d^2/(4*a*c-
b^2)^2/(c*x^2+b*x+a)*b*c*g-6/d^2/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*
c-b^2)^(1/2))*b*h+12/d^2/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)
^(1/2))*g
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93127, size = 2399, normalized size = 17.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3
*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b
*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^
2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g -
 (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^
3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
```

```
 - 64*a^3*b*c^4)*d^2*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*
c^3 - 128*a^4*c^4)*d^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d^2*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d
^2), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*
(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g -
a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b
^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c
*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*
c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*
b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^
4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^3 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x^2 + 2*
(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2*x + (a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d^2)]
```

Sympy [B] time = 3.77541, size = 709, normalized size = 5.06

$$\frac{3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)+144a^2b^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)-36ab^4c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)+3b^6c\sqrt{-\frac{1}{\left(4ac-\right.}}}{6bc^2h-12c^3g}\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x**2+b*x+a)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*
c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b
*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d**2 - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b
*h - 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)
 - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*
c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b
**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))
/d**2 - (8*a**2*c*h + a*b**2*h - 10*a*b*c*g + b**3*g + x**3*(6*b*c**2*h - 1
2*c**3*g) + x**2*(9*b**2*c*h - 18*b*c**2*g) + x*(10*a*b*c*h - 20*a*c**2*g +
 2*b**3*h - 4*b**2*c*g))/(32*a**4*c**2*d**2 - 16*a**3*b**2*c*d**2 + 2*a**2*
b**4*d**2 + x**4*(32*a**2*c**4*d**2 - 16*a*b**2*c**3*d**2 + 2*b**4*c**2*d**
2) + x**3*(64*a**2*b*c**3*d**2 - 32*a*b**3*c**2*d**2 + 4*b**5*c*d**2) + x**
```

```
2*(64*a**3*c**3*d**2 - 12*a*b**4*c*d**2 + 2*b**6*d**2) + x*(64*a**3*b*c**2*
d**2 - 32*a**2*b**3*c*d**2 + 4*a*b**5*d**2))
```

Giac [A] time = 1.208, size = 296, normalized size = 2.11

$$\frac{6\left(2\,c^2g - bch\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4d^2 - 8\,ab^2cd^2 + 16\,a^2c^2d^2\right)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3gx^3 - 6\,bc^2hx^3 + 18\,bc^2gx^2 - 9\,b^2chx^2 + 4\,b^2cgx + 20\,ac^2gx - 2\,b^3hx - }{2\left(b^4d^2 - 8\,ab^2cd^2 + 16\,a^2c^2d^2\right)\left(cx^2 + bx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="giac")
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d^2 - 8*a*
b^2*c*d^2 + 16*a^2*c^2*d^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c
^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*
b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d^
2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*(c*x^2 + b*x + a)^2)
```

3.18 $\int \frac{g+hx}{(a+bx+cx^2)^2 (ad+bdx+cdx^2)} dx$

Optimal. Leaf size=140

$$\frac{3(b+2cx)(2cg-bh)}{2d(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d(b^2-4ac)^{5/2}}$$

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) +
(3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) - (6*
c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2
)*d)
```

Rubi [A] time = 0.103563, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {998, 638, 614, 618, 206}

$$\frac{3(b+2cx)(2cg-bh)}{2d(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d(b^2-4ac)(a+bx+cx^2)^2} - \frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] -(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*(b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2) +
(3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) - (6*
c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2
)*d)
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{g+hx}{\left(a+bx+cx^2\right)^2\left(ad+bdx+cdx^2\right)}\,dx = \frac{\int \frac{g+hx}{\left(a+bx+cx^2\right)^3}\,dx}{d}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} - \frac{(3(2cg-bh))\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)d}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} + \frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)} + \frac{(3c(2cg - \cdots}{(\cdots}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} + \frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)} - \frac{(6c(2cg - \cdots}{\cdots}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d\left(a+bx+cx^2\right)^2} + \frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d\left(a+bx+cx^2\right)} - \frac{6c(2cg - \cdots}{(\cdots}$$

Mathematica [A] time = 0.027777, size = 131, normalized size = 0.94

$$\frac{\frac{\left(b^2-4ac\right)(2ah-bg+bhx-2cgx)}{(a+x(b+cx))^2} - \frac{12c(bh-2cg)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \frac{3(b+2cx)(2cg-bh)}{a+x(b+cx)}}{2d\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (
3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d
)
```

Maple [B] time = 0.167, size = 340, normalized size = 2.4

$$-\frac{bxh}{2d\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2} + \frac{cxg}{d\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2} - \frac{ah}{d\left(4ac-b^2\right)\left(cx^2+bx+a\right)^2} + \frac{bg}{2d\left(4ac-b^2\right)\left(cx^2+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x)
```

```
[Out] -1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*b*h+1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*x*
c*g-1/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*a*h+1/2/d/(4*a*c-b^2)/(c*x^2+b*x+a)^2*b
*g-3/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*x*c*b*h+6/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*
x*c^2*g-3/2/d/(4*a*c-b^2)^2/(c*x^2+b*x+a)*b^2*h+3/d/(4*a*c-b^2)^2/(c*x^2+b*
x+a)*b*c*g-6/d/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+
12/d/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*g
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.93572, size = 2372, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="fricas"
)
```

```
[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3
*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b
*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^
2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g -
 (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^
3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
```

```
- 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*
c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d), 1/2*(
6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 -
 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g - a^2*b*c*h
+ (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2
*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^
5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2
*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*
x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c
 - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2 - 64*a^5*c^3)*d)]
```

Sympy [B] time = 3.19199, size = 680, normalized size = 4.86

$$\frac{3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)+144a^2b^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)-36ab^4c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(bh-2cg\right)+3b^6c\sqrt{-\frac{1}{\left(4ac-\cdots\right.}}}{6bc^2h-12c^3g}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x**2+b*x+a)**2/(c*d*x**2+b*d*x+a*d),x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*
c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b
*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h
- 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) -
144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*c**
2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b**2
)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d
- (8*a**2*c*h + a*b**2*h - 10*a*b*c*g + b**3*g + x**3*(6*b*c**2*h - 12*c**3
*g) + x**2*(9*b**2*c*h - 18*b*c**2*g) + x*(10*a*b*c*h - 20*a*c**2*g + 2*b**
3*h - 4*b**2*c*g))/(32*a**4*c**2*d - 16*a**3*b**2*c*d + 2*a**2*b**4*d + x**
4*(32*a**2*c**4*d - 16*a*b**2*c**3*d + 2*b**4*c**2*d) + x**3*(64*a**2*b*c**
3*d - 32*a*b**3*c**2*d + 4*b**5*c*d) + x**2*(64*a**3*c**3*d - 12*a*b**4*c*d
```

```
+ 2*b**6*d) + x*(64*a**3*b*c**2*d - 32*a**2*b**3*c*d + 4*a*b**5*d))
```

Giac [A] time = 1.15878, size = 279, normalized size = 1.99

$$\frac{6\left(2\,c^2g-bch\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4d-8\,ab^2cd+16\,a^2c^2d\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3gx^3-6\,bc^2hx^3+18\,bc^2gx^2-9\,b^2chx^2+4\,b^2cgx+20\,ac^2gx-2\,b^3hx-10\,a}{2\left(b^4d-8\,ab^2cd+16\,a^2c^2d\right)\left(cx^2+bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="giac")
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d - 8*a*b^
2*c*d + 16*a^2*c^2*d)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c^2*h*x
^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3*h*
x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d - 8*a*
b^2*c*d + 16*a^2*c^2*d)*(c*x^2 + b*x + a)^2)
```

3.19 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=617

$$\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(B\left(f(be-af)-c\left(e^2-df\right)\right)+Af(ce-bf)\right)\right)\tanh^{-1}\left(\frac{4af+2x(bf\ldots}{2\sqrt{2}\sqrt{a+bx+cx^2}\ldots}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] (B*Sqrt[a + b*x + c*x^2])/f - ((2*B*c*e - b*B*f - 2*A*c*f)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) + ((2*f*(A*f*(c*d -
 a*f) - B*d*(c*e - b*f)) - (e - Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f) + B*(f*
(b*e - a*f) - c*(e^2 - d*f))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e
 + Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f) + B*(f*(b*e - a*f) - c*(e^2 - d*f)))
)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4
*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 8.99756, antiderivative size = 615, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x(bf-\ldots}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{\ldots}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (B*Sqrt[a + b*x + c*x^2])/f - ((2*B*c*e - b*B*f - 2*A*c*f)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) + ((2*f*(A*f*(c*d -
 a*f) - B*d*(c*e - b*f)) - (e - Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(
c*e - b*f) - B*c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
```

```
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e
 + Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))
)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4
*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
```

```
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{1}{2}(bBd-2aAf)-\frac{1}{2}(2Abf-B(2cd+be-2af))x+\frac{1}{2}(2Bce-bBf-2Acf)x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{f}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{1}{2}f(bBd-2aAf)-\frac{1}{2}d(2Bce-bBf-2Acf)+\left(-\frac{1}{2}e(2Bce-bBf-2Acf)+\frac{1}{2}f(-2Abf+B(2cd+b}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}}{f^2}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} + \frac{\left(2f(Af(c\right.}{}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{\left(2\left(2f(Af(cd-af)\right.\right.}{}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} + \frac{\left(2f(Af(cd-af)-\right.}{}$$

Mathematica [A] time = 2.15169, size = 517, normalized size = 0.84

$$\frac{-\sqrt{2}\left(B\left(\sqrt{e^2-4df}+e\right)-2Af\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-\right.}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f(2a}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] ((-2*B*c*e + b*B*f + 2*A*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])])/(2*Sqrt[c]*f^2) + (4*B*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)]
 - Sqrt[2]*(-2*A*f + B*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2
*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2
]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 -
 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[2]*(2*A*f + B*(-e + Sqrt[e^2 - 4*
d*f]))*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt
[e^2 - 4*d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e +
Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(
4*f^2*Sqrt[e^2 - 4*d*f])
```

Maple [B] time = 0.309, size = 16209, normalized size = 26.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((A + B*x)*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.20 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=1092

result too large to display

```
[Out] -((2*A*c*f*(4*c*e - 5*b*f) - B*(b^2*f^2 - 2*c*f*(5*b*e - 4*a*f) + 8*c^2*(e^
2 - d*f)) + 2*c*f*(2*B*c*e - b*B*f - 2*A*c*f)*x)*Sqrt[a + b*x + c*x^2])/(8*
c*f^3) + (B*(a + b*x + c*x^2)^(3/2))/(3*f) + ((2*A*c*f*(3*b^2*f^2 - 12*c*f*
(b*e - a*f) + 8*c^2*(e^2 - d*f)) - B*(b^3*f^3 + 6*b*c*f^2*(b*e - 2*a*f) - 2
4*c^2*f*(b*e^2 - b*d*f - a*e*f) + 16*c^3*(e^3 - 2*d*e*f)))*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^4) - ((2*c*f*(B*d*(c*
e - b*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f
^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - c*(e - Sqrt[e^2 - 4*d*f])*(A*f*(
c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f +
d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 -
d*f) - b*(e^3 - 2*d*e*f)))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*c*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*f*(B*d*(c*e - b*f)*(c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2
- d*f))) - (e + Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e
^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^
2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f)))))*ArcTa
nh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))
*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^
2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 18.8666, antiderivative size = 1092, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1019, 1066, 1076, 621, 206, 1032, 724}

$$\frac{B\left(cx^2+bx+a\right)^{3/2}}{3f}-\frac{\left(2Acf(4ce-5bf)-B\left(8\left(e^2-df\right)c^2-2f(5be-4af)c+b^2f^2\right)+2cf(2Bce-bBf-2Acf)x\right)\sqrt{cx^2}}{8cf^3}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -((2*A*c*f*(4*c*e - 5*b*f) - B*(b^2*f^2 - 2*c*f*(5*b*e - 4*a*f) + 8*c^2*(e^
2 - d*f)) + 2*c*f*(2*B*c*e - b*B*f - 2*A*c*f)*x)*Sqrt[a + b*x + c*x^2])/(8*
c*f^3) + (B*(a + b*x + c*x^2)^(3/2))/(3*f) + ((2*A*c*f*(3*b^2*f^2 - 12*c*f*
(b*e - a*f) + 8*c^2*(e^2 - d*f)) - B*(b^3*f^3 + 6*b*c*f^2*(b*e - 2*a*f) - 2
4*c^2*f*(b*e^2 - b*d*f - a*e*f) + 16*c^3*(e^3 - 2*d*e*f)))*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^4) - ((2*c*f*(B*d*(c*
e - b*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f
^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - c*(e - Sqrt[e^2 - 4*d*f])*(A*f*(
c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f +
d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 -
d*f) - b*(e^3 - 2*d*e*f)))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*c*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*f*(B*d*(c*e - b*f)*(c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^2
- d*f))) - (e + Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e
^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^
2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f)))))*ArcTa
nh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))
*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^
2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
```

```
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3}{2}(bBd-2aAf)-\frac{3}{2}(2Abf-B(2cd+be-2af))x+\frac{3}{2}(2Bce-bBf-2Acf)x}{d+ex+fx^2}}{3f}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce-bBf}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce-bBf}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce-bBf}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce-bBf}{8cf^3}$$

$$= -\frac{\left(2Acf(4ce-5bf)-B\left(b^2f^2-2cf(5be-4af)+8c^2\left(e^2-df\right)\right)+2cf(2Bce-bBf}{8cf^3}$$

Mathematica [A] time = 6.53623, size = 1627, normalized size = 1.49

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] ((B - (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) + ((B
 + (B*e - 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2))/(6*f) - ((B +
(-(B*e) + 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x*(b + c*x))^(3/2)*(((4*c*f*(-4*a*
f + b*(e - Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(-(b*f
) + 2*c*(e - Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*
```

```
x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2 - 4*d*f])
)*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(
e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e
*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(4*(e - Sqrt[e^2 - 4*d*f])*(b*f
 - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 -
4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*f*(4*a*f -
b*(e - Sqrt[e^2 - 4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e
^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(
4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f
] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e
 - Sqrt[e^2 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/(4*f*
(a + b*x + c*x^2)^(3/2)) - ((B - (-(B*e) + 2*A*f)/Sqrt[e^2 - 4*d*f])*(a + x
*(b + c*x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f -
c*(e + Sqrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2
*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]
]*(4*(e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4
*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 -
 4*d*f]))) + 4*f*(2*c*f*(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^2 - (e + Sqrt[e
^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e
+ Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*
f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2
- 4*d*f])^2)))/(16*c*f^2)))/(4*f*(a + b*x + c*x^2)^(3/2))
```

Maple [B] time = 0.306, size = 59465, normalized size = 54.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.21 $\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=416

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}}}$$

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(4*c*d - (b - Sqrt[b^2 - 4*a*c
])*e + 2*(c*e - (b - Sqrt[b^2 - 4*a*c])*f)*x)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c
*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2]
)])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqr
t[b^2 - 4*a*c]*(c*e - b*f)]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh
[(4*c*d - (b + Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b + Sqrt[b^2 - 4*a*c])*f)*x
)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*
e - b*f)]*Sqrt[d + e*x + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d
- b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)])
```

Rubi [A] time = 2.70211, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1032, 724, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\right.}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(4*c*d - (b - Sqrt[b^2 - 4*a*c
])*e + 2*(c*e - (b - Sqrt[b^2 - 4*a*c])*f)*x)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c
*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2]
)])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqr
t[b^2 - 4*a*c]*(c*e - b*f)]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh
[(4*c*d - (b + Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b + Sqrt[b^2 - 4*a*c])*f)*x
)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*
e - b*f)]*Sqrt[d + e*x + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d
```

```
- b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+ex+fx^2}}\,dx = \frac{\left(2Ac-B\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex+fx^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2Ac-B\left(b+\sqrt{b^2-\right.\right.}{}$$

$$= \frac{\left(2\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{16c^2d-8c\left(b-\sqrt{b^2-4ac}\right)e+4\left(b-\sqrt{b^2-4ac}\right)^2f-x^2}\,dx,x\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{4cd-\left(b-\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b-\sqrt{b^2-4ac}\right)f\right)x}{2\sqrt{2}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}(ce-bf)}\sqrt{d+ex+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}(ce-bf)}}$$

Mathematica [A] time = 4.23536, size = 393, normalized size = 0.94

$$\frac{-\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tanh^{-1}\left(\frac{\left(\sqrt{b^2-4ac}-b\right)(e+2fx)+2c(2d+ex)}{2\sqrt{2}\sqrt{d+x(e+fx)}\sqrt{c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d}}\right)}{\sqrt{c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d}}-\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2c(2d+ex)-\left(\sqrt{b^2-4ac}+b\right)(e \cdots}{2\sqrt{d+x(e+fx)}\sqrt{-2c\left(e\sqrt{b^2-4ac}+2af+be\right)+2 \cdots}}\right)}{\sqrt{-c\left(e\sqrt{b^2-4ac}+2af+be\right)+bf\left(\sqrt{b^2-4ac}+b\right)+2c^2 \cdots}}}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (-(((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*(2*d + e*x) + (-b +
 Sqrt[b^2 - 4*a*c])*(e + 2*f*x))/(2*Sqrt[2]*Sqrt[2*c^2*d + b*(b - Sqrt[b^2
- 4*a*c])*f + c*(-(b*e) + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[d + x*(e + f*x
)])])/Sqrt[2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*e) + Sqrt[b^2 - 4
*a*c]*e - 2*a*f)]) - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*(2*d
 + e*x) - (b + Sqrt[b^2 - 4*a*c])*(e + 2*f*x))/(2*Sqrt[4*c^2*d + 2*b*(b + S
qrt[b^2 - 4*a*c])*f - 2*c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)]*Sqrt[d + x*(
e + f*x)])])/Sqrt[2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2
 - 4*a*c]*e + 2*a*f)])/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.431, size = 2269, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -2/(-4*a*c+b^2)^(1/2)/(-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*
a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+
b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-
c*e)/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^2)^(1/2)-(-
4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*(-
b+(-4*a*c+b^2)^(1/2)))^2*f-4*(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*
c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))*A-1
/c/(-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2
*c^2*d)/c^2)^(1/2)*ln((-(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*
c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/c*(-b
```

```
+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*
e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))^2*f-4*(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
))-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c
^2*d)/c^2)^(1/2))/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))*B+1/(-4*a*c+b^2)^(1/2)
/c/(-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2
*c^2*d)/c^2)^(1/2)*ln((-(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*
c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*
e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))^2*f-4*(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
))-2*(b*f*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c
^2*d)/c^2)^(1/2))/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))*B*b+2/(-4*a*c+b^2)^(1/
2)/(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-
2*c^2*d)/c^2)^(1/2)*ln((-(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*
a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(
-4*a*c+b^2)^(1/2))/c)+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c
*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)^2*f-4*(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^
2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*A-1/c/(-2*(-b*f*(-4*a*c+
b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*l
n((-(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c
^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*
e-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*(f*(-4*a*c+
b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-b*f*(-4*a*c+b^2)
^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x+1
/2*(b+(-4*a*c+b^2)^(1/2))/c))*B-1/(-4*a*c+b^2)^(1/2)/c/(-2*(-b*f*(-4*a*c+b^
2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln(
(-(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2
*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1
/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-
2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*(f*(-4*a*c+b^
2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-b*f*(-4*a*c+b^2)^(
1/2)+(-4*a*c+b^2)^(1/2)*c*e+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x+1/2
*(b+(-4*a*c+b^2)^(1/2))/c))*B*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.22 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$

Optimal. Leaf size=780

$$\frac{\sqrt{A\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\tanh^{-1}\left(\cdots\right.}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-2df\right)+c\cdots}}$$

```
[Out] (Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*
Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(
e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a*B*e + A*(c*d - a*f - Sq
rt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f +
Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sqr
t[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]) -
 (Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)
])]*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]
)]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c
*(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[-(A*c*e) + B*(c*d - a*f
- Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c*d - a*f +
 Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sq
rt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
```

Rubi [A] time = 5.16229, antiderivative size = 780, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1036, 1030, 208}

$$\frac{\sqrt{A\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\tanh^{-1}\left(\cdots\right.}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+ac\left(e^2-2df\right)+c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*
Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(
e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a*B*e + A*(c*d - a*f - Sq
rt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f +
Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sqr
t[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]) -
 (Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)
])]*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]
)]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c
*(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[-(A*c*e) + B*(c*d - a*f
- Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c*d - a*f +
 Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sq
rt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+cx^2\right)\sqrt{d+ex+fx^2}}\,dx = -\frac{\int \frac{-aBe-A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)+\left(-Ace+B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)x}{\left(a+cx^2\right)\sqrt{d+ex+fx^2}}\,dx}{2\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}} + \frac{\int \frac{-aB}{}}{}$$

$$= \frac{\left(a\left(Ace-B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\right)\left(aBe+A\left(cd-af+\sqrt{c^2d^2+}\right.\right.}{}$$

$$= \frac{\sqrt{aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)}\sqrt{-Ace+B\left(cd-af+\sqrt{c^2d^2+}\right.}}{\sqrt{2}\sqrt{}}$$

Mathematica [A] time = 0.43835, size = 254, normalized size = 0.33

$$\frac{\frac{\left(A\sqrt{c}-\sqrt{-a}B\right)\tanh^{-1}\left(\frac{\sqrt{c}(2d+ex)-\sqrt{-a}(e+2fx)}{2\sqrt{d+x(e+fx)}\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{-\sqrt{-a}\sqrt{c}e-af+cd}}-\frac{\left(\sqrt{-a}B+A\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{-a}(e+2fx)+\sqrt{c}(2d+ex)}{2\sqrt{d+x(e+fx)}\sqrt{\sqrt{-a}\sqrt{c}e-af+cd}}\right)}{\sqrt{\sqrt{-a}\sqrt{c}e-af+cd}}}{2\sqrt{-a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (((-(Sqrt[-a]*B) + A*Sqrt[c])*ArcTanh[(Sqrt[c]*(2*d + e*x) - Sqrt[-a]*(e +
2*f*x))/(2*Sqrt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f]*Sqrt[d + x*(e + f*x)])]])/Sq
rt[c*d - Sqrt[-a]*Sqrt[c]*e - a*f] - ((Sqrt[-a]*B + A*Sqrt[c])*ArcTanh[(Sqr
t[c]*(2*d + e*x) + Sqrt[-a]*(e + 2*f*x))/(2*Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e -
 a*f]*Sqrt[d + x*(e + f*x)])]])/Sqrt[c*d + Sqrt[-a]*Sqrt[c]*e - a*f])/(2*Sqr
t[-a]*Sqrt[c])
```

Maple [A] time = 0.355, size = 784, normalized size = 1.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -1/2/(-a*c)^(1/2)/(-(-(-a*c)^(1/2)*e+a*f-c*d)/c)^(1/2)*ln((-2*(-(-a*c)^(1/2
)*e+a*f-c*d)/c+(2*f*(-a*c)^(1/2)+c*e)/c*(x-1/c*(-a*c)^(1/2))+2*(-(-(-a*c)^(
1/2)*e+a*f-c*d)/c)^(1/2)*((x-1/c*(-a*c)^(1/2))^2*f+(2*f*(-a*c)^(1/2)+c*e)/c
*(x-1/c*(-a*c)^(1/2))-(-(-a*c)^(1/2)*e+a*f-c*d)/c)^(1/2))/(x-1/c*(-a*c)^(1/
2)))*A-1/2/c/(-(-(-a*c)^(1/2)*e+a*f-c*d)/c)^(1/2)*ln((-2*(-(-a*c)^(1/2)*e+a
*f-c*d)/c+(2*f*(-a*c)^(1/2)+c*e)/c*(x-1/c*(-a*c)^(1/2))+2*(-(-(-a*c)^(1/2)*
e+a*f-c*d)/c)^(1/2)*((x-1/c*(-a*c)^(1/2))^2*f+(2*f*(-a*c)^(1/2)+c*e)/c*(x-1
/c*(-a*c)^(1/2))-(-(-a*c)^(1/2)*e+a*f-c*d)/c)^(1/2))/(x-1/c*(-a*c)^(1/2)))*
B+1/2/(-a*c)^(1/2)/(-((-a*c)^(1/2)*e+a*f-c*d)/c)^(1/2)*ln((-2*((-a*c)^(1/2)
*e+a*f-c*d)/c+1/c*(-2*f*(-a*c)^(1/2)+c*e)*(x+1/c*(-a*c)^(1/2))+2*(-((-a*c)^
(1/2)*e+a*f-c*d)/c)^(1/2)*((x+1/c*(-a*c)^(1/2))^2*f+1/c*(-2*f*(-a*c)^(1/2)+
c*e)*(x+1/c*(-a*c)^(1/2))-((-a*c)^(1/2)*e+a*f-c*d)/c)^(1/2))/(x+1/c*(-a*c)^
(1/2)))*A-1/2/c/(-((-a*c)^(1/2)*e+a*f-c*d)/c)^(1/2)*ln((-2*((-a*c)^(1/2)*e+
a*f-c*d)/c+1/c*(-2*f*(-a*c)^(1/2)+c*e)*(x+1/c*(-a*c)^(1/2))+2*(-((-a*c)^(1/
2)*e+a*f-c*d)/c)^(1/2)*((x+1/c*(-a*c)^(1/2))^2*f+1/c*(-2*f*(-a*c)^(1/2)+c*e
)*(x+1/c*(-a*c)^(1/2))-((-a*c)^(1/2)*e+a*f-c*d)/c)^(1/2))/(x+1/c*(-a*c)^(1/
2)))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.23 $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx$

Optimal. Leaf size=302

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{2cd\cdots}{\sqrt{2}\sqrt{d+fx^2}\sqrt{\cdots}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-\cdots}}$$

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f]*Sqrt
[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - S
qrt[b^2 - 4*a*c])*f]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sq
rt[b^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^
2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
```

Rubi [A] time = 0.842636, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1034, 725, 206}

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\tanh^{-1}\left(\frac{2cd\cdots}{\sqrt{2}\sqrt{d+fx^2}\sqrt{\cdots}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f]*Sqrt
[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - S
qrt[b^2 - 4*a*c])*f]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sq
rt[b^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^
2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)\sqrt{d+fx^2}}\,dx = \frac{\left(2Ac-B\left(b-\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+fx^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2Ac-B\left(b+\sqrt{b^2-4ac}\right)\right)\int}{\sqrt{b^2-4}}$$

$$= -\frac{\left(2Ac-B\left(b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{4c^2d+\left(b-\sqrt{b^2-4ac}\right)^2f-x^2}\,dx, x, \frac{2cd-\left(b-\sqrt{b^2-4ac}\right)fx}{\sqrt{d+fx^2}}\right)}{\sqrt{b^2-4ac}} +$$

$$= \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)fx}{\sqrt{2}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}\sqrt{d+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}} + \frac{\left(2Ac-B\left(b+\right.\right.}{\sqrt{2}}$$

Mathematica [A] time = 0.49046, size = 283, normalized size = 0.94

$$\frac{\sqrt{2}\left(-\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tanh^{-1}\left(\frac{fx\left(\sqrt{b^2-4ac}-b\right)+2cd}{\sqrt{d+fx^2}\sqrt{2bf\left(b-\sqrt{b^2-4ac}\right)-4acf+4c^2d}}\right)}{2\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}-\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tanh^{-1}\left(\frac{2cd-fx\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{d+fx^2}\sqrt{2bf\left(\sqrt{b^2-4ac}+b\right)-4acf+4c^2d}}\right)}{2\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (Sqrt[2]*(-((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d + (-b + S
qrt[b^2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b - Sqrt[b^2 - 4*a*c]
)*f]*Sqrt[d + f*x^2])]])/(2*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c
])*f]) - ((b*B - 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*f*x)/(Sqrt[4*c^2*d - 4*a*c*f + 2*b*(b + Sqrt[b^2 - 4*a*c])*f]*S
qrt[d + f*x^2])]])/(2*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
))/Sqrt[b^2 - 4*a*c]
```

Maple [B] time = 0.344, size = 1771, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x)
```

```
[Out] -2/(-4*a*c+b^2)^(1/2)/(-2*(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^
2)^(1/2)*ln((-(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*a
*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^2
)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))^2*f-4*f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))-2*(b
*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))*A-1/c/(-2*(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c
^2)^(1/2)*ln((-(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b-(-4*
a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*(b*f*(-4*a*c+b^
2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/
2)))^2*f-4*f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))-2*(
b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x-1/2/c*(-b+(-4*
```

```
a*c+b^2)^(1/2))))*B+1/(-4*a*c+b^2)^(1/2)/c/(-2*(b*f*(-4*a*c+b^2)^(1/2)+2*a*
c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*
c^2*d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2
*(-2*(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x-1/2/c*
(-b+(-4*a*c+b^2)^(1/2)))^2*f-4*f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*
a*c+b^2)^(1/2)))-2*(b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2
))/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))*B*b+2/(-4*a*c+b^2)^(1/2)/(-2*(-b*f*(-
4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(-b*f*(-4*a*c+b^2)^
(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c
^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b+(-4*a*c+b^2)^(1/2)
)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2
*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*A-1/c/(-2*(-b*f*(
-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(-b*f*(-4*a*c+b^2)
^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/
c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*(b+(-4*a*c+b^2)^(1/2
))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^
2*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))*B-1/(-4*a*c+b^2)
^(1/2)/c/(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln(
(-(-b*f*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^(1/
2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+1/2*(-2*(-b*f*(-4*a*c+b^2)^(1/2)+2*a
*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*f-4*f*
(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-b*f*(-4*a*c+b
^2)^(1/2)+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/
c))*B*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx}{\sqrt{d+fx^2}\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/(sqrt(d + f*x**2)*(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.24 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$

Optimal. Leaf size=101

$$\frac{A\tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

```
[Out] (A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])])/(Sqrt[a]*Sqrt[c*d
 - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*d - a*f]])/(Sqrt[c]*
Sqrt[c*d - a*f])
```

Rubi [A] time = 0.1271, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1010, 377, 205, 444, 63, 208}

$$\frac{A\tan^{-1}\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```


```
[Out] (A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])])/(Sqrt[a]*Sqrt[c*d
 - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*d - a*f]])/(Sqrt[c]*
Sqrt[c*d - a*f])
```


Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{\left(a+cx^2\right)\sqrt{d+fx^2}}\,dx &= A\int \frac{1}{\left(a+cx^2\right)\sqrt{d+fx^2}}\,dx + B\int \frac{x}{\left(a+cx^2\right)\sqrt{d+fx^2}}\,dx \\
&= A\,\mathrm{Subst}\left(\int \frac{1}{a-(-cd+af)x^2}\,dx, x, \frac{x}{\sqrt{d+fx^2}}\right) + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{1}{(a+cx)\sqrt{d+fx}}\,dx, x, x^2\right) \\
&= \frac{A\tan^{-1}\left(\frac{\sqrt{cd-af}\,x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} + \frac{B\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{cd}{f}+\frac{cx^2}{f}}\,dx, x, \sqrt{d+fx^2}\right)}{f} \\
&= \frac{A\tan^{-1}\left(\frac{\sqrt{cd-af}\,x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}
\end{aligned}$$

Mathematica [A] time = 0.176682, size = 154, normalized size = 1.52

$$\frac{\left(A\sqrt{c}-\sqrt{-a}B\right)\tanh^{-1}\left(\frac{\sqrt{c}d-\sqrt{-a}fx}{\sqrt{d+fx^2}\sqrt{cd-af}}\right)-\left(\sqrt{-a}B+A\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{-a}fx+\sqrt{c}d}{\sqrt{d+fx^2}\sqrt{cd-af}}\right)}{2\sqrt{-a}\sqrt{c}\sqrt{cd-af}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] ((-(Sqrt[-a]*B) + A*Sqrt[c])*ArcTanh[(Sqrt[c]*d - Sqrt[-a]*f*x)/(Sqrt[c*d -
 a*f]*Sqrt[d + f*x^2])] - (Sqrt[-a]*B + A*Sqrt[c])*ArcTanh[(Sqrt[c]*d + Sqr
t[-a]*f*x)/(Sqrt[c*d - a*f]*Sqrt[d + f*x^2])])/(2*Sqrt[-a]*Sqrt[c]*Sqrt[c*d
 - a*f])
```

Maple [B] time = 0.326, size = 608, normalized size = 6.

$$-\frac{A}{2}\ln\left(\left(-2\,\frac{af-cd}{c}+2\,\frac{f\sqrt{-ac}}{c}\left(x-\frac{\sqrt{-ac}}{c}\right)+2\,\sqrt{-\frac{af-cd}{c}}\sqrt{\left(x-\frac{\sqrt{-ac}}{c}\right)^2f+2\,\frac{f\sqrt{-ac}}{c}\left(x-\frac{\sqrt{-ac}}{c}\right)-\frac{af-cd}{c}}\right)\left(x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x)
```

```
[Out] -1/2/(-a*c)^(1/2)/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/2)/
c*(x-1/c*(-a*c)^(1/2))+2*(-(a*f-c*d)/c)^(1/2)*((x-1/c*(-a*c)^(1/2))^2*f+2*f
*(-a*c)^(1/2)/c*(x-1/c*(-a*c)^(1/2))-(a*f-c*d)/c)^(1/2))/(x-1/c*(-a*c)^(1/2
)))*A-1/2/c/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/2)/c*(x-1
/c*(-a*c)^(1/2))+2*(-(a*f-c*d)/c)^(1/2)*((x-1/c*(-a*c)^(1/2))^2*f+2*f*(-a*c
)^(1/2)/c*(x-1/c*(-a*c)^(1/2))-(a*f-c*d)/c)^(1/2))/(x-1/c*(-a*c)^(1/2)))*B+
1/2/(-a*c)^(1/2)/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c-2*f*(-a*c)^(1/2)/c
*(x+1/c*(-a*c)^(1/2))+2*(-(a*f-c*d)/c)^(1/2)*((x+1/c*(-a*c)^(1/2))^2*f-2*f*
(-a*c)^(1/2)/c*(x+1/c*(-a*c)^(1/2))-(a*f-c*d)/c)^(1/2))/(x+1/c*(-a*c)^(1/2)
))*A-1/2/c/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-c*d)/c-2*f*(-a*c)^(1/2)/c*(x+1/
c*(-a*c)^(1/2))+2*(-(a*f-c*d)/c)^(1/2)*((x+1/c*(-a*c)^(1/2))^2*f-2*f*(-a*c)
^(1/2)/c*(x+1/c*(-a*c)^(1/2))-(a*f-c*d)/c)^(1/2))/(x+1/c*(-a*c)^(1/2)))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.49673, size = 2943, normalized size = 29.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 -
 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)
*f*x + (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*
c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 +
 d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 -
2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqrt(-A^2*B^2/(a*c^3*d^
2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A
^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-
A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(
((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*
a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c
*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A
^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqr
t(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*
d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) - 1/4*sqrt((B^2*a - A^2*c - 2*(a*c^2
*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^
2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f -
 (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2
*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c - 2*(a*c^2*
d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2
*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B
^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*
a - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f
+ a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c
^2*d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^
2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a
 - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f +
 a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d
```

```
*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))
/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx}{\left(a+cx^2\right)\sqrt{d+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + f*x**2)), x)
```

Giac [C] time = 3.82123, size = 11297, normalized size = 111.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^
2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arcco
s(d/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a
*f/(c*abs(d)))))^3*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))
- (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*
a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cosh(1/2*imag_part(arccos(d/a
bs(d) - 2*a*f/(c*abs(d)))))^3*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*
abs(d)))))^3 - 9*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sq
rt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_p
art(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(1/2*imag_part(arccos(d/abs
(d) - 2*a*f/(c*abs(d)))))^2*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*ab
s(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) + 3*(2*a*
c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqr
t(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cosh(1/2*imag_part(arccos(d/abs(d) -
 2*a*f/(c*abs(d)))))^2*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))
)))^3*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) + 9*(2*a*c^2
*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-
```

d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 - 3*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 - 3*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3 + (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3 - 4*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) + 8*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) - 4*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 + (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) - (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))))*arctan(-(sqrt(f)*x + (d^2)^(1/4)*cos(1/2*arccos((c*d - 2*a*f)/(c*abs(d)))) - sqrt(f*x^2 + d))/((d^2)^(1/4)*sin(1/2*arccos((c*d - 2*a*f)/(c*abs(d))))))/(a*c^3*d^3*f - a^2*c^2*d^2*f^2) - 1/2*(3*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) - (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3 - 9*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(

1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sin(1/2*real_part(arc cos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a *f/(c*abs(d))))) + 3*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^ 2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cosh(1/2*i mag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sin(1/2*real_part(arccos(d /abs(d) - 2*a*f/(c*abs(d)))))^3*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/ (c*abs(d))))) + 9*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*s qrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_ part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(1/2*imag_part(arccos(d/ab s(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs (d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 - 3*(2*a *c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sq rt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 - 3*(2*a*c^ 2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2* a*f/(c*abs(d)))))^2*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) *sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3 + (2*a*c^2*sqrt (-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d* f)*sqrt(-a*c*d*f + a^2*f^2))*B*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c *abs(d)))))^3*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3 + 4*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f /(c*abs(d)))))*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*s in(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) - 8*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d* f + a^2*f^2))*A*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*cos h(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arc cos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a *f/(c*abs(d))))) + 4*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sq rt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*real_part(arcc os(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f /(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 + (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a *c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cosh(1/2*imag_part(arccos(d/ab s(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs (d))))) - (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)* d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*sin(1/2*real_part(arc cos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a *f/(c*abs(d))))))*arctan(-(sqrt(f)*x - (d^2)^(1/4)*cos(1/2*arccos((c*d - 2* a*f)/(c*abs(d)))) - sqrt(f*x^2 + d))/((d^2)^(1/4)*sin(1/2*arccos((c*d - 2*a *f)/(c*abs(d))))))/(a*c^3*d^3*f - a^2*c^2*d^2*f^2) + 1/4*((2*a*c^2*sqrt(-d) *d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*s qrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs

(d)))))^3*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3 - 3*(2
*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*
sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d)
 - 2*a*f/(c*abs(d)))))*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)
))))^3*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 - 3*(2*a*c
^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt
(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2
*a*f/(c*abs(d)))))^3*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))
))^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) + 9*(2*a*c^2*
sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d
)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*
f/(c*abs(d)))))*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*
sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sinh(1/2*imag_par
t(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) + 3*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2
*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f +
 a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*cosh
(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arc
cos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 - 9*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*
sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^
2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*cosh(1/2*
imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/
abs(d) - 2*a*f/(c*abs(d)))))^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(
c*abs(d)))))^2 - (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sq
rt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_p
art(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*sinh(1/2*imag_part(arccos(d/abs
(d) - 2*a*f/(c*abs(d)))))^3 + 3*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*
d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B
*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(
arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sinh(1/2*imag_part(arccos(d/abs(d)
- 2*a*f/(c*abs(d)))))^3 - 2*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2
*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*real_pa
rt(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(1/2*imag_part(arccos(d/abs(
d) - 2*a*f/(c*abs(d)))))^2 + 2*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (
c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cosh(1/2*ima
g_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sin(1/2*real_part(arccos(d/a
bs(d) - 2*a*f/(c*abs(d)))))^2 + 4*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2)
+ (c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*r
eal_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(1/2*imag_part(arccos(
d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(
c*abs(d))))) - 4*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f
) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cosh(1/2*imag_part(arccos(
d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c
*abs(d)))))^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) - 2*
(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/
2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(

c*abs(d)))))^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 +
 2*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d*f^
(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*sin(1/2*real_part(arccos(d/abs(d) - 2*a*
f/(c*abs(d)))))^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^
2 + (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 -
2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/
abs(d) - 2*a*f/(c*abs(d)))))*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*
abs(d))))) - (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-
d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(
arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) -
2*a*f/(c*abs(d))))))*log(2*(d^2)^(1/4)*(sqrt(f)*x - sqrt(f*x^2 + d))*cos(1/
2*arccos((c*d - 2*a*f)/(c*abs(d)))) + (sqrt(f)*x - sqrt(f*x^2 + d))^2 + sqr
t(d^2))/(a*c^3*d^3*f - a^2*c^2*d^2*f^2) - 1/4*((2*a*c^2*sqrt(-d)*d^2*f - 2*
a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*
f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*c
osh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3 - 3*(2*a*c^2*sqrt
(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*
f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c
*abs(d)))))*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*sin(
1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 - 3*(2*a*c^2*sqrt(-d)
*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*s
qrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs
(d)))))^3*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sinh(1
/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) + 9*(2*a*c^2*sqrt(-d)*d^
2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt
(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)
))))*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sin(1/2*rea
l_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sinh(1/2*imag_part(arccos(d/
abs(d) - 2*a*f/(c*abs(d))))) + 3*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)
*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*
B*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^3*cosh(1/2*imag_p
art(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d
) - 2*a*f/(c*abs(d)))))^2 - 9*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*
f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*c
os(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*cosh(1/2*imag_part(a
rccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*
a*f/(c*abs(d)))))^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))
)^2 - (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2
- 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(
d/abs(d) - 2*a*f/(c*abs(d)))))^3*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f
/(c*abs(d)))))^3 + 3*(2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^
2*sqrt(-d)*d^2 - 2*a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*re
al_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/ab
s(d) - 2*a*f/(c*abs(d)))))^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*
abs(d)))))^3 + 2*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f

```
) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*real_part(arccos(d
/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/
(c*abs(d)))))^2 - 2*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqr
t(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cosh(1/2*imag_part(arcc
os(d/abs(d) - 2*a*f/(c*abs(d)))))^2*sin(1/2*real_part(arccos(d/abs(d) - 2*a
*f/(c*abs(d)))))^2 - 4*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*
sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cos(1/2*real_part(ar
ccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2*cosh(1/2*imag_part(arccos(d/abs(d) -
2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))
) + 4*(2*a*c^2*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d
*f^(3/2))*sqrt(-a*c*d*f + a^2*f^2))*A*cosh(1/2*imag_part(arccos(d/abs(d) -
2*a*f/(c*abs(d)))))*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))
^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))) + 2*(2*a*c^2*d^
2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt(-a
*c*d*f + a^2*f^2))*A*cos(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d))))
)^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 - 2*(2*a*c^2
*d^2*f^(3/2) - 2*a^2*c*d*f^(5/2) + (c^2*d^2*sqrt(f) - 2*a*c*d*f^(3/2))*sqrt
(-a*c*d*f + a^2*f^2))*A*sin(1/2*real_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)
))))^2*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))^2 + (2*a*c^
2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*a*c*sqrt(
-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/abs(d) - 2*
a*f/(c*abs(d)))))*cosh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*abs(d)))))
- (2*a*c^2*sqrt(-d)*d^2*f - 2*a^2*c*sqrt(-d)*d*f^2 + (c^2*sqrt(-d)*d^2 - 2*
a*c*sqrt(-d)*d*f)*sqrt(-a*c*d*f + a^2*f^2))*B*cos(1/2*real_part(arccos(d/ab
s(d) - 2*a*f/(c*abs(d)))))*sinh(1/2*imag_part(arccos(d/abs(d) - 2*a*f/(c*ab
s(d))))))*log(-2*(d^2)^(1/4)*(sqrt(f)*x - sqrt(f*x^2 + d))*cos(1/2*arccos((
c*d - 2*a*f)/(c*abs(d)))) + (sqrt(f)*x - sqrt(f*x^2 + d))^2 + sqrt(d^2))/(a
*c^3*d^3*f - a^2*c^2*d^2*f^2)
```

3.25 $\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx$

Optimal. Leaf size=139

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

```
[Out] (Sqrt[-13/5 + Sqrt[10]]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*S
qrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (Sqrt[13/5 + Sqrt[10]]*ArcTa
nh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 +
3*x - 2*x^2])])/2
```

Rubi [A] time = 0.220049, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1032, 724, 204, 206}

$$\frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] (Sqrt[-13/5 + Sqrt[10]]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*S
qrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (Sqrt[13/5 + Sqrt[10]]*ArcTa
nh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 +
3*x - 2*x^2])])/2
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx = \frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx + \frac{1}{5}\left(5+4\sqrt{10}\right)\int \frac{}{\left(4+2\sqrt{10}\right.}$$

$$= -\left(\frac{1}{5}\left(2\left(5-4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)-8\left(4-2\sqrt{10}\right)^2-x^2}\,dx, x, -\right.\right.$$

$$= \frac{1}{10}\sqrt{-65+25\sqrt{10}}\tan^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{1}{10}\sqrt{65+25\sqrt{10}}\tanh^{-}$$

Mathematica [A] time = 0.295987, size = 140, normalized size = 1.01

$$\frac{\left(4\sqrt{10}-5\right)\tan^{-1}\left(\frac{4\sqrt{10}x+x-3\sqrt{10}+12}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+3\sqrt{5\left(7+2\sqrt{10}\right)}\tanh^{-1}\left(\frac{-4\sqrt{10}x+x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{1+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] ((-5 + 4*Sqrt[10])*ArcTan[(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 +
Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])] + 3*Sqrt[5*(7 + 2*Sqrt[10])]*ArcTanh[(3*(
```

```
4 + Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x
^2])])/(10*Sqrt[1 + Sqrt[10]])
```

Maple [B] time = 0.135, size = 324, normalized size = 2.3

$$\frac{2\sqrt{10}}{5\sqrt{-1+\sqrt{10}}}\operatorname{Artanh}\left(\frac{9}{2\sqrt{-1+\sqrt{10}}}\left(-\frac{2}{9}+\frac{2\sqrt{10}}{9}+\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)\right)\frac{1}{\sqrt{-18\left(x-2/3-1/3\sqrt{10}\right)^2+9\left(1/\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x)
```

```
[Out] 2/5*10^(1/2)/(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10
^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))
^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))+1/2/(-1+10^
(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*1
0^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/
2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))+2/5*10^(1/2)/(1+10^(1/2))^(1/2)
*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+
10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3
*10^(1/2))-1-10^(1/2))^(1/2))-1/2/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*1
0^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x
-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^
(1/2))
```

Maxima [B] time = 1.56001, size = 487, normalized size = 3.5

$$-\frac{1}{20}\sqrt{10}\left(\frac{\sqrt{10}\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17\left|6x+2\sqrt{10}-4\right|}+\frac{2\sqrt{17}x}{17\left|6x+2\sqrt{10}-4\right|}-\frac{6\sqrt{17}\sqrt{10}}{17\left|6x+2\sqrt{10}-4\right|}+\frac{24\sqrt{17}}{17\left|6x+2\sqrt{10}-4\right|}\right)}{\sqrt{\sqrt{10}+1}}-\frac{\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{-2x^2+3x}}{3\left|6x-2\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(10)*(sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(1
0) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10
```

```
)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqr
t(sqrt(10) + 1) - sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*s
qrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt
(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/sqrt(sqrt(10) - 1) - 8*ar
csin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/a
bs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4)
 + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqrt(sqrt(10) + 1) - 8*log(-2/
9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt
(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(1
0) - 4) + 1/18)/sqrt(sqrt(10) - 1))
```

Fricas [B] time = 1.64926, size = 948, normalized size = 6.82

$$\frac{2}{5}\sqrt{5}\sqrt{5\sqrt{5}\sqrt{2}-13}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{5}x-\sqrt{2}x\right)\sqrt{5\sqrt{5}\sqrt{2}-13}\sqrt{\frac{\sqrt{5}\sqrt{2}\left(3x^2+2x\right)+6x^2-2\left(\sqrt{5}\sqrt{2}x+2x+2\right)\sqrt{-2x^2+3x+1}+10x+4}{x^2}}+2\left(\cdots\right.}{18x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) - 13)*arctan(1/18*(sqrt(2)*(2*sqrt(5)*x
- sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) - 13)*sqrt((sqrt(5)*sqrt(2)*(3*x^2 + 2*
x) + 6*x^2 - 2*(sqrt(5)*sqrt(2)*x + 2*x + 2)*sqrt(-2*x^2 + 3*x + 1) + 10*x
+ 4)/x^2) + 2*(sqrt(2)*(4*x - 1) + sqrt(5)*(x + 2) - sqrt(-2*x^2 + 3*x + 1)
*(2*sqrt(5) - sqrt(2)))*sqrt(5*sqrt(5)*sqrt(2) - 13))/x) - 1/10*sqrt(5)*sqr
t(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x + (4*sqrt(5)*x - 7*sqrt(
2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)
/x) + 1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x -
(4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-
2*x^2 + 3*x + 1) - 18)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx-\int\frac{2}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -Integral(x/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) -
 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(-2*x**2 + 3*x + 1
) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.26 $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(\sqrt{10}-3\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+}{2\sqrt{\sqrt{10}-1}\sqrt{-2}}\right.$$

```
[Out] (-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) - (9*Sqrt[(-3 + Sqrt[10])/5]*Ar
cTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 +
 3*x - 2*x^2])])/2 + (9*Sqrt[(3 + Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) +
(1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rubi [A] time = 0.221062, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1016, 12, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(\sqrt{10}-3\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(1-4\sqrt{10}\right)x+}{2\sqrt{\sqrt{10}-1}\sqrt{-2}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```


```
[Out] (-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) - (9*Sqrt[(-3 + Sqrt[10])/5]*Ar
cTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 +
 3*x - 2*x^2])])/2 + (9*Sqrt[(3 + Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) +
(1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```


Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
```

```
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{3/2}}\,dx = -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}+\frac{2}{17}\int \frac{153x}{2\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}+9\int \frac{x}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}+\frac{1}{5}\left(9\left(5-\sqrt{10}\right)\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx+\frac{1}{5}$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}-\frac{1}{5}\left(18\left(5-\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)-8\left(4-\right.}\right.$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}-\frac{9}{2}\sqrt{\frac{1}{5}\left(-3+\sqrt{10}\right)}\tan^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)+$$

Mathematica [A] time = 0.379718, size = 167, normalized size = 1.01

$$\frac{1}{170}\left(153\sqrt{5\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{-4\sqrt{10}x+x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)-\frac{153\sqrt{5\left(\sqrt{10}-3\right)}\sqrt{-2x^2+3x+1}\tan^{-1}\left(\frac{4\sqrt{10}x+x-3\sqrt{10}+}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3}}\right.}{\sqrt{-2x^2+3x+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (-((300 + 280*x + 153*Sqrt[5*(-3 + Sqrt[10])]*Sqrt[1 + 3*x - 2*x^2]*ArcTan[
(12 - 3*Sqrt[10] + x + 4*Sqrt[10]*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2
*x^2])])/Sqrt[1 + 3*x - 2*x^2]) + 153*Sqrt[5*(3 + Sqrt[10])]*ArcTanh[(3*(4
+ Sqrt[10]) + x - 4*Sqrt[10]*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2
])])/170
```

Maple [B] time = 0.106, size = 760, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x)
```

```
[Out] -26/255*10^(1/2)/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10
^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)-32/765/(-1/9+1/9*10^(1
/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9
+1/9*10^(1/2))^(1/2)*x*10^(1/2)-62/153/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*1
0^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*
x+7/51/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x
-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)+2/5*10^(1/2)/(-1/9+1/9*10^(1/2))
/(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2
/3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/
3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))+1/2/(-1/9+1/9*10^(1/2))
/(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2
/3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/
3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))+26/255*10^(1/2)/(-1/9-1
/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1
/2))-1/9-1/9*10^(1/2))^(1/2)+32/765/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(
1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x*1
0^(1/2)-62/153/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(
1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x+7/51/(-1/9-1/9*10^(1/2
))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1
/9*10^(1/2))^(1/2)+2/5*10^(1/2)/(-1/9-1/9*10^(1/2))/(1+10^(1/2))^(1/2)*arct
an(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1
/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(
1/2))-1-10^(1/2))^(1/2))-1/2/(-1/9-1/9*10^(1/2))/(1+10^(1/2))^(1/2)*arctan(
9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2)
)^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2
))-1-10^(1/2))^(1/2))
```

Maxima [B] time = 1.58402, size = 915, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/340*sqrt(10)*(124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x
^2 + 3*x + 1)) - 124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*
x^2 + 3*x + 1)) + 153*sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*
sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*s
qrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4
```

```
))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) - 128*x/(sqrt(10)*sqr
t(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 128*x/(sqrt(10)*sqrt(-2*x^2
 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 1224*arcsin(8/17*sqrt(17)*sqrt(10)*
x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6
/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x +
2*sqrt(10) - 4))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) + 153*s
qrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/a
bs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs
(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(3/2) - 42*sqrt(10)/(sqrt(10)
*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 42*sqrt(10)/(sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 1224*log(-2/9*sqrt(10) +
2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2
/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/1
8)/(sqrt(10) - 1)^(3/2) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^
2 + 3*x + 1)) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x +
1)))
```

Fricas [B] time = 1.45106, size = 1029, normalized size = 6.2

$$-\frac{612\sqrt{5}\left(2x^2-3x-1\right)\sqrt{\sqrt{10}-3}\arctan\left(\frac{\sqrt{10}\sqrt{5}\sqrt{2}x\sqrt{\sqrt{10}-3}\sqrt{\frac{6x^2+\sqrt{10}\left(3x^2+2x\right)-2\sqrt{-2x^2+3x+1}\left(\sqrt{10}x+2x+2\right)+10x+4}{x^2}}+2\left(\sqrt{10}\sqrt{5}(x+1)-\sqrt{10}\sqrt{5}\sqrt{\cdots}\right.}{10x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/170*(612*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) - 3)*arctan(1/10*(sqrt(
10)*sqrt(5)*sqrt(2)*x*sqrt(sqrt(10) - 3)*sqrt((6*x^2 + sqrt(10)*(3*x^2 + 2*
x) - 2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*x + 2*x + 2) + 10*x + 4)/x^2) + 2*(
sqrt(10)*sqrt(5)*(x + 1) - sqrt(10)*sqrt(5)*sqrt(-2*x^2 + 3*x + 1) + 5*sqrt
(5)*x)*sqrt(sqrt(10) - 3))/x) + 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10)
 + 3)*log(9*(5*sqrt(10)*x + (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt
(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/x) - 153*sqrt(5)*(2*x^2
- 3*x - 1)*sqrt(sqrt(10) + 3)*log(9*(5*sqrt(10)*x - (3*sqrt(10)*sqrt(5)*x -
 10*sqrt(5)*x)*sqrt(sqrt(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/
x) + 600*x^2 - 20*sqrt(-2*x^2 + 3*x + 1)*(14*x + 15) - 900*x - 300)/(2*x^2
- 3*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(3/2),x)
```

```
[Out] -Integral(x/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 17*x**3*sqrt(-2*x**2 + 3*x +
 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqr
t(-2*x**2 + 3*x + 1)), x) - Integral(2/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 1
7*x**3*sqrt(-2*x**2 + 3*x + 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt
(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.27 $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}-\frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(17\sqrt{10}-53\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(53+\right.}$$

```
[Out] (-2*(15 + 14*x))/(51*(1 + 3*x - 2*x^2)^(3/2)) - (2*(291 + 4814*x))/(867*Sqr
t[1 + 3*x - 2*x^2]) + (9*Sqrt[(-53 + 17*Sqrt[10])/5]*ArcTan[(3*(4 - Sqrt[10
]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 +
 (9*Sqrt[(53 + 17*Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])
*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rubi [A] time = 0.265946, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1016, 1060, 1032, 724, 204, 206}

$$-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}-\frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(17\sqrt{10}-53\right)}\tan^{-1}\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)+\frac{9}{2}\sqrt{\frac{1}{5}\left(53+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(51*(1 + 3*x - 2*x^2)^(3/2)) - (2*(291 + 4814*x))/(867*Sqr
t[1 + 3*x - 2*x^2]) + (9*Sqrt[(-53 + 17*Sqrt[10])/5]*ArcTan[(3*(4 - Sqrt[10
]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 +
 (9*Sqrt[(53 + 17*Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])
*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
```

```
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{5/2}}\,dx = -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}}+\frac{2}{51}\int \frac{-56+\frac{235x}{2}+84x^2}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{3/2}}\,dx$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}}-\frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}}+\frac{4}{867}\int \frac{\frac{7803}{2}+\frac{23409x}{4}}{\left(2+4x-3x^2\right)\sqrt{1+3x-}}$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}}-\frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}}+\frac{1}{5}\left(27\left(5-2\sqrt{10}\right)\right)\int \frac{}{\left(4-2\sqrt{10}-\right.}$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}}-\frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}}-\frac{1}{5}\left(54\left(5-2\sqrt{10}\right)\right)\mathrm{Subst}\left(\int \frac{}{144}\right.$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}}-\frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}}+\frac{9}{2}\sqrt{\frac{1}{5}\left(-53+17\sqrt{10}\right)}\tan^{-1}\left(\frac{3(4}{2\sqrt{}}\right.$$

Mathematica [A] time = 0.608677, size = 185, normalized size = 0.96

$$-\frac{2\left(-9628x^3+13860x^2+5925x+546\right)}{867\left(-2x^2+3x+1\right)^{3/2}}+\frac{3}{10}\sqrt{1+\sqrt{10}}\left(7\sqrt{10}-25\right)\tan^{-1}\left(\frac{3\left(\sqrt{10}-4\right)-\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)-\frac{3}{10}\sqrt{\sqrt{10}-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(546 + 5925*x + 13860*x^2 - 9628*x^3))/(867*(1 + 3*x - 2*x^2)^(3/2)) +
(3*Sqrt[1 + Sqrt[10]]*(-25 + 7*Sqrt[10])*ArcTan[(3*(-4 + Sqrt[10]) - (1 + 4
*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])]])/10 - (3*Sqrt[-
1 + Sqrt[10]]*(25 + 7*Sqrt[10])*ArcTanh[(-3*(4 + Sqrt[10]) + (-1 + 4*Sqrt[1
0])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])]])/10
```

Maple [B] time = 0.107, size = 1560, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x)
```

```
[Out] -992/7803/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))
*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)*x+2/5*10^(1/2)/(-1/9-1/9*10^(
1/2))^2/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))
*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/
3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))-128/13005*10^(1/2)/
(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/
3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)+2/5*10^(1/2)/(-1/9+1/9*10^(1/2))^2/(-1+
10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/
3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^
(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))+32/765/(-1/9-1/9*10^(1/2))^2
/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9
*10^(1/2))^(1/2)*x*10^(1/2)+32/2295/(-1/9-1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3
+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^
(3/2)*x-32/2295/(-1/9+1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3-1/3*10^(1/2))^2+(1/
3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)*x+248/2601/(-1
/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*1
0^(1/2))-1/9-1/9*10^(1/2))^(1/2)+248/2601/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/
3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/
2)-62/153/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2
))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*x-26/255/(-1/9+1/9*10^(1/2)
)^2/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+
1/9*10^(1/2))^(1/2)*10^(1/2)+1/2/(-1/9+1/9*10^(1/2))^2/(-1+10^(1/2))^(1/2)*
arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1
+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/
```

```
3*10^(1/2))-1+10^(1/2))^(1/2))-62/459/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10
^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)*x
-26/765/(-1/9+1/9*10^(1/2))*10^(1/2)/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10
^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(3/2)+26/255/(-1/9-1/9*10^(1
/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1
/9-1/9*10^(1/2))^(1/2)*10^(1/2)-1/2/(-1/9-1/9*10^(1/2))^2/(1+10^(1/2))^(1/2
)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1
+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/
3*10^(1/2))-1-10^(1/2))^(1/2))+128/13005*10^(1/2)/(-1/9+1/9*10^(1/2))/(-2*(
x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1
/2))^(1/2)-992/7803/(-1/9+1/9*10^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3
*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)*x-62/153/(-1/9-1/9*
10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/
2))-1/9-1/9*10^(1/2))^(1/2)*x+26/765/(-1/9-1/9*10^(1/2))*10^(1/2)/(-2*(x-2/
3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))
^(3/2)-62/459/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1
/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(3/2)*x-32/765/(-1/9+1/9*10^(1/
2))^2/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/
9+1/9*10^(1/2))^(1/2)*x*10^(1/2)+7/51/(-1/9+1/9*10^(1/2))^2/(-2*(x-2/3-1/3*
10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/2))^(1/2)
+7/153/(-1/9-1/9*10^(1/2))/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x
-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(3/2)+7/153/(-1/9+1/9*10^(1/2))/(-2*(x
-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1/9+1/9*10^(1/
2))^(3/2)+7/51/(-1/9-1/9*10^(1/2))^2/(-2*(x-2/3+1/3*10^(1/2))^2+(1/3+4/3*10
^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^(1/2)-512/39015/(-1/9+1/9*10
^(1/2))/(-2*(x-2/3-1/3*10^(1/2))^2+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-
1/9+1/9*10^(1/2))^(1/2)*x*10^(1/2)+512/39015/(-1/9-1/9*10^(1/2))/(-2*(x-2/3
+1/3*10^(1/2))^2+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1/9-1/9*10^(1/2))^
(1/2)*x*10^(1/2)
```

Maxima [B] time = 1.73312, size = 1723, normalized size = 8.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17340*sqrt(10)*(2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*
x^2 + 3*x + 1)^(3/2)) - 2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2)
- (-2*x^2 + 3*x + 1)^(3/2)) - 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*
x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*
```

```
x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1984*sqrt(10)*x/(sqrt(10)*sqr
t(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 1984*sqrt(10)*x/(sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 70227*sqrt(10)*arcsin(8/1
7*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x +
 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17
*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) + 1) + 11*sq
rt(sqrt(10) + 1)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3
*x + 1)^(3/2)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x
+ 1)^(3/2)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 +
 3*x + 1)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 +
3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1
)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 56
1816*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(
17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(1
0) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(1
0) + 1) + 11*sqrt(sqrt(10) + 1)) - 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x + 1
)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) + 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x
+ 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 19278*sqrt(10)/(2*sqrt(10)*sqrt(-2
*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) - 19278*sqrt(10)/(2*sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) - 1488*sqrt(10)/(sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 1488*sqrt(10)/(sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 5304/(sqrt(10)*(-2*x^2
 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 5304/(sqrt(10)*(-2*x^2 + 3*
x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 +
 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 + 3
*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1
) + sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(
-2*x^2 + 3*x + 1)) + 70227*sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3
*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x
 - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(
5/2) + 561816*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10)
- 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2
/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(5/2))
```

Fricas [B] time = 1.49064, size = 1304, normalized size = 6.76

$$-\frac{43680\,x^4 - 131040\,x^3 - 31212\,\sqrt{5}\left(4\,x^4 - 12\,x^3 + 5\,x^2 + 6\,x + 1\right)\sqrt{17\,\sqrt{10} - 53}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{10}\sqrt{5}x + 10\,\sqrt{5}x\right)\sqrt{17\,\sqrt{10} - 53}\sqrt{\frac{6\,x^{\cdots}}{\cdots}}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8670*(43680*x^4 - 131040*x^3 - 31212*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6
*x + 1)*sqrt(17*sqrt(10) - 53)*arctan(1/90*(sqrt(2)*(sqrt(10)*sqrt(5)*x + 1
0*sqrt(5)*x)*sqrt(17*sqrt(10) - 53)*sqrt((6*x^2 + sqrt(10)*(3*x^2 + 2*x) -
2*sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*x + 2*x + 2) + 10*x + 4)/x^2) + 2*(sqrt(
10)*sqrt(5)*(6*x + 1) - sqrt(-2*x^2 + 3*x + 1)*(sqrt(10)*sqrt(5) + 10*sqrt(
5)) + 5*sqrt(5)*(3*x + 2))*sqrt(17*sqrt(10) - 53))/x) - 7803*sqrt(5)*(4*x^4
 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*sqrt(10) + 53)*log(9*(45*sqrt(10)*x +
(13*sqrt(10)*sqrt(5)*x - 40*sqrt(5)*x)*sqrt(17*sqrt(10) + 53) - 90*x + 90*s
qrt(-2*x^2 + 3*x + 1) - 90)/x) + 7803*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x
 + 1)*sqrt(17*sqrt(10) + 53)*log(9*(45*sqrt(10)*x - (13*sqrt(10)*sqrt(5)*x
- 40*sqrt(5)*x)*sqrt(17*sqrt(10) + 53) - 90*x + 90*sqrt(-2*x^2 + 3*x + 1) -
 90)/x) + 54600*x^2 - 20*(9628*x^3 - 13860*x^2 - 5925*x - 546)*sqrt(-2*x^2
+ 3*x + 1) + 65520*x + 10920)/(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.28 $\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$

Optimal. Leaf size=151

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

```
[Out] -(Sqrt[1 + (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x
)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2 + (Sqrt[1 - (7*Sqrt[
2/5])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*S
qrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2
```

Rubi [A] time = 0.228555, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1032, 724, 206}

$$\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] -(Sqrt[1 + (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x
)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2 + (Sqrt[1 - (7*Sqrt[
2/5])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*S
qrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\sqrt{1+3x+2x^2}}\,dx = \frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x+2x^2}}\,dx + \frac{1}{5}\left(5+4\sqrt{10}\right)\int \frac{1}{\left(4+2\sqrt{10}\right.}$$

$$= -\left(\frac{1}{5}\left(2\left(5-4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)+8\left(4-2\sqrt{10}\right)^2-x^2}\,dx, x, -\right.\right.$$

$$= -\frac{1}{10}\sqrt{25+7\sqrt{10}}\tanh^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(17-4\sqrt{10}\right)x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) + \frac{1}{10}\sqrt{25-7\sqrt{10}}\tanh$$

Mathematica [A] time = 0.354072, size = 148, normalized size = 0.98

$$\frac{\left(5-4\sqrt{10}\right)\tanh^{-1}\left(\frac{-4\sqrt{10}x+17x-3\sqrt{10}+12}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+3\sqrt{285-90\sqrt{10}}\tanh^{-1}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)}{10\sqrt{55-17\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] ((5 - 4*Sqrt[10])*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]*x)/(2*Sqrt[5
5 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])] + 3*Sqrt[285 - 90*Sqrt[10]]*ArcTan
h[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1
 + 3*x + 2*x^2])])/(10*Sqrt[55 - 17*Sqrt[10]])
```

Maple [A] time = 0.125, size = 186, normalized size = 1.2

$$\frac{\left(8+\sqrt{10}\right)\sqrt{10}}{20\sqrt{55+17\sqrt{10}}}\mathrm{Artanh}\left(\frac{9}{2\sqrt{55+17\sqrt{10}}}\left(\frac{110}{9}+\frac{34\sqrt{10}}{9}+\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)\right)\frac{1}{\sqrt{18\left(x-2/3-1/3\sqrt{10}\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x)
```

```
[Out] 1/20*(8+10^(1/2))*10^(1/2)/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(110/9+34/9*1
0^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2))^(1/2)/(1
8*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55+17*1
0^(1/2))^(1/2))+1/20*(-8+10^(1/2))*10^(1/2)/(55-17*10^(1/2))^(1/2)*arctanh(
9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*1
0^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*
10^(1/2))+55-17*10^(1/2))^(1/2))
```

Maxima [B] time = 1.53812, size = 490, normalized size = 3.25

$$\frac{1}{60}\sqrt{10}\left(\frac{3\sqrt{10}\log\left(\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{17\sqrt{10}+55}}{3\left|6x-2\sqrt{10}-4\right|}+\frac{34\sqrt{10}}{9\left|6x-2\sqrt{10}-4\right|}+\frac{110}{9\left|6x-2\sqrt{10}-4\right|}+\frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}}+\frac{\sqrt{10}\log\left(-\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x}}{|6x}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(10)*(3*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt
(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sq
rt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 5
5) + sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(1
0) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) -
 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9)
+ 24*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) + 55)/ab
s(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/a
bs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 55) - 8*log(-2/9*sqrt(
10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(
10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt
```

```
(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9))
```

Fricas [B] time = 1.28405, size = 709, normalized size = 4.7

$$\frac{1}{10}\sqrt{7\sqrt{10}+25}\log\left(-\frac{3\sqrt{10}x+\left(\sqrt{10}x-4x\right)\sqrt{7\sqrt{10}+25}+6x-6\sqrt{2x^2+3x+1}+6}{x}\right)-\frac{1}{10}\sqrt{7\sqrt{10}+25}\log\left(-\frac{3\sqrt{\ }}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(7*sqrt(10) + 25)*log(-(3*sqrt(10)*x + (sqrt(10)*x - 4*x)*sqrt(7*s
qrt(10) + 25) + 6*x - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) - 1/10*sqrt(7*sqrt(10
) + 25)*log(-(3*sqrt(10)*x - (sqrt(10)*x - 4*x)*sqrt(7*sqrt(10) + 25) + 6*x
 - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) + 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqr
t(10)*x + (sqrt(10)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 +
3*x + 1) - 6)/x) - 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqrt(10)*x - (sqrt(10
)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 + 3*x + 1) - 6)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx-\int\frac{2}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -Integral(x/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) - 2
*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(2*x**2 + 3*x + 1) -
4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.29 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}-\frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}\tanh^{-1}$$

```
[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(3*(2065 + 653*Sqrt[10]))
/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[1
0]]*Sqrt[1 + 3*x + 2*x^2])])/10 + (Sqrt[(3*(2065 - 653*Sqrt[10]))/5]*ArcTan
h[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1
 + 3*x + 2*x^2])])/10
```

Rubi [A] time = 0.254702, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1016, 1032, 724, 206}

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}-\frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(3*(2065 + 653*Sqrt[10]))
/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[1
0]]*Sqrt[1 + 3*x + 2*x^2])])/10 + (Sqrt[(3*(2065 - 653*Sqrt[10]))/5]*ArcTan
h[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1
 + 3*x + 2*x^2])])/10
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
```

```
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx = \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{2}{15}\int \frac{-72+\frac{81x}{2}}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{5}\left(9\left(3-\sqrt{10}\right)\right)\int \frac{1}{\left(4+2\sqrt{10}-6x\right)\sqrt{1+3x+2x^2}}\,dx - \frac{1}{5}($$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} + \frac{1}{5}\left(18\left(3-\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4+2\sqrt{10}\right)+8\left(4+2\right.}\right.$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(17-4\sqrt{1}\right.}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+}}\right.$$

Mathematica [A] time = 0.575963, size = 172, normalized size = 0.99

$$\frac{1}{50}\left(\frac{\sqrt{30975-9795\sqrt{10}}\sqrt{2x^2+3x+1}\tanh^{-1}\left(\frac{4\sqrt{10}x+17x+3\sqrt{10}+12}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+440x+420}{\sqrt{2x^2+3x+1}} - \sqrt{30975+9795\sqrt{10}}\tanh^{-1}\left(\frac{-4}{2\sqrt{5}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (-(Sqrt[30975 + 9795*Sqrt[10]]*ArcTanh[(12 - 3*Sqrt[10] + 17*x - 4*Sqrt[10]
*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])]) + (420 + 440*x + Sqr
t[30975 - 9795*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2]*ArcTanh[(12 + 3*Sqrt[10] + 1
7*x + 4*Sqrt[10]*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/Sqrt
[1 + 3*x + 2*x^2])/50
```

Maple [B] time = 0.11, size = 466, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x)
```

```
[Out] -1/20*(8+10^(1/2))*10^(1/2)*(1/3/(55/9+17/9*10^(1/2))/(2*(x-2/3-1/3*10^(1/2
))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)-1/3
*(17/3+4/3*10^(1/2))/(55/9+17/9*10^(1/2))*(4*x+3)/(440/9+136/9*10^(1/2)-(17
/3+4/3*10^(1/2))^2)/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/
3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)-1/(55/9+17/9*10^(1/2))/(55+17*10^(1/2
))^(1/2)*arctanh(9/2*(110/9+34/9*10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10
^(1/2)))/(55+17*10^(1/2))^(1/2)/(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(
1/2))*(x-2/3-1/3*10^(1/2))+55+17*10^(1/2))^(1/2)))-1/20*(-8+10^(1/2))*10^(1
/2)*(1/3/(55/9-17/9*10^(1/2))/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))
*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-1/3*(17/3-4/3*10^(1/2))/(55
/9-17/9*10^(1/2))*(4*x+3)/(440/9-136/9*10^(1/2)-(17/3-4/3*10^(1/2))^2)/(2*(
x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10
^(1/2))^(1/2)-1/(55/9-17/9*10^(1/2))/(55-17*10^(1/2))^(1/2)*arctanh(9/2*(11
0/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*10^(1/2)
)^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2
))+55-17*10^(1/2))^(1/2)))
```

Maxima [B] time = 1.56529, size = 902, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/60*sqrt(10)*(588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt
(2*x^2 + 3*x + 1)) - 588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55
*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sq
rt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(
2*x^2 + 3*x + 1)) - 27*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
- 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) +
 55)^(3/2) - sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/
9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sq
rt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55
/9)^(3/2) + 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2
 + 3*x + 1)) - 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*
x^2 + 3*x + 1)) - 216*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*
sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(1
0) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(3/2)
 + 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9
)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110
```

```
/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(3/2) + 1656/
(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 1656/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)))
```

Fricas [B] time = 1.477, size = 1131, normalized size = 6.5

$$\frac{\sqrt{5}\left(2\,x^2+3\,x+1\right)\sqrt{1959\,\sqrt{10}+6195}\log\left(-\frac{45\,\sqrt{10}x+\left(41\,\sqrt{10}\sqrt{5}x-130\,\sqrt{5}x\right)\sqrt{1959\,\sqrt{10}+6195}+90\,x-90\,\sqrt{2\,x^2+3\,x+1}+90}{x}\right)-\sqrt{5}\left(2\,x^2+3\,x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1959*sqrt(10) + 6195)*log(-(45*sqrt(10
)*x + (41*sqrt(10)*sqrt(5)*x - 130*sqrt(5)*x)*sqrt(1959*sqrt(10) + 6195) +
90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1
959*sqrt(10) + 6195)*log(-(45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x - 130*sqr
t(5)*x)*sqrt(1959*sqrt(10) + 6195) + 90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/
x) + sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-1959*sqrt(10) + 6195)*log((45*sqrt(10)
*x + (41*sqrt(10)*sqrt(5)*x + 130*sqrt(5)*x)*sqrt(-1959*sqrt(10) + 6195) -
90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-
1959*sqrt(10) + 6195)*log((45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x + 130*sqr
t(5)*x)*sqrt(-1959*sqrt(10) + 6195) - 90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)
/x) + 840*x^2 + 20*sqrt(2*x^2 + 3*x + 1)*(22*x + 21) + 1260*x + 420)/(2*x^2
 + 3*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.30 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=197

$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}+\frac{2(230x+273)}{15\sqrt{2x^2+3x+1}}-\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+$$

```
[Out] (2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + (2*(273 + 230*x))/(15*Sqrt[1
 + 3*x + 2*x^2]) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 - Sq
rt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x
^2])])/50 + (Sqrt[(4885115 - 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 + Sqrt[10])
 + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/
50
```

Rubi [A] time = 0.303302, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1016, 1060, 1032, 724, 206}

$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}+\frac{2(230x+273)}{15\sqrt{2x^2+3x+1}}-\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```


```
[Out] (2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + (2*(273 + 230*x))/(15*Sqrt[1
 + 3*x + 2*x^2]) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 - Sq
rt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x
^2])])/50 + (Sqrt[(4885115 - 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 + Sqrt[10])
 + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/
50
```


Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
```

```
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

<u>Rule 1060</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

<u>Rule 1032</u>

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
```

```
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x+2x^2\right)^{5/2}}\,dx = \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} - \frac{2}{45}\int \frac{-480-\frac{813x}{2}+396x^2}{\left(2+4x-3x^2\right)\left(1+3x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{4}{675}\int \frac{\frac{23355}{2}-\frac{27135x}{4}}{\left(2+4x-3x^2\right)\sqrt{1+3x+2x}}$$

$$= \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{25}\left(3\left(335-106\sqrt{10}\right)\right)\int \frac{}{\left(4+2\sqrt{1}\right.}$$

$$= \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{1}{25}\left(6\left(335-106\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{}{1}\right.$$

$$= \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\,\text{ta}$$

Mathematica [A] time = 0.72194, size = 190, normalized size = 0.96

$$\frac{1}{450}\left(\frac{60\left(460x^3+1236x^2+1071x+294\right)}{\left(2x^2+3x+1\right)^{3/2}}+\sqrt{55-17\sqrt{10}}\left(7289+2305\sqrt{10}\right)\tanh^{-1}\left(\frac{\left(4\sqrt{10}-17\right)x+3\left(\sqrt{10}-4\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

```
[Out] ((60*(294 + 1071*x + 1236*x^2 + 460*x^3))/(1 + 3*x + 2*x^2)^(3/2) + Sqrt[55
 - 17*Sqrt[10]]*(7289 + 2305*Sqrt[10])*ArcTanh[(3*(-4 + Sqrt[10]) + (-17 +
4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])] - Sqrt[55
+ 17*Sqrt[10]]*(-7289 + 2305*Sqrt[10])*ArcTanh[(-3*(4 + Sqrt[10]) - (17 + 4
*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/450
```

Maple [B] time = 0.101, size = 878, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x)
```

```
[Out] -1/20*(8+10^(1/2))*10^(1/2)*(1/9/(55/9+17/9*10^(1/2))/(2*(x-2/3-1/3*10^(1/2
))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(3/2)-1/6
*(17/3+4/3*10^(1/2))/(55/9+17/9*10^(1/2))*(2/3*(4*x+3)/(440/9+136/9*10^(1/2
)-(17/3+4/3*10^(1/2))^2)/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2
/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(3/2)+32/3/(440/9+136/9*10^(1/2)-(17/3
+4/3*10^(1/2))^2)^2*(4*x+3)/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(
x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2))+1/3/(55/9+17/9*10^(1/2))*(1/
(55/9+17/9*10^(1/2))/(2*(x-2/3-1/3*10^(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1
/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)-(17/3+4/3*10^(1/2))/(55/9+17/9*10^(1
/2))*(4*x+3)/(440/9+136/9*10^(1/2)-(17/3+4/3*10^(1/2))^2)/(2*(x-2/3-1/3*10^
(1/2))^2+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55/9+17/9*10^(1/2))^(1/2)
-3/(55/9+17/9*10^(1/2))/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(110/9+34/9*10^(
1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2))^(1/2)/(18*(
x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55+17*10^(
1/2))^(1/2)))-1/20*(-8+10^(1/2))*10^(1/2)*(1/9/(55/9-17/9*10^(1/2))/(2*(x-
2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(
1/2))^(3/2)-1/6*(17/3-4/3*10^(1/2))/(55/9-17/9*10^(1/2))*(2/3*(4*x+3)/(440/
9-136/9*10^(1/2)-(17/3-4/3*10^(1/2))^2)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3
*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(3/2)+32/3/(440/9-136/9
*10^(1/2)-(17/3-4/3*10^(1/2))^2)^2*(4*x+3)/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-
4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2))+1/3/(55/9-17/
9*10^(1/2))*(1/(55/9-17/9*10^(1/2))/(2*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^
(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*10^(1/2))^(1/2)-(17/3-4/3*10^(1/2))/(
55/9-17/9*10^(1/2))*(4*x+3)/(440/9-136/9*10^(1/2)-(17/3-4/3*10^(1/2))^2)/(2
*(x-2/3+1/3*10^(1/2))^2+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55/9-17/9*
10^(1/2))^(1/2)-3/(55/9-17/9*10^(1/2))/(55-17*10^(1/2))^(1/2)*arctanh(9/2*(
```

```
110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*10^(1/
2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1
/2))+55-17*10^(1/2))^(1/2))))
```

Maxima [B] time = 1.76204, size = 1723, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/300*sqrt(10)*(980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(
2*x^2 + 3*x + 1)^(3/2)) - 980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/
2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 5292*sqrt(10)*x/(374*sqrt(10)*sqrt(2*x^2
 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) - 5292*sqrt(10)*x/(374*sqrt(10)*s
qrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 15680*sqrt(10)*x/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 15680*sqrt(10)*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 3520*x/(
17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) + 3520*x/
(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 19008*
x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) + 19008
*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 5632
0*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) - 56320*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 750*sqrt
(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) - 7
50*sqrt(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/
2)) + 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 +
 3*x + 1)) - 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(
2*x^2 + 3*x + 1)) - 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*
sqrt(2*x^2 + 3*x + 1)) + 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1)
- 55*sqrt(2*x^2 + 3*x + 1)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 5
5*(2*x^2 + 3*x + 1)^(3/2)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55
*(2*x^2 + 3*x + 1)^(3/2)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 118
3*sqrt(2*x^2 + 3*x + 1)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183
*sqrt(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqr
t(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*
x^2 + 3*x + 1)) - 1215*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
- 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) +
 55)^(5/2) - 5*sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-1
7/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*
```

```
sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) +
55/9)^(5/2) - 9720*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqr
t(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10)
- 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(5/2) +
40*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/
abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9
/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(5/2))
```

Fricas [B] time = 1.45697, size = 1422, normalized size = 7.22

$$\frac{23520\,x^4 + 70560\,x^3 + \sqrt{3}\left(4\,x^4 + 12\,x^3 + 13\,x^2 + 6\,x + 1\right)\sqrt{1544809\,\sqrt{10} + 4885115}\,\log\left(-\frac{243\,\sqrt{10}x + \left(893\,\sqrt{10}\sqrt{3}x - 2824\,\sqrt{3}x\right)}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/150*(23520*x^4 + 70560*x^3 + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*
sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x + (893*sqrt(10)*sqrt(
3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(
2*x^2 + 3*x + 1) + 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sq
rt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)
*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(2*
x^2 + 3*x + 1) + 486)/x) + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt
(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x + (893*sqrt(10)*sqrt(3)*x
 + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x
^2 + 3*x + 1) - 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(
-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)*x
+ 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x^
2 + 3*x + 1) - 486)/x) + 76440*x^2 + 20*(460*x^3 + 1236*x^2 + 1071*x + 294)
*sqrt(2*x^2 + 3*x + 1) + 35280*x + 5880)/(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.31 $\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=15

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

```
[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]
```

Rubi [A] time = 0.016535, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1024, 206}

$$-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sqrt{5+2x+x^2}\right)\right)$$
$$= -\tanh^{-1}\left(\sqrt{5+2x+x^2}\right)$$

Mathematica [C] time = 0.0441422, size = 79, normalized size = 5.27

$$\frac{1}{2}\left(-\tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] (-ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]] - ArcTanh[(4 +
 I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]])/2
```

Maple [A] time = 0.046, size = 14, normalized size = 0.9

$$-\text{Artanh}\left(\sqrt{x^2+2\,x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] -arctanh((x^2+2*x+5)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^2+2\,x+5}\,(x^2+2\,x+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

Fricas [B] time = 1.32252, size = 136, normalized size = 9.07

$$\frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}(x+2)+3x+6\right)-\frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}x+x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x
^2 + 2*x + 5)*x + x + 4)
```

Sympy [B] time = 3.44338, size = 36, normalized size = 2.4

$$\frac{\log\left(-1+\frac{1}{\sqrt{x^2+2x+5}}\right)}{2}-\frac{\log\left(1+\frac{1}{\sqrt{x^2+2x+5}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] log(-1 + 1/sqrt(x**2 + 2*x + 5))/2 - log(1 + 1/sqrt(x**2 + 2*x + 5))/2
```

Giac [B] time = 1.15753, size = 78, normalized size = 5.2

$$\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2x+5}\right)^2+4x-4\sqrt{x^2+2x+5}+7\right)-\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2x+5}\right)^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log((x - sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) - 1/
2*log((x - sqrt(x^2 + 2*x + 5))^2 + 3)
```

3.32 $\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=44

$$\sqrt{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

```
[Out] Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[Sqrt[5 + 2*
x + x^2]]
```

Rubi [A] time = 0.0522627, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1025, 982, 204, 1024, 206}

$$\sqrt{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[Sqrt[5 + 2*
x + x^2]]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{4+x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx = \frac{1}{2}\int \frac{2+2x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx + 3\int \frac{1}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sqrt{5+2x+x^2}\right)\right) - 12\,\text{Subst}\left(\int \frac{1}{-24-2x^2}\,dx, x, \frac{2}{\sqrt{5}}\right.$$
$$= \sqrt{3}\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right) - \tanh^{-1}\left(\sqrt{5+2x+x^2}\right)$$

Mathematica [C] time = 0.0539322, size = 101, normalized size = 2.3

$$-\frac{1}{2}\left(1+i\sqrt{3}\right)\tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)-\frac{1}{2}\left(1-i\sqrt{3}\right)\tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -((1 + I*Sqrt[3])*ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]
])/2 - ((1 - I*Sqrt[3])*ArcTanh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x
```

```
+ x^2]])/2
```

Maple [A] time = 0.046, size = 40, normalized size = 0.9

$$-\text{Artanh}\left(\sqrt{x^2+2x+5}\right)+\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(2x+2)}{6}\frac{1}{\sqrt{x^2+2x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] -arctanh((x^2+2*x+5)^(1/2))+3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2
*x+2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+4}{\sqrt{x^2+2x+5}\left(x^2+2x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

Fricas [B] time = 1.28286, size = 327, normalized size = 7.43

$$-\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)+\frac{1}{2}\log\left(x^2-\sqrt{x^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + s
qrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + 1/2*log(x
^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^2 + 2*x
```

```
+ 5)*x + x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+4}{\left(x^2+2x+4\right)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral((x + 4)/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

Giac [B] time = 1.12917, size = 146, normalized size = 3.32

$$-\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)+\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2x+5}\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + sqrt(3)*arcta
n(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/2*log((x - sqrt(x^2 + 2*x + 5
))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) - 1/2*log((x - sqrt(x^2 + 2*x + 5))
^2 + 3)
```

3.33 $\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

Optimal. Leaf size=24

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

Rubi [A] time = 0.0187828, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1024, 206}

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{5+x+x^2}\right)\right)$$
$$= -\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)$$

Mathematica [C] time = 0.0627908, size = 90, normalized size = 3.75

$$-\frac{\tanh^{-1}\left(\frac{-2i\sqrt{11}x-i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)+\tanh^{-1}\left(\frac{2i\sqrt{11}x+i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -((ArcTanh[(19 - I*Sqrt[11] - (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2
])] + ArcTanh[(19 + I*Sqrt[11] + (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*Sqrt[5 + x +
x^2])])/Sqrt[2])
```

Maple [A] time = 0.099, size = 20, normalized size = 0.8

$$-\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{x^2+x+5}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x)
```

```
[Out] -arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x+1}{\sqrt{x^2+x+5}\,(x^2+x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

Fricas [A] time = 1.26232, size = 103, normalized size = 4.29

$$\frac{1}{2}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}\sqrt{x^2+x+5}+x+7}{x^2+x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((x^2 - 2*sqrt(2)*sqrt(x^2 + x + 5) + x + 7)/(x^2 + x + 3))
```

Sympy [A] time = 3.17531, size = 68, normalized size = 2.83

$$2\left(\begin{cases}-\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} & \text{for}\ \frac{1}{x^2+x+5}>\frac{1}{2}\\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{x^2+x+5}}\right)}{2} & \text{for}\ \frac{1}{x^2+x+5}<\frac{1}{2}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

```
[Out] 2*Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5)
 > 1/2), (-sqrt(2)*atanh(sqrt(2)/sqrt(x**2 + x + 5))/2, 1/(x**2 + x + 5) <
1/2))
```

Giac [B] time = 1.25939, size = 127, normalized size = 5.29

$$\frac{1}{2}\sqrt{2}\log\left(\left(x-\sqrt{x^2+x+5}\right)^2+\left(x-\sqrt{x^2+x+5}\right)\left(2\sqrt{2}+1\right)+\sqrt{2}+5\right)-\frac{1}{2}\sqrt{2}\log\left(\left(x-\sqrt{x^2+x+5}\right)^2-\left(x-\sqrt{x^2+x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log((x - sqrt(x^2 + x + 5))^2 + (x - sqrt(x^2 + x + 5))*(2*sqrt
(2) + 1) + sqrt(2) + 5) - 1/2*sqrt(2)*log((x - sqrt(x^2 + x + 5))^2 - (x -
sqrt(x^2 + x + 5))*(2*sqrt(2) - 1) - sqrt(2) + 5)
```

3.34 $\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(Sqrt[2/11]*(1 + 2*x))/Sqrt[5 + x + x^2]]/Sqrt[22]) - ArcTanh[Sqrt
[5 + x + x^2]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0501775, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1025, 982, 204, 1024, 206}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```


```
[Out] -(ArcTan[(Sqrt[2/11]*(1 + 2*x))/Sqrt[5 + x + x^2]]/Sqrt[22]) - ArcTanh[Sqrt
[5 + x + x^2]/Sqrt[2]]/Sqrt[2]
```


Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx\right)+\frac{1}{2}\int \frac{1+2x}{\left(3+x+x^2\right)\sqrt{5+x+x^2}}\,dx\\
&= \text{Subst}\left(\int \frac{1}{-11-2x^2}\,dx, x, \frac{1+2x}{\sqrt{5+x+x^2}}\right)-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{5+x+x^2}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(1+2x)}{\sqrt{5+x+x^2}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.0631257, size = 114, normalized size = 2.04

$$\frac{-\left(\sqrt{11}-i\right)\tanh^{-1}\left(\frac{-2i\sqrt{11}x-i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)-\left(\sqrt{11}+i\right)\tanh^{-1}\left(\frac{2i\sqrt{11}x+i\sqrt{11}+19}{4\sqrt{2}\sqrt{x^2+x+5}}\right)}{2\sqrt{22}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] (-((-I + Sqrt[11])*ArcTanh[(19 - I*Sqrt[11] - (2*I)*Sqrt[11]*x)/(4*Sqrt[2]*
Sqrt[5 + x + x^2])]) - (I + Sqrt[11])*ArcTanh[(19 + I*Sqrt[11] + (2*I)*Sqrt
[11]*x)/(4*Sqrt[2]*Sqrt[5 + x + x^2])])/(2*Sqrt[22])
```

Maple [A] time = 0.049, size = 45, normalized size = 0.8

$$-\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{x^2+x+5}\right)-\frac{\sqrt{22}}{22}\arctan\left(\frac{(1+2x)\sqrt{22}}{11}\frac{1}{\sqrt{x^2+x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+3)/(x^2+x+5)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)*
22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2+x+5}\left(x^2+x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

Fricas [B] time = 1.47429, size = 1018, normalized size = 18.18

$$-\frac{1}{33}\sqrt{11}\sqrt{6}\sqrt{3}\arctan\left(\frac{2}{33}\sqrt{11}\sqrt{3}\sqrt{\sqrt{6}\sqrt{3}(2x+1)+6x^2-\sqrt{x^2+x+5}\left(2\sqrt{6}\sqrt{3}+6x+3\right)+6x+30}+\frac{1}{33}\sqrt{11}\left(2\sqrt{6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/33*sqrt(11)*sqrt(6)*sqrt(3)*arctan(2/33*sqrt(11)*sqrt(3)*sqrt(sqrt(6)*sq
rt(3)*(2*x + 1) + 6*x^2 - sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) + 6*x + 3) +
 6*x + 30) + 1/33*sqrt(11)*(2*sqrt(6)*sqrt(3) + 6*x + 3) - 2/11*sqrt(11)*sq
rt(x^2 + x + 5)) + 1/33*sqrt(11)*sqrt(6)*sqrt(3)*arctan(-1/33*sqrt(11)*(2*s
qrt(6)*sqrt(3) - 6*x - 3) + 1/33*sqrt(11)*sqrt(-12*sqrt(6)*sqrt(3)*(2*x + 1
) + 72*x^2 + 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) - 6*x - 3) + 72*x + 36
0) - 2/11*sqrt(11)*sqrt(x^2 + x + 5)) + 1/12*sqrt(6)*sqrt(3)*log(12*sqrt(6)
*sqrt(3)*(2*x + 1) + 72*x^2 - 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) + 6*x
 + 3) + 72*x + 360) - 1/12*sqrt(6)*sqrt(3)*log(-12*sqrt(6)*sqrt(3)*(2*x + 1
) + 72*x^2 + 12*sqrt(x^2 + x + 5)*(2*sqrt(6)*sqrt(3) - 6*x - 3) + 72*x + 36
0)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^2+x+3)\sqrt{x^2+x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

```
[Out] Integral(x/((x**2 + x + 3)*sqrt(x**2 + x + 5)), x)
```

Giac [C] time = 1.24447, size = 217, normalized size = 3.88

$$-\frac{1}{44}\sqrt{2}\left(-i\sqrt{11}-11\right)\log\left(9\sqrt{2}\left(i\sqrt{22}-4\right)-36x+36\sqrt{x^2+x+5}-18\right)+\frac{1}{44}\sqrt{2}\left(i\sqrt{11}-11\right)\log\left(-9\sqrt{2}\left(i\sqrt{22}-4\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/44*sqrt(2)*(-I*sqrt(11) - 11)*log(9*sqrt(2)*(I*sqrt(22) - 4) - 36*x + 36
*sqrt(x^2 + x + 5) - 18) + 1/44*sqrt(2)*(I*sqrt(11) - 11)*log(-9*sqrt(2)*(I
*sqrt(22) - 4) - 36*x + 36*sqrt(x^2 + x + 5) - 18) - 1/44*sqrt(2)*(I*sqrt(1
1) - 11)*log(9*sqrt(2)*(-I*sqrt(22) - 4) - 36*x + 36*sqrt(x^2 + x + 5) - 18
) + 1/44*sqrt(2)*(-I*sqrt(11) - 11)*log(-9*sqrt(2)*(-I*sqrt(22) - 4) - 36*x
 + 36*sqrt(x^2 + x + 5) - 18)
```

3.35 $\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$

Optimal. Leaf size=249

$$\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{\sqrt{d+ex+fx^2}(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}+\frac{B\tanh}{2\sqrt{}}$$

```
[Out] -((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a
*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + ((B*e - 2*A*f)*(8*a*e*f - b*(
e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a
*f]*Sqrt[d + e*x + f*x^2])])/(2*e^(3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(
3/2)) + (B*ArcTanh[(Sqrt[b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d - a*e]])/(2*Sqr
t[b]*(b*d - a*e)^(3/2)*f)
```

Rubi [A] time = 0.909722, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1016, 1025, 982, 208, 1024}

$$\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{\sqrt{d+ex+fx^2}(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}+\frac{B\tanh}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a
*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + ((B*e - 2*A*f)*(8*a*e*f - b*(
e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a
*f]*Sqrt[d + e*x + f*x^2])])/(2*e^(3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(
3/2)) + (B*ArcTanh[(Sqrt[b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d - a*e]])/(2*Sqr
t[b]*(b*d - a*e)^(3/2)*f)
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
```

```
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx = -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{\int \frac{-\frac{1}{2}b(bd-ae)f^2\left(2bBde-2ae(Be-4A\right.}{\sqrt{d+ex+}}}{be(bd-}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{B\int \frac{e+2fx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)}\,dx}{4(bd-ae)f}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{(Be)\,\mathrm{Subst}\left(\int \frac{1}{bde-ae^2-bex^2}\,dx\right.}{2(bd-ae)}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{(Be-2Af)\left(8aef-b\left(e^2+\right.\right.}{2e^{3/2}(bd-a}$$

Mathematica [B] time = 1.65353, size = 767, normalized size = 3.08

$$-\frac{-(ae+bx(e+fx))\log\left(b(e+2fx)-\sqrt{b}\sqrt{e}\sqrt{be-4af}\right)\left(-8abef(Be-2Af)-b^{3/2}Be^{5/2}\sqrt{be-4af}+4a\sqrt{b}Be^{3/2}f\sqrt{be-4a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -(4*b*Sqrt[e]*Sqrt[b*d - a*e]*f*Sqrt[b*e - 4*a*f]*Sqrt[d + x*(e + f*x)]*(-(
B*e*(2*a + b*x)) + A*b*(e + 2*f*x)) - (-(b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4*a*f
]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f) +
b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[-(Sqrt[b]*Sqrt[e
]*Sqrt[b*e - 4*a*f]) + b*(e + 2*f*x)] + (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4*a*f
] - 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f) + b
^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*Sqrt[e]*S
qrt[b*e - 4*a*f] + b*(e + 2*f*x)] - (b^(3/2)*B*e^(5/2)*Sqrt[b*e - 4*a*f] -
4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*A*f) + b^2*(
B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*(e^(3/2)*Sqrt
[b*e - 4*a*f] + Sqrt[b]*(e^2 - 4*d*f) + 2*Sqrt[e]*f*Sqrt[b*e - 4*a*f]*x - 4
*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])] + (-(b^(3/2)*B*e^(5/2)*Sqrt[b*e
- 4*a*f]) + 4*a*Sqrt[b]*B*e^(3/2)*f*Sqrt[b*e - 4*a*f] - 8*a*b*e*f*(B*e - 2*
A*f) + b^2*(B*e - 2*A*f)*(e^2 + 4*d*f))*(a*e + b*x*(e + f*x))*Log[Sqrt[b]*(
e^(3/2)*Sqrt[b*e - 4*a*f] - Sqrt[b]*(e^2 - 4*d*f) + 2*Sqrt[e]*f*Sqrt[b*e -
```

```
4*a*f]*x + 4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])])/(4*b*e^(3/2)*(b*d -
 a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)*(a*e + b*x*(e + f*x)))
```

Maple [B] time = 0.325, size = 3606, normalized size = 14.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2*e/f-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x
-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x
-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)*A+1/2/f/(4*a*f
-b*e)/(a*e-b*d)/(x+1/2*e/f-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x-1/2*(-b*e+
(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+
(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)*B-1/2/f/e/(4*a*f-b*e)/b/(
a*e-b*d)/(x+1/2*e/f-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x-1/2*(-b*e+(-b*e*(
4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(
4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)*B*(-b*e*(4*a*f-b*e))^(1/2)+1/2/e
/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((
-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(
1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b
/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b
/f)-(a*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*A-1/4/
f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln(
(-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^
(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f)-(a*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B-1/4
/f/b/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(
1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*(
(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*
(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x-1/2*(-b*
e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B-2/(-b*e*(4*a*f-b*e))^(1/2)/e/(4*a*f-b*e
)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2
*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+
(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+
(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*
f-b*e))^(1/2))/b/f))*A*f+1/(-b*e*(4*a*f-b*e))^(1/2)/(4*a*f-b*e)/(-(a*e-b*d)
/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*
(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-
```

```
b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-
b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2)
)/b/f))*B+2/(-b*e*(4*a*f-b*e))^(1/2)/e/(4*a*f-b*e)/(-(a*e-b*d)/b)^(1/2)*ln(
(-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(
1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/
f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f
)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*A*f-1/(-b
*e*(4*a*f-b*e))^(1/2)/(4*a*f-b*e)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(
-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a
*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a
*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1
/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B-1/e/(4*a*f-b*e)/(a*e-b*d
)/(x+1/2*e/f+1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*
e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e
))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)*A+1/2/f/(4*a*f-b*e)/(a*e-b*d)/(x+1/2*e/f+
1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/
f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f
)-(a*e-b*d)/b)^(1/2)*B+1/2/f/e/(4*a*f-b*e)/b/(a*e-b*d)/(x+1/2*e/f+1/2/b/f*(
-b*e*(4*a*f-b*e))^(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-
b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*
d)/b)^(1/2)*B*(-b*e*(4*a*f-b*e))^(1/2)-1/2/e/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e
))^(1/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e
))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2
)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/
b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*
e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*A+1/4/f/(4*a*f-b*e)/b*(-b*e*(4*a*f-b*e))^
(1/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^
(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*(
(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(
x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(
-b*e*(4*a*f-b*e))^(1/2))/b/f))*B-1/4/f/b/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln(
(-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(
1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/
f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f
)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx + A}{\left(bfx^2 + bex + ae\right)^2 \sqrt{fx^2 + ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError
```

3.36 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{2(-2ah+x(2cg-bh)+bg)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] (-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^
2])
```

Rubi [A] time = 0.0498577, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {998, 636}

$$-\frac{2(-2ah+x(2cg-bh)+bg)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

```
[Out] (-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^
2])
```

Rule 998

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^2}\,dx = \frac{\int \frac{g+hx}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{d^2}$$
$$= -\frac{2(bg-2ah+(2cg-bh)x)}{\left(b^2-4ac\right)d^2\sqrt{a+bx+cx^2}}$$

Mathematica [A] time = 0.198364, size = 46, normalized size = 0.96

$$\frac{4ah-2bg+2bhx-4cgx}{d^2\left(b^2-4ac\right)\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

```
[Out] (-2*b*g + 4*a*h - 4*c*g*x + 2*b*h*x)/((b^2 - 4*a*c)*d^2*Sqrt[a + x*(b + c*x
)])
```

Maple [A] time = 0.044, size = 48, normalized size = 1.

$$-2\,\frac{bhx-2\,cgx+2\,ah-bg}{\sqrt{cx^2+bx+a}d^2\left(4\,ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x)
```

```
[Out] -2/(c*x^2+b*x+a)^(1/2)*(b*h*x-2*c*g*x+2*a*h-b*g)/d^2/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}\left(hx+g\right)}{\left(cdx^2+bdx+ad\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2, x)
```

Fricas [A] time = 3.7852, size = 181, normalized size = 3.77

$$-\frac{2\sqrt{cx^2+bx+a}\left(bg-2\,ah+\left(2\,cg-bh\right)x\right)}{\left(b^2c-4\,ac^2\right)d^2x^2+\left(b^3-4\,abc\right)d^2x+\left(ab^2-4\,a^2c\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="f
ricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2*c - 4*a*c^2)
*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x + (a*b^2 - 4*a^2*c)*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16985, size = 109, normalized size = 2.27

$$-\frac{2\left(\frac{\left(2\,cd^2g-bd^2h\right)x}{b^2d^4-4\,acd^4}+\frac{bd^2g-2\,ad^2h}{b^2d^4-4\,acd^4}\right)}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="g
iac")
```

```
[Out] -2*((2*c*d^2*g - b*d^2*h)*x/(b^2*d^4 - 4*a*c*d^4) + (b*d^2*g - 2*a*d^2*h)/(
b^2*d^4 - 4*a*c*d^4))/sqrt(c*x^2 + b*x + a)
```

3.37 $\int \frac{3+2x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$

Optimal. Leaf size=17

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rubi [A] time = 0.0224239, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1027, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = 3\,\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right)$$
$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

Mathematica [C] time = 0.298228, size = 165, normalized size = 9.71

$$\frac{1}{6}\left(\sqrt{1-2i\sqrt{2}}\left(\sqrt{2}+i\right)\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)+\sqrt{1+2i\sqrt{2}}\left(\sqrt{2}-i\right)\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (Sqrt[1 - (2*I)*Sqrt[2]]*(I + Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*
Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + Sqrt[1 + (2*I
)*Sqrt[2]]*(-I + Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(
Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/6
```

Maple [B] time = 0.091, size = 94, normalized size = 5.5

$$-\frac{\sqrt{4}\sqrt{3}}{6}\sqrt{3\,\frac{x^2}{\left(-3/2-x\right)^2}-12}\,\text{Artanh}\left(3\,\frac{x}{-3/2-x}\,\frac{1}{\sqrt{3\,\frac{x^2}{\left(-3/2-x\right)^2}-12}}\right)\frac{1}{\sqrt{\left(x^2\left(-\frac{3}{2}-x\right)^{-2}-4\right)\left(1+x\left(-\frac{3}{2}-x\right)^{-1}\right)^{-2}}}\left(1+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/6*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2
-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x+3}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [B] time = 1.79492, size = 142, normalized size = 8.35

$$-\frac{1}{4}\,\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right)+\frac{1}{4}\,\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((2*sqrt(-x^2
- 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+3}{\sqrt{-\left(x+1\right)\left(x+3\right)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral((2*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Giac [B] time = 1.16539, size = 132, normalized size = 7.76

$$\frac{1}{2}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{\left(x+2\right)^2}+1\right)-\frac{1}{2}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{\left(x+2\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)
^2/(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x
^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.38 $\int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=86

$$\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] - Sqrt[2]*ArcTan
[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] + ArcTanh[x/Sqrt[-3 - 4*x - x^
2]]
```

Rubi [A] time = 0.184598, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] - Sqrt[2]*ArcTan
[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] + ArcTanh[x/Sqrt[-3 - 4*x - x^
2]]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
```

```
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
```

```
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = -\left(3\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right) - \int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

$$= \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \frac{1}{2}\int -\frac{4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

$$= 2\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 2\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - 3\,\text{Subst}\left(\int\right.$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 16\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\,\text{S}$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)$$

$$= \sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \sqrt{2}\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right.$$

Mathematica [C] time = 0.11334, size = 150, normalized size = 1.74

$$-\frac{1}{2}i\left(\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)-\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-I/2)*(Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2]
)*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 - (2*I)*Sqrt[
2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]
]*Sqrt[-3 - 4*x - x^2])])
```

Maple [A] time = 0.092, size = 123, normalized size = 1.4

$$\frac{\sqrt{4}\sqrt{3}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)-\mathrm{Artanh}\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{}{\sqrt{\left(x^2\left(-\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x+3)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/
(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1
/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4\,x+3}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 1.64487, size = 360, normalized size = 4.19

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)-\frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4\,x-3}x}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
3)) + 1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((
2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x+3}{\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral((4*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Giac [B] time = 1.17591, size = 220, normalized size = 2.56

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-\cdots}\right.}{x+2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + sq
rt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*lo
g(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x
+ 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4
*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.39 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=136

$$\frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}-\frac{\sqrt{a+bx+cx^2}(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\sqrt{ad+bdx+cdx^2}}$$

```
[Out] -(((2*c*g - b*h)*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c*Sqrt[b^2 - 4*a*c]*d*Sqrt[a*d + b*d*x + c*d*x^2])) + (h*Sqrt[a + b*x
+ c*x^2]*Log[a + b*x + c*x^2])/(2*c*d*Sqrt[a*d + b*d*x + c*d*x^2])
```

Rubi [A] time = 0.11423, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {999, 634, 618, 206, 628}

$$\frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}-\frac{\sqrt{a+bx+cx^2}(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\sqrt{ad+bdx+cdx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x]
```

```
[Out] -(((2*c*g - b*h)*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c*Sqrt[b^2 - 4*a*c]*d*Sqrt[a*d + b*d*x + c*d*x^2])) + (h*Sqrt[a + b*x
+ c*x^2]*Log[a + b*x + c*x^2])/(2*c*d*Sqrt[a*d + b*d*x + c*d*x^2])
```

Rule 999

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b
*x + c*x^2)^FracPart[p])/(d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]), Int[
(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && !IntegerQ[p] &
& !IntegerQ[q] && !GtQ[c/f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^{3/2}}\,dx &= \frac{\sqrt{a+bx+cx^2}\int \frac{g+hx}{ad+bdx+cdx^2}\,dx}{\sqrt{ad+bdx+cdx^2}}\\
&= \frac{\left(h\sqrt{a+bx+cx^2}\right)\int \frac{bd+2cdx}{ad+bdx+cdx^2}\,dx}{2cd\sqrt{ad+bdx+cdx^2}} + \frac{\left((2cdg-bdh)\sqrt{a+bx+cx^2}\right)\int \frac{1}{ad+bdx+cdx^2}\,dx}{2cd\sqrt{ad+bdx+cdx^2}}\\
&= \frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}} - \frac{\left((2cdg-bdh)\sqrt{a+bx+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\left(b^2-4ac\right)d^2-x^2}\right.}{cd\sqrt{ad+bdx+cdx^2}}\\
&= -\frac{(2cg-bh)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,d\sqrt{ad+bdx+cdx^2}} + \frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}
\end{aligned}
$$

Mathematica [A] time = 0.095123, size = 108, normalized size = 0.79

$$
\frac{(a+x(b+cx))^{3/2}\left((4cg-2bh)\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right)+h\sqrt{4ac-b^2}\log(a+x(b+cx))\right)}{2c\sqrt{4ac-b^2}(d(a+x(b+cx)))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x
]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*((4*c*g - 2*b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*
a*c]] + Sqrt[-b^2 + 4*a*c]*h*Log[a + x*(b + c*x)]))/(2*c*Sqrt[-b^2 + 4*a*c]
*(d*(a + x*(b + c*x)))^(3/2))
```

Maple [A] time = 0.234, size = 122, normalized size = 0.9

$$-\frac{1}{2\,cd^{2}}\sqrt{d\left(cx^{2}+bx+a\right)}\left(2\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^{2}}}\right)bh-4\arctan\left(\frac{2\,cx+b}{\sqrt{4\,ac-b^{2}}}\right)cg-h\ln\left(cx^{2}+bx+a\right)\sqrt{4\,ac-b^{2}}\right)\frac{}{\sqrt{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x)
```

```
[Out] -1/2/(c*x^2+b*x+a)^(1/2)*(d*(c*x^2+b*x+a))^(1/2)*(2*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2))*b*h-4*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*c*g-h*ln(c*x^2+b*x+a
)*(4*a*c-b^2)^(1/2))/d^2/c/(4*a*c-b^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(d\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**(3/2),x)
```

```
[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)/(d*(a + b*x + c*x**2))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+bx+a}\,(hx+g)}{\left(cdx^2+bdx+ad\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2), x)
```

3.40 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=212

$$-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}(8bc-15adx)}{60d^2(a+bx)}-\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+}}{8d(a+bx)}$$

```
[Out] -(a*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + (b
*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((8
*b*c - 15*a*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(60*d^2*(
a + b*x)) - (a*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(8*d^(3/2)*(a + b*x))
```

Rubi [A] time = 0.116095, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {1001, 833, 780, 195, 217, 206}

$$-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}(8bc-15adx)}{60d^2(a+bx)}-\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+}}{8d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -(a*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + (b
*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((8
*b*c - 15*a*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(60*d^2*(
a + b*x)) - (a*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(8*d^(3/2)*(a + b*x))
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int x^2\left(2ab+2b^2x\right)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\int x\left(-4b^2c+10abdx\right)}{5d\left(2ab+2b^2x\right)}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{(8bc-15adx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)}{60d^2(a+bx)}$$

$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{(8bc-}{}$$

$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{(8bc-}{}$$

$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{(8bc-}{}$$

Mathematica [A] time = 0.156995, size = 129, normalized size = 0.61

$$\frac{\sqrt{(a+bx)^2}\sqrt{c+dx^2}\left(\sqrt{\frac{dx^2}{c}+1}\left(15adx\left(c+2dx^2\right)+8b\left(-2c^2+cdx^2+3d^2x^4\right)\right)-15ac^{3/2}\sqrt{d}\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{120d^2(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[1 + (d*x^2)/c]*(15*a*d*x*(c + 2*d*
x^2) + 8*b*(-2*c^2 + c*d*x^2 + 3*d^2*x^4)) - 15*a*c^(3/2)*Sqrt[d]*ArcSinh[(
Sqrt[d]*x)/Sqrt[c]]))/(120*d^2*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

Maple [C] time = 0.211, size = 103, normalized size = 0.5

$$-\frac{\operatorname{csgn}(bx+a)}{120}\left(-24\,d^{3/2}\left(dx^2+c\right)^{3/2}x^2b-30\,d^{3/2}\left(dx^2+c\right)^{3/2}xa+16\,\sqrt{d}\left(dx^2+c\right)^{3/2}bc+15\,d^{3/2}\sqrt{dx^2+c}xac+15\,\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] -1/120*csgn(b*x+a)*(-24*d^(3/2)*(d*x^2+c)^(3/2)*x^2*b-30*d^(3/2)*(d*x^2+c)^
(3/2)*x*a+16*d^(1/2)*(d*x^2+c)^(3/2)*b*c+15*d^(3/2)*(d*x^2+c)^(1/2)*x*a*c+1
5*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*a*c^2*d)/d^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\sqrt{(bx+a)^2}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2, x)
```

Fricas [A] time = 1.96571, size = 435, normalized size = 2.05

$$\left[\frac{15\,ac^2\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(24\,bd^2x^4+30\,ad^2x^3+8\,bcdx^2+15\,acdx-16\,bc^2\right)\sqrt{dx^2+c}}{240\,d^2}, \frac{15\,ac^2\sqrt{-d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*a*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
2*(24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqrt(
d*x^2 + c))/d^2, 1/120*(15*a*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)
) + (24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqr
t(d*x^2 + c))/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

Giac [A] time = 1.17183, size = 158, normalized size = 0.75

$$\frac{ac^2\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)}{8d^{\frac{3}{2}}}+\frac{1}{120}\sqrt{dx^2+c}\left(\left(2\left(3\left(4bx\operatorname{sgn}(bx+a)+5a\operatorname{sgn}(bx+a)\right)x+\frac{4bc\operatorname{sgn}(bx+\right.\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*a*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/1
20*sqrt(d*x^2 + c)*((2*(3*(4*b*x*sgn(b*x + a) + 5*a*sgn(b*x + a))*x + 4*b*c
*sgn(b*x + a)/d)*x + 15*a*c*sgn(b*x + a)/d)*x - 16*b*c^2*sgn(b*x + a)/d^2)
```

3.41 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=161

$$-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)}$$

```
[Out] -(b*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + ((
4*a + 3*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(12*d*(a + b*
x)) - (b*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(8*d^(3/2)*(a + b*x))
```

Rubi [A] time = 0.0673629, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1001, 780, 195, 217, 206}

$$-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -(b*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(8*d*(a + b*x)) + ((
4*a + 3*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(12*d*(a + b*
x)) - (b*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(8*d^(3/2)*(a + b*x))
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
```

```
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int x\left(2ab+2b^2x\right)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)} - \frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)\int\sqrt{c+dx^2}}{2d\left(2ab+2b^2x\right)}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)} -$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)} -$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{12d(a+bx)} -$$

Mathematica [A] time = 0.11876, size = 117, normalized size = 0.73

$$\frac{\sqrt{(a+bx)^2}\sqrt{c+dx^2}\left(\sqrt{d}\sqrt{\frac{dx^2}{c}+1}\left(8a\left(c+dx^2\right)+3bx\left(c+2dx^2\right)\right)-3bc^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{24d^{3/2}(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*Sqrt[1 + (d*x^2)/c]*(8*a*(c + d
*x^2) + 3*b*x*(c + 2*d*x^2)) - 3*b*c^(3/2)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(
24*d^(3/2)*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

Maple [C] time = 0.223, size = 83, normalized size = 0.5

$$\frac{\operatorname{csgn}(bx+a)}{24}\left(6\sqrt{d}\left(dx^2+c\right)^{3/2}xb+8a\left(dx^2+c\right)^{3/2}\sqrt{d}-3\sqrt{d}\sqrt{dx^2+c}xbc-3\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)bc^2\right)d^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/24*csgn(b*x+a)*(6*d^(1/2)*(d*x^2+c)^(3/2)*x*b+8*a*(d*x^2+c)^(3/2)*d^(1/2)
-3*d^(1/2)*(d*x^2+c)^(1/2)*x*b*c-3*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*b*c^2)/d^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{dx^2+c}\sqrt{(bx+a)^2}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x, x)
```

Fricas [A] time = 1.90422, size = 381, normalized size = 2.37

$$\left[\frac{3\,bc^2\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(6\,bd^2x^3+8\,ad^2x^2+3\,bcdx+8\,acd\right)\sqrt{dx^2+c}}{48\,d^2},\ \frac{3\,bc^2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*b*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*
(6*b*d^2*x^3 + 8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2, 1/2
4*(3*b*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (6*b*d^2*x^3 + 8*a
*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

Giac [A] time = 1.14935, size = 132, normalized size = 0.82

$$\frac{bc^2\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)}{8\,d^{\frac{3}{2}}}+\frac{1}{24}\,\sqrt{dx^2+c}\left(\left(2\left(3\,bx\mathrm{sgn}\left(bx+a\right)+4\,a\mathrm{sgn}\left(bx+a\right)\right)x+\frac{3\,bc\mathrm{sgn}\left(bx+a\right)}{d}\right)x\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/2
4*sqrt(d*x^2 + c)*((2*(3*b*x*sgn(b*x + a) + 4*a*sgn(b*x + a))*x + 3*b*c*sgn
(b*x + a)/d)*x + 8*a*c*sgn(b*x + a)/d)
```

3.42 $\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal. Leaf size=148

$$\frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)}+\frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)}+\frac{ac\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}$$

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) + (b*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(3*d*(a + b*x)) + (a*c*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a
+ b*x))
```

Rubi [A] time = 0.0567116, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {970, 641, 195, 217, 206}

$$\frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)}+\frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)}+\frac{ac\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) + (b*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(3*d*(a + b*x)) + (a*c*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a
+ b*x))
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \left(2ab+2b^2x\right)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(2ab\sqrt{a^2+2abx+b^2x^2}\right)\int \sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(abc\sqrt{a^2+2\right.}{}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(abc\sqrt{a^2+2\right.}{}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{ac\sqrt{a^2+2ab}}{}$$

Mathematica [A] time = 0.0588663, size = 85, normalized size = 0.57

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{c+dx^2}\left(3adx+2b\left(c+dx^2\right)\right)+3ac\sqrt{d}\log\left(\sqrt{d}\sqrt{c+dx^2}+dx\right)\right)}{6d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(3*a*d*x + 2*b*(c + d*x^2)) + 3*a*c*Sqr
t[d]*Log[d*x + Sqrt[d]*Sqrt[c + d*x^2]]))/(6*d*(a + b*x))
```

Maple [C] time = 0.216, size = 65, normalized size = 0.4

$$\frac{\operatorname{csgn}(bx+a)}{6}\left(2\,b\left(dx^2+c\right)^{3/2}\sqrt{d}+3\,d^{3/2}\sqrt{dx^2+c}xa+3\,\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)acd\right)d^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x)
```

```
[Out] 1/6*csgn(b*x+a)*(2*b*(d*x^2+c)^(3/2)*d^(1/2)+3*d^(3/2)*(d*x^2+c)^(1/2)*x*a+
3*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*a*c*d)/d^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\sqrt{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2), x)
```

Fricas [A] time = 1.87309, size = 316, normalized size = 2.14

$$\left[\frac{3\,ac\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(2\,bdx^2+3\,adx+2\,bc\right)\sqrt{dx^2+c}}{12\,d},\,-\frac{3\,ac\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-\left(2\,bdx^2+3\right.}{6\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2
*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c))/d, -1/6*(3*a*c*sqrt(-d)*arctan
(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c
))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx^2}\sqrt{(a + bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

Giac [A] time = 1.20071, size = 107, normalized size = 0.72

$$-\frac{ac \log\left(\left|-\sqrt{d}x + \sqrt{dx^2 + c}\right|\right) \operatorname{sgn}(bx + a)}{2\sqrt{d}} + \frac{1}{6}\sqrt{dx^2 + c}\left((2\,bx\operatorname{sgn}(bx + a) + 3\,a\operatorname{sgn}(bx + a))x + \frac{2\,bc\operatorname{sgn}(bx + a)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/6*
sqrt(d*x^2 + c)*((2*b*x*sgn(b*x + a) + 3*a*sgn(b*x + a))*x + 2*b*c*sgn(b*x
+ a)/d)
```

3.43 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx+b^2x^2}(2a+bx)\sqrt{c+dx^2}}{2(a+bx)}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}-\frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

```
[Out] ((2*a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) +
 (b*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
2*Sqrt[d]*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqr
t[c + d*x^2]/Sqrt[c]])/(a + b*x)
```

Rubi [A] time = 0.119417, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1001, 815, 844, 217, 206, 266, 63, 208}

$$\frac{\sqrt{a^2+2abx+b^2x^2}(2a+bx)\sqrt{c+dx^2}}{2(a+bx)}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}-\frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] ((2*a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) +
 (b*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
2*Sqrt[d]*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqr
t[c + d*x^2]/Sqrt[c]])/(a + b*x)
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x}\,dx}{2ab+2b^2x}$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{4abcd+2b^2cdx}{x\sqrt{c+dx^2}}\,dx}{2d\left(2ab+2b^2x\right)}$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{\left(2abc\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+dx^2}}\,dx}{2ab+2b^2x} + \cdots$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{\left(abc\sqrt{a^2+2abx+b^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} \cdots\right.}{2ab+2b^2x}$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} + \cdots$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \cdots$$

Mathematica [A] time = 0.146834, size = 139, normalized size = 0.87

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{d}\sqrt{\frac{dx^2}{c}+1}\left((2a+bx)\sqrt{c+dx^2}-2a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)+b\sqrt{c}\sqrt{c+dx^2}\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{2\sqrt{d}(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*Sqrt[c]*Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]
+ Sqrt[d]*Sqrt[1 + (d*x^2)/c]*((2*a + b*x)*Sqrt[c + d*x^2] - 2*a*Sqrt[c]*Ar
cTanh[Sqrt[c + d*x^2]/Sqrt[c]])))/(2*Sqrt[d]*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

Maple [C] time = 0.205, size = 94, normalized size = 0.6

$$-\frac{\operatorname{csgn}(bx+a)}{2}\left(2\sqrt{d}\ln\left(2\frac{\sqrt{c}\sqrt{dx^2+c}+c}{x}\right)\sqrt{c}a-\sqrt{d}\sqrt{dx^2+c}xb-2\sqrt{d}\sqrt{dx^2+c}a-\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)bc\right)\frac{1}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x)
```

```
[Out] -1/2*csgn(b*x+a)*(2*d^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*c^(1/2)*a-d
^(1/2)*(d*x^2+c)^(1/2)*x*b-2*d^(1/2)*(d*x^2+c)^(1/2)*a-ln(x*d^(1/2)+(d*x^2+
c)^(1/2))*b*c)/d^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x, x)
```

Fricas [A] time = 1.85328, size = 859, normalized size = 5.37

$$\left[\frac{bc\sqrt{d}\log\left(-2\,dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,a\sqrt{c}d\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(bdx+2\,ad)\sqrt{dx^2+c}}{4\,d},-\frac{bc\sqrt{-d}\arctan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*a*sqr
t(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(b*d*x + 2*a
*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c
```

```
)) - a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - (b*d
*x + 2*a*d)*sqrt(d*x^2 + c))/d, 1/4*(4*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*
x^2 + c)) + b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2
*(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/s
qrt(d*x^2 + c)) - 2*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (b*d*x
+ 2*a*d)*sqrt(d*x^2 + c))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x, x)
```

Giac [A] time = 1.20625, size = 138, normalized size = 0.86

$$\frac{2\,ac\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}\left(bx+a\right)}{\sqrt{-c}}-\frac{bc\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}\left(bx+a\right)}{2\sqrt{d}}+\frac{1}{2}\sqrt{dx^2+c}(bx\mathrm{sgn}\left(bx+a\right)+2\,a\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c)
 - 1/2*b*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/
2*sqrt(d*x^2 + c)*(b*x*sgn(b*x + a) + 2*a*sgn(b*x + a))
```

3.44 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2}}{x(a+bx)}+\frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(x*(a + b*x)))
+ (a*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(a + b*x) - (b*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(a + b*x)
```

Rubi [A] time = 0.113191, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1001, 813, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2}}{x(a+bx)}+\frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(x*(a + b*x)))
+ (a*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(a + b*x) - (b*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(a + b*x)
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
```

```
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(2ab+2b^2x\right)\sqrt{c+dx^2}}{x^2}\,dx}{2ab+2b^2x}$$
$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-4b^2c-4abdx}{x\sqrt{c+dx^2}}\,dx}{2\left(2ab+2b^2x\right)}$$
$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{\left(2b^2c\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+dx^2}}\,dx}{2ab+2b^2x} +$$
$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\right.}{2ab+2b^2x}$$
$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}$$
$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}$$

Mathematica [A] time = 0.283455, size = 118, normalized size = 0.76

$$\frac{\sqrt{(a+bx)^2}\left(\frac{(bx-a)\sqrt{c+dx^2}}{x}+\frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{dx^2}{c}+1}\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c+dx^2}}-b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{a+bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(((-a + b*x)*Sqrt[c + d*x^2])/x + (a*Sqrt[c]*Sqrt[d]*Sqr
t[1 + (d*x^2)/c]*ArcSinh[(Sqrt[d]*x)/Sqrt[c]])/Sqrt[c + d*x^2] - b*Sqrt[c]*
ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(a + b*x)
```

Maple [C] time = 0.228, size = 120, normalized size = 0.8

$$-\frac{\text{csgn}\,(bx+a)}{cx}\left(c^{\frac{3}{2}}\ln\left(2\,\frac{\sqrt{c}\sqrt{dx^2+c}+c}{x}\right)\sqrt{d}xb-d^{\frac{3}{2}}\sqrt{dx^2+c}x^2a+a\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{d}-\sqrt{d}\sqrt{dx^2+c}xbc-\ln\left(x\sqrt{d}+\sqrt{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x)
```

```
[Out] -csgn(b*x+a)*(c^(3/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*d^(1/2)*x*b-d^(3/
2)*(d*x^2+c)^(1/2)*x^2*a+a*(d*x^2+c)^(3/2)*d^(1/2)-d^(1/2)*(d*x^2+c)^(1/2)*
x*b*c-ln(x*d^(1/2)+(d*x^2+c)^(1/2))*x*a*c*d)/x/c/d^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2, x)
```

Fricas [A] time = 1.71983, size = 821, normalized size = 5.26

$$\left[\frac{a\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+b\sqrt{c}x\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,\sqrt{dx^2+c}(bx-a)}{2\,x}, -\frac{2\,a\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}}{\sqrt{dx^2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + b*sqrt(
c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c
)*(b*x - a))/x, -1/2*(2*a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b
*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x
^2 + c)*(b*x - a))/x, 1/2*(2*b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c))
+ a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*sqrt(d*x^
2 + c)*(b*x - a))/x, -(a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b*
sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*(b*x - a))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**2, x)
```

Giac [A] time = 1.18116, size = 170, normalized size = 1.09

$$\frac{2\,bc\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sgn}(bx+a)}{\sqrt{-c}} - a\sqrt{d}\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a) + \sqrt{dx^2+c}\,b\operatorname{sgn}(bx+a) + \frac{2\,ac}{\left(\sqrt{d}x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*b*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c)
 - a*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + sqrt(d*x
^2 + c)*b*sgn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a)/((sqrt(d)*x - sqrt(d*x^
2 + c))^2 - c)
```

3.45 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{ad\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx)}$$

```
[Out] -((a + 2*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*x^2*(a + b*
x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(a + b*x) - (a*d*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c + d
*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x))
```

Rubi [A] time = 0.116269, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1001, 811, 844, 217, 206, 266, 63, 208}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}-\frac{ad\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] -((a + 2*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*x^2*(a + b*
x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c +
 d*x^2]])/(a + b*x) - (a*d*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c + d
*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x))
```

Rule 1001

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x^3}\,dx}{2ab+2b^2x}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-4abcd-8b^2cdx}{x\sqrt{c+dx^2}}\,dx}{4c\left(2ab+2b^2x\right)}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{\left(abd\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+dx^2}}\,dx}{2ab+2b^2x} + \cdots$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{\left(abd\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} \cdots\right.}{2\left(2ab+2b^2x\right)}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}$$

Mathematica [A] time = 0.116849, size = 126, normalized size = 0.78

$$-\frac{\sqrt{(a+bx)^2}\sqrt{c+dx^2}\left(c(a+2bx)\sqrt{\frac{dx^2}{c}+1}+adx^2\tanh^{-1}\left(\sqrt{\frac{dx^2}{c}+1}\right)-2b\sqrt{c}\sqrt{d}x^2\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{2cx^2(a+bx)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*Sqrt[c + d*x^2]*(c*(a + 2*b*x)*Sqrt[1 + (d*x^2)/c] - 2*
b*Sqrt[c]*Sqrt[d]*x^2*ArcSinh[(Sqrt[d]*x)/Sqrt[c]] + a*d*x^2*ArcTanh[Sqrt[1
 + (d*x^2)/c]]))/(2*c*x^2*(a + b*x)*Sqrt[1 + (d*x^2)/c])
```

Maple [C] time = 0.24, size = 141, normalized size = 0.9

$$-\frac{\operatorname{csgn}(bx+a)}{2\,cx^2}\left(\sqrt{c}\ln\left(2\,\frac{\sqrt{c}\sqrt{dx^2+c}+c}{x}\right)d^{\frac{3}{2}}x^2a-2\,d^{3/2}\sqrt{dx^2+c}x^3b+2\,\sqrt{d}\left(dx^2+c\right)^{3/2}xb-d^{\frac{3}{2}}\sqrt{dx^2+c}x^2a-2\,\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*csgn(b*x+a)*(c^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*d^(3/2)*x^2*a
-2*d^(3/2)*(d*x^2+c)^(1/2)*x^3*b+2*d^(1/2)*(d*x^2+c)^(3/2)*x*b-d^(3/2)*(d*x
^2+c)^(1/2)*x^2*a-2*ln(x*d^(1/2)+(d*x^2+c)^(1/2))*x^2*b*c*d+a*(d*x^2+c)^(3/
2)*d^(1/2))/x^2/c/d^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3, x)
```

Fricas [A] time = 1.72258, size = 934, normalized size = 5.8

$$\left[\frac{2\,bc\sqrt{d}x^2\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+a\sqrt{c}dx^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,(2\,bcx+ac)\sqrt{dx^2+c}}{4\,cx^2},-\frac{4\,bc\sqrt{-d}x^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c*sqrt(d)*x^2*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + a
*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*b
```

```
*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), -1/4*(4*b*c*sqrt(-d)*x^2*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) - a*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(c) + 2*c)/x^2) + 2*(2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*(a*sqrt
(-c)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + b*c*sqrt(d)*x^2*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^
2), -1/2*(2*b*c*sqrt(-d)*x^2*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - a*sqrt(-c
)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x + a*c)*sqrt(d*x^2 + c))
/(c*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**3, x)
```

Giac [A] time = 1.14724, size = 269, normalized size = 1.67

$$\frac{ad\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sgn}(bx+a)}{\sqrt{-c}} - b\sqrt{d}\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a) + \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^3 ad\operatorname{sgn}(bx+a)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] a*d*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c) -
 b*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + ((sqrt(d)*
x - sqrt(d*x^2 + c))^3*a*d*sgn(b*x + a) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2
*b*c*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + c))*a*c*d*sgn(b*x + a
) - 2*b*c^2*sqrt(d)*sgn(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^2
```

3.46 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal. Leaf size=317

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}\left(-6dx(10ad-7be)+50ade+32bcd-35be^2\right)}{240d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+}}{1}$$

```
[Out] -((2*a*d*(4*c*d - 5*e^2) - b*(12*c*d*e - 7*e^3))*(e + 2*d*x)*Sqrt[a^2 + 2*a
*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(128*d^4*(a + b*x)) + (b*x^2*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((32*b*c*
d + 50*a*d*e - 35*b*e^2 - 6*d*(10*a*d - 7*b*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]*(c + e*x + d*x^2)^(3/2))/(240*d^3*(a + b*x)) - ((4*c*d - e^2)*(8*a*c*d
^2 - 12*b*c*d*e - 10*a*d*e^2 + 7*b*e^3)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTa
nh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b*x))
```

Rubi [A] time = 0.333016, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 832, 779, 612, 621, 206}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2+ex\right)^{3/2}\left(-6dx(10ad-7be)+50ade+32bcd-35be^2\right)}{240d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```


```
[Out] -((2*a*d*(4*c*d - 5*e^2) - b*(12*c*d*e - 7*e^3))*(e + 2*d*x)*Sqrt[a^2 + 2*a
*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(128*d^4*(a + b*x)) + (b*x^2*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((32*b*c*
d + 50*a*d*e - 35*b*e^2 - 6*d*(10*a*d - 7*b*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]*(c + e*x + d*x^2)^(3/2))/(240*d^3*(a + b*x)) - ((4*c*d - e^2)*(8*a*c*d
^2 - 12*b*c*d*e - 10*a*d*e^2 + 7*b*e^3)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTa
nh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b*x))
```


Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int x^2\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+ex+dx^2\right)^{3/2}}{5d(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\int x\left(-4b^2c+\right.}{5d\left(2ab\right.}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+ex+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{\left(32bcd+50ade-35be^2-6d(10\right.}{}$$

$$= -\frac{\left(8acd^2-12bcde-10ade^2+7be^3\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+\right.}{128d^4(a+bx)}$$

$$= -\frac{\left(8acd^2-12bcde-10ade^2+7be^3\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+\right.}{128d^4(a+bx)}$$

$$= -\frac{\left(8acd^2-12bcde-10ade^2+7be^3\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+\right.}{128d^4(a+bx)}$$

Mathematica [A] time = 0.281873, size = 198, normalized size = 0.62

$$\frac{\sqrt{(a+bx)^2}\left(-\frac{5\left(2ad\left(4cd-5e^2\right)+b\left(7e^3-12cde\right)\right)\left(\left(4cd-e^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right)+2\sqrt{d}(2dx+e)\sqrt{c+x(dx+e)}\right)}{256d^{7/2}} + \frac{(c+x(dx+e))^{3/2}(10ad(6dx-5e)-32bcd+7be(}{48d^2}\right.}{5d(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(b*x^2*(c + x*(e + d*x))^(3/2) + ((c + x*(e + d*x))^(3/2
)*(-32*b*c*d + 7*b*e*(5*e - 6*d*x) + 10*a*d*(-5*e + 6*d*x)))/(48*d^2) - (5*
(2*a*d*(4*c*d - 5*e^2) + b*(-12*c*d*e + 7*e^3))*(2*Sqrt[d]*(e + 2*d*x)*Sqrt
[c + x*(e + d*x)] + (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x
*(e + d*x)])]))/(256*d^(7/2))))/(5*d*(a + b*x))
```

Maple [C] time = 0.214, size = 530, normalized size = 1.7

$$\frac{\operatorname{csgn}(bx+a)}{3840}\left(768\,d^{9/2}\left(dx^2+ex+c\right)^{3/2}x^2b+960\,d^{9/2}\left(dx^2+ex+c\right)^{3/2}xa-672\,d^{7/2}\left(dx^2+ex+c\right)^{3/2}xbe-800\,d^{7/2}\left(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/3840*csgn(b*x+a)*(768*d^(9/2)*(d*x^2+e*x+c)^(3/2)*x^2*b+960*d^(9/2)*(d*x^
2+e*x+c)^(3/2)*x*a-672*d^(7/2)*(d*x^2+e*x+c)^(3/2)*x*b*e-800*d^(7/2)*(d*x^2
+e*x+c)^(3/2)*a*e-512*d^(7/2)*(d*x^2+e*x+c)^(3/2)*b*c+560*d^(5/2)*(d*x^2+e*
x+c)^(3/2)*b*e^2-480*d^(9/2)*(d*x^2+e*x+c)^(1/2)*x*a*c+600*d^(7/2)*(d*x^2+e
*x+c)^(1/2)*x*a*e^2+720*d^(7/2)*(d*x^2+e*x+c)^(1/2)*x*b*c*e-420*d^(5/2)*(d*
x^2+e*x+c)^(1/2)*x*b*e^3-240*d^(7/2)*(d*x^2+e*x+c)^(1/2)*a*c*e+300*d^(5/2)*
(d*x^2+e*x+c)^(1/2)*a*e^3+360*d^(5/2)*(d*x^2+e*x+c)^(1/2)*b*c*e^2-210*d^(3/
2)*(d*x^2+e*x+c)^(1/2)*b*e^4-480*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*
x+e)/d^(1/2))*a*c^2*d^4+720*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/
d^(1/2))*a*c*d^3*e^2-150*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(
1/2))*a*d^2*e^4+720*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))
*b*c^2*d^3*e-600*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*
c*d^2*e^3+105*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d*e
^5)/d^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2 + ex + c}\sqrt{(bx + a)^2}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2, x)
```

Fricas [A] time = 1.84483, size = 1250, normalized size = 3.94

$$\left[-\frac{15\left(32\,ac^2d^3 - 48\,bc^2d^2e - 48\,acd^2e^2 + 40\,bcde^3 + 10\,ade^4 - 7\,be^5\right)\sqrt{d}\log\left(8\,d^2x^2 + 8\,dex + 4\,\sqrt{dx^2 + ex + c}(2\,dx + e)\sqrt{\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^3
 + 10*a*d*e^4 - 7*b*e^5)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e
*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - 4*(384*b*d^5*x^4 - 256*b*c^2*d
^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4 + 48*(10
*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x^2 + 2*(
120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(d*x^2 +
e*x + c))/d^5, 1/3840*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 +
 40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x
+ c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(384*b*d^5*x^4 - 256
*b*c^2*d^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4
+ 48*(10*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x
^2 + 2*(120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(
d*x^2 + e*x + c))/d^5]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18472, size = 497, normalized size = 1.57

$$\frac{1}{1920}\sqrt{dx^2+xe+c}\left(2\left(4\left(6\left(8\,bx\mathrm{sgn}\left(bx+a\right)+\frac{10\,ad^4\mathrm{sgn}\left(bx+a\right)+bd^3e\mathrm{sgn}\left(bx+a\right)}{d^4}\right)x+\frac{16\,bcd^3\mathrm{sgn}\left(bx+a\right)+10\,ad\right.}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(d*x^2 + x*e + c)*(2*(4*(6*(8*b*x*sgn(b*x + a) + (10*a*d^4*sgn(b
*x + a) + b*d^3*e*sgn(b*x + a))/d^4)*x + (16*b*c*d^3*sgn(b*x + a) + 10*a*d^
3*e*sgn(b*x + a) - 7*b*d^2*e^2*sgn(b*x + a))/d^4)*x + (120*a*c*d^3*sgn(b*x
+ a) - 116*b*c*d^2*e*sgn(b*x + a) - 50*a*d^2*e^2*sgn(b*x + a) + 35*b*d*e^3*
sgn(b*x + a))/d^4)*x - (256*b*c^2*d^2*sgn(b*x + a) + 520*a*c*d^2*e*sgn(b*x
+ a) - 460*b*c*d*e^2*sgn(b*x + a) - 150*a*d*e^3*sgn(b*x + a) + 105*b*e^4*sg
```

```
n(b*x + a))/d^4) + 1/256*(32*a*c^2*d^3*sgn(b*x + a) - 48*b*c^2*d^2*e*sgn(b*
x + a) - 48*a*c*d^2*e^2*sgn(b*x + a) + 40*b*c*d*e^3*sgn(b*x + a) + 10*a*d*e
^4*sgn(b*x + a) - 7*b*e^5*sgn(b*x + a))*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2
+ x*e + c))*sqrt(d) - e))/d^(9/2)
```

3.47 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal. Leaf size=227

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+dx^2+ex}\left(8ade+4bcd-5be^2\right)}{64d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)\left(8ade+4bcd-5be^2\right)}{128d^{7/2}(a+bx)}$$

```
[Out] -((4*b*c*d + 8*a*d*e - 5*b*e^2)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*S
qrt[c + e*x + d*x^2])/(64*d^3*(a + b*x)) + ((8*a*d - 5*b*e + 6*b*d*x)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(24*d^2*(a + b*x)) - ((4*
c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcT
anh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x))
```

Rubi [A] time = 0.126839, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1000, 779, 612, 621, 206}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+dx^2+ex}\left(8ade+4bcd-5be^2\right)}{64d^3(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)\left(8ade+4bcd-5be^2\right)}{128d^{7/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] -((4*b*c*d + 8*a*d*e - 5*b*e^2)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*S
qrt[c + e*x + d*x^2])/(64*d^3*(a + b*x)) + ((8*a*d - 5*b*e + 6*b*d*x)*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(24*d^2*(a + b*x)) - ((4*
c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcT
anh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x))
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int x\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{(8ad-5be+6bdx)\sqrt{a^2+2abx+b^2x^2}\left(c+ex+dx^2\right)^{3/2}}{24d^2(a+bx)} - \frac{\left(b\left(4bcd+8ade-\right.\right.}{}$$

$$= -\frac{\left(4bcd+8ade-5be^2\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{64d^3(a+bx)} + \frac{(8ad-}{}$$

$$= -\frac{\left(4bcd+8ade-5be^2\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{64d^3(a+bx)} + \frac{(8ad-}{}$$

$$= -\frac{\left(4bcd+8ade-5be^2\right)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{64d^3(a+bx)} + \frac{(8ad-}{}$$

Mathematica [A] time = 0.136698, size = 147, normalized size = 0.65

$$\frac{\sqrt{(a+bx)^2}\left((c+x(dx+e))^{3/2}(8ad+6bdx-5be)-\frac{3\left(8ade+4bcd-5be^2\right)\left(\left(4cd-e^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right)+2\sqrt{d}(2dx+e)\sqrt{c+x(dx+e)}\right)}{16d^{3/2}}\right)}{24d^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((8*a*d - 5*b*e + 6*b*d*x)*(c + x*(e + d*x))^(3/2) - (3*
(4*b*c*d + 8*a*d*e - 5*b*e^2)*(2*Sqrt[d]*(e + 2*d*x)*Sqrt[c + x*(e + d*x)]
+ (4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]))/(1
6*d^(3/2))))/(24*d^2*(a + b*x))
```

Maple [C] time = 0.213, size = 381, normalized size = 1.7

$$\frac{\operatorname{csgn}(bx+a)}{384}\left(96\,d^{7/2}\left(dx^2+ex+c\right)^{3/2}xb+128\,d^{7/2}\left(dx^2+ex+c\right)^{3/2}a-80\,d^{5/2}\left(dx^2+ex+c\right)^{3/2}be-96\,d^{7/2}\sqrt{dx^2+ex}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/384*csgn(b*x+a)*(96*d^(7/2)*(d*x^2+e*x+c)^(3/2)*x*b+128*d^(7/2)*(d*x^2+e*
x+c)^(3/2)*a-80*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b*e-96*d^(7/2)*(d*x^2+e*x+c)^(1
/2)*x*a*e-48*d^(7/2)*(d*x^2+e*x+c)^(1/2)*x*b*c+60*d^(5/2)*(d*x^2+e*x+c)^(1/
2)*x*b*e^2-48*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a*e^2-24*d^(5/2)*(d*x^2+e*x+c)^(1
/2)*b*c*e+30*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e^3-96*ln(1/2*(2*(d*x^2+e*x+c)^(
1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*a*c*d^3*e+24*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d
^(1/2)+2*d*x+e)/d^(1/2))*a*d^2*e^3-48*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)
+2*d*x+e)/d^(1/2))*b*c^2*d^3+72*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x
+e)/d^(1/2))*b*c*d^2*e^2-15*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/
d^(1/2))*b*d*e^4)/d^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2 + ex + c}\sqrt{(bx + a)^2}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x, x)
```

Fricas [A] time = 1.78244, size = 930, normalized size = 4.1

$$\left[\frac{3\left(16\,bc^2d^2 + 32\,acd^2e - 24\,bcde^2 - 8\,ade^3 + 5\,be^4\right)\sqrt{d}\log\left(8\,d^2x^2 + 8\,dex - 4\,\sqrt{dx^2 + ex + c}(2\,dx + e)\sqrt{d} + 4\,cd + e^2\right) + 4}{76}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4
)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqr
t(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^
2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e
```

```
- 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 + 32*a
*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*sqrt(-d)*arctan(1/2*sqrt(d*x
^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*b*d^4*x
^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b
*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e - 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x
+ c))/d^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

Giac [A] time = 1.19218, size = 362, normalized size = 1.59

$$\frac{1}{192}\sqrt{dx^2+xe+c}\left(2\left(4\left(6\,bx\mathrm{sgn}\left(bx+a\right)+\frac{8\,ad^3\mathrm{sgn}\left(bx+a\right)+bd^2e\mathrm{sgn}\left(bx+a\right)}{d^3}\right)x+\frac{12\,bcd^2\mathrm{sgn}\left(bx+a\right)+8\,ad^2e\mathrm{sgn}}{d^3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(d*x^2 + x*e + c)*(2*(4*(6*b*x*sgn(b*x + a) + (8*a*d^3*sgn(b*x +
a) + b*d^2*e*sgn(b*x + a))/d^3)*x + (12*b*c*d^2*sgn(b*x + a) + 8*a*d^2*e*sg
n(b*x + a) - 5*b*d*e^2*sgn(b*x + a))/d^3)*x + (64*a*c*d^2*sgn(b*x + a) - 52
*b*c*d*e*sgn(b*x + a) - 24*a*d*e^2*sgn(b*x + a) + 15*b*e^3*sgn(b*x + a))/d^
3) + 1/128*(16*b*c^2*d^2*sgn(b*x + a) + 32*a*c*d^2*e*sgn(b*x + a) - 24*b*c*
d*e^2*sgn(b*x + a) - 8*a*d*e^3*sgn(b*x + a) + 5*b*e^4*sgn(b*x + a))*log(abs
(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(7/2)
```

3.48 $\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal. Leaf size=198

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)(2ad-be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)(2ad-be)\sqrt{c+dx^2+ex}}{8d^2(a+bx)}+$$

```
[Out] ((2*a*d - b*e)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x
^2])/(8*d^2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)
^(3/2))/(3*d*(a + b*x)) + ((2*a*d - b*e)*(4*c*d - e^2)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(16*d^(5/
2)*(a + b*x))
```

Rubi [A] time = 0.101076, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {969, 640, 612, 621, 206}

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4cd-e^2\right)(2ad-be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)(2ad-be)\sqrt{c+dx^2+ex}}{8d^2(a+bx)}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] ((2*a*d - b*e)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x
^2])/(8*d^2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)
^(3/2))/(3*d*(a + b*x)) + ((2*a*d - b*e)*(4*c*d - e^2)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(16*d^(5/
2)*(a + b*x))
```

Rule 969

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p
]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !In
tegerQ[p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+ex+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{\left(b(2ad-be)\sqrt{a^2+2abx+b^2x^2}\right)\int \sqrt{\ldots}}{d\left(2ab+2b^2x\right)}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(\ldots\right.}{3d(a+b\ldots}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(\ldots\right.}{3d(a+b\ldots}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(\ldots\right.}{3d(a+b\ldots}$$

Mathematica [A] time = 0.128185, size = 134, normalized size = 0.68

$$\frac{\sqrt{(a+bx)^2}\left(2\sqrt{d}\sqrt{c+x(dx+e)}\left(6ad(2dx+e)+b\left(8cd+8d^2x^2+2dex-3e^2\right)\right)+3\left(4cd-e^2\right)(2ad-be)\tanh^{-1}\left(\frac{2dx+}{2\sqrt{d}\sqrt{c+x}}\right.\right.}{48d^{5/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(6*a*d*(e + 2*d*x) + b*
(8*c*d - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 3*(2*a*d - b*e)*(4*c*d - e^2)*ArcT
anh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])]))/(48*d^(5/2)*(a + b*x))
```

Maple [C] time = 0.202, size = 257, normalized size = 1.3

$$\frac{\mathrm{csgn}\,(bx+a)}{48}\left(16\,d^{5/2}\left(dx^2+ex+c\right)^{3/2}b+24\,d^{7/2}\sqrt{dx^2+ex+c}xa-12\,d^{5/2}\sqrt{dx^2+ex+c}xbe+12\,d^{5/2}\sqrt{dx^2+ex+c}ae-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x)
```

```
[Out] 1/48*csgn(b*x+a)*(16*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b+24*d^(7/2)*(d*x^2+e*x+c)
^(1/2)*x*a-12*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b*e+12*d^(5/2)*(d*x^2+e*x+c)^(1
/2)*a*e-6*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e^2+24*ln(1/2*(2*(d*x^2+e*x+c)^(1/2
)*d^(1/2)+2*d*x+e)/d^(1/2))*a*c*d^3-6*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)
+2*d*x+e)/d^(1/2))*a*d^2*e^2-12*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x
+e)/d^(1/2))*b*c*d^2*e+3*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(
1/2))*b*d*e^3)/d^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2), x)
```

Fricas [A] time = 1.71057, size = 683, normalized size = 3.45

$$\left[\frac{3\left(8\,acd^2-4\,bcde-2\,ade^2+be^3\right)\sqrt{d}\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)+4\left(8\,bd^3x^2+8\,b\right.}{96\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 + b*e^3)*sqrt(d)*log(8*d^2*x^2
+ 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*
(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 + 2*(6*a*d^3 + b*d^2*e)*x)
*sqrt(d*x^2 + e*x + c))/d^3, -1/48*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 +
b*e^3)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*
x^2 + d*e*x + c*d)) - 2*(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 +
2*(6*a*d^3 + b*d^2*e)*x)*sqrt(d*x^2 + e*x + c))/d^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

Giac [A] time = 1.21137, size = 250, normalized size = 1.26

$$\frac{1}{24}\sqrt{dx^2+xe+c}\left(2\left(4\,bx\mathrm{sgn}\left(bx+a\right)+\frac{6\,ad^2\mathrm{sgn}\left(bx+a\right)+bde\mathrm{sgn}\left(bx+a\right)}{d^2}\right)x+\frac{8\,bcd\mathrm{sgn}\left(bx+a\right)+6\,ade\mathrm{sgn}\left(bx+a\right)}{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(d*x^2 + x*e + c)*(2*(4*b*x*sgn(b*x + a) + (6*a*d^2*sgn(b*x + a) +
 b*d*e*sgn(b*x + a))/d^2)*x + (8*b*c*d*sgn(b*x + a) + 6*a*d*e*sgn(b*x + a)
- 3*b*e^2*sgn(b*x + a))/d^2) - 1/16*(8*a*c*d^2*sgn(b*x + a) - 4*b*c*d*e*sgn
(b*x + a) - 2*a*d*e^2*sgn(b*x + a) + b*e^3*sgn(b*x + a))*log(abs(-2*(sqrt(d
)*x - sqrt(d*x^2 + x*e + c))*sqrt(d) - e))/d^(5/2)
```

3.49 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx$

Optimal. Leaf size=211

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4ade+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d(a+bx)}-$$

```
[Out] ((4*a*d + b*e + 2*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2
])/(4*d*(a + b*x)) + ((4*b*c*d + 4*a*d*e - b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2)*(a
+ b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*S
qrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x)
```

Rubi [A] time = 0.221434, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(4ade+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d(a+bx)}-$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```



```
[Out] ((4*a*d + b*e + 2*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2
])/(4*d*(a + b*x)) + ((4*b*c*d + 4*a*d*e - b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2)*(a
+ b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*S
qrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x)
```


Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x}\,dx}{2ab+2b^2x}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int -}{4d\left(2ab\right.}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)} + \frac{\left(2abc\sqrt{a^2+2abx+b^2\right.}{2ab+2}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)} - \frac{\left(4abc\sqrt{a^2+2abx+b^2\right.}{}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)} + \frac{\left(4bcd+4ade-be^2\right)\sqrt{}}{}$$

Mathematica [A] time = 0.202133, size = 149, normalized size = 0.71

$$\frac{\sqrt{(a+bx)^2}\left(\left(4ade+4bcd-be^2\right)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right)+2\sqrt{d}\left(\sqrt{c+x(dx+e)}(4ad+b(2dx+e))-4a\sqrt{c}d\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.\right.\right.}{8d^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((4*b*c*d + 4*a*d*e - b*e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt
[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c + x*(e + d*x)]*(4*a*d + b*(
e + 2*d*x)) - 4*a*Sqrt[c]*d*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e +
d*x)])])))/(8*d^(3/2)*(a + b*x))
```

Maple [C] time = 0.244, size = 214, normalized size = 1.

$$-\frac{\mathrm{csgn}\,(bx+a)}{8}\left(8\sqrt{c}d^{5/2}\ln\left(\frac{2c+ex+2\sqrt{c}\sqrt{dx^2+ex+c}}{x}\right)a-4\,d^{5/2}\sqrt{dx^2+ex+c}xb-8\,d^{5/2}\sqrt{dx^2+ex+c}a-2\,d^{3/2}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x)
```

```
[Out] -1/8*csgn(b*x+a)*(8*c^(1/2)*d^(5/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/
2))/x)*a-4*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x*b-8*d^(5/2)*(d*x^2+e*x+c)^(1/2)*a-
2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*b*e-4*d^2*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/
2)+2*d*x+e)/d^(1/2))*a*e-4*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d
^(1/2))*b*c*d^2+ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d
*e^2)/d^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + ex + c}\sqrt{(bx + a)^2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x, x)
```

Fricas [A] time = 6.14859, size = 1581, normalized size = 7.49

$$\left[\frac{8\,a\sqrt{c}d^2\log\left(\frac{8\,cex+\left(4\,cd+e^2\right)x^2-4\,\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right)-\left(4\,bcd+4\,ade-be^2\right)\sqrt{d}\log\left(8\,d^2x^2+8\,dex-4\,\sqrt{dx^2+ex+c}(2\,d\right.}{16\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*
x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt
(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) +
 4*c*d + e^2) + 4*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2,
 1/8*(4*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x
 + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(
-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x
+ c*d)) + 2*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/16*
```

```
(16*a*sqrt(-c)*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c
*d*x^2 + c*e*x + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(d)*log(8*d^2*x^2
+ 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*
(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/8*(8*a*sqrt(-c)
*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x
 + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x
 + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*d^2*x + 4*a*d^
2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2 + ex}\sqrt{(a + bx)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.50 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(}{}$$

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(x*(a + b
*x))) + ((2*a*d + b*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2
*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*(a + b*x)) - ((2*b*c + a*e)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d
*x^2])])/(2*Sqrt[c]*(a + b*x))
```

Rubi [A] time = 0.194411, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(x*(a + b
*x))) + ((2*a*d + b*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2
*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*(a + b*x)) - ((2*b*c + a*e)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d
*x^2])])/(2*Sqrt[c]*(a + b*x))
```

Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x^2}\,dx}{2ab+2b^2x}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-2b(2bc+ae)-}{x\sqrt{c+ex}}}{2\left(2ab+2b^2x\right)}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} + \frac{\left(b(2bc+ae)\sqrt{a^2+2abx+b^2x^2}\right)}{2ab+2b^2x}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} - \frac{\left(2b(2bc+ae)\sqrt{a^2+2abx+b^2x^2}\right.}{2ab+}$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} + \frac{(2ad+be)\sqrt{a^2+2abx+b^2x^2}\tan}{2\sqrt{d}(a+bx)}$$

Mathematica [A] time = 0.195053, size = 155, normalized size = 0.77

$$\frac{\sqrt{(a+bx)^2}\left(\sqrt{c}x(2ad+be)\tanh^{-1}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+x(dx+e)}}\right)+\sqrt{d}\left(2\sqrt{c}(bx-a)\sqrt{c+x(dx+e)}-x(ae+2bc)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+x(dx+e)}}\right)\right)\right)}{2\sqrt{c}\sqrt{d}x(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c]*(2*a*d + b*e)*x*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*
Sqrt[c + x*(e + d*x)])] + Sqrt[d]*(2*Sqrt[c]*(-a + b*x)*Sqrt[c + x*(e + d*x
)] - (2*b*c + a*e)*x*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + x*(e + d*x)])]
)))/(2*Sqrt[c]*Sqrt[d]*x*(a + b*x))
```

Maple [C] time = 0.216, size = 249, normalized size = 1.2

$$\frac{\operatorname{csgn}(bx+a)}{2cx}\left(2d^{5/2}\sqrt{dx^2+ex+c}x^2a-2d^{3/2}c^{3/2}\ln\left(\frac{2c+ex+2\sqrt{c}\sqrt{dx^2+ex+c}}{x}\right)xb-d^{\frac{3}{2}}\sqrt{c}\ln\left(\frac{1}{x}\left(2c+ex+2\sqrt{c}\sqrt{d}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*csgn(b*x+a)*(2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^2*a-2*d^(3/2)*c^(3/2)*ln((
2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*x*b-d^(3/2)*c^(1/2)*ln((2*c+e*x+2
*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*x*a*e-2*d^(3/2)*(d*x^2+e*x+c)^(3/2)*a+2*d^
(3/2)*(d*x^2+e*x+c)^(1/2)*x*a*e+2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x*b*c+2*ln(1/
2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*x*a*c*d^2+ln(1/2*(2*(d*x
^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*d*x*b*c*e)/x/c/d^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2, x)
```

Fricas [A] time = 3.45487, size = 1562, normalized size = 7.73

$$\left[\frac{(2acd+bce)\sqrt{d}x\log\left(8d^2x^2+8dex+4\sqrt{dx^2+ex+c}(2dx+e)\sqrt{d}+4cd+e^2\right)+(2bcd+ade)\sqrt{c}x\log\left(\frac{8cex+\left(4cd+e^2\right)x}{}\right.}{4cdx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 +
e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (2*b*c*d + a*d*e)*sqrt(c)*x*l
og((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
c) + 8*c^2)/x^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), -1/4
*(2*(2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x +
e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (2*b*c*d + a*d*e)*sqrt(c)*x*log((8*c
*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*
c^2)/x^2) - 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/4*(2*(2*b
*c*d + a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
-c)/(c*d*x^2 + c*e*x + c^2)) + (2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 +
```

```
8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(b
*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/2*((2*b*c*d + a*d*e)*sqrt
(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e
*x + c^2)) - (2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*
(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(b*c*d*x - a*c*d)*sqrt(d*
x^2 + e*x + c))/(c*d*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2 + ex}\sqrt{(a + bx)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**2, x)
```

Giac [A] time = 1.21186, size = 284, normalized size = 1.41

$$\sqrt{dx^2 + xe + c}\,b\mathrm{sgn}\,(bx + a) + \frac{(2\,bc\mathrm{sgn}\,(bx + a) + ae\mathrm{sgn}\,(bx + a))\arctan\left(-\frac{\sqrt{d}x - \sqrt{dx^2 + xe + c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{(2\,ad\mathrm{sgn}\,(bx + a) + be\mathrm{sgn}\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] sqrt(d*x^2 + x*e + c)*b*sgn(b*x + a) + (2*b*c*sgn(b*x + a) + a*e*sgn(b*x +
a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + x*e + c))/sqrt(-c))/sqrt(-c) - 1/2*(2
*a*d*sgn(b*x + a) + b*e*sgn(b*x + a))*log(abs(2*(sqrt(d)*x - sqrt(d*x^2 + x
*e + c))*sqrt(d) + e))/sqrt(d) + ((sqrt(d)*x - sqrt(d*x^2 + x*e + c))*a*e*s
gn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 + x*e +
c))^2 - c)
```

3.51 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx$

Optimal. Leaf size=215

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4acd-ae^2+4bce\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2(a+bx)}$$

```
[Out] -((2*a*c + (4*b*c + a*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*
x^2])/(4*c*x^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTan
h[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(a + b*x) - ((4*a*c*d + 4
*b*c*e - a*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c
]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)*(a + b*x))
```

Rubi [A] time = 0.183417, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1000, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{a^2+2abx+b^2x^2}\left(4acd-ae^2+4bce\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```


```
[Out] -((2*a*c + (4*b*c + a*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*
x^2])/(4*c*x^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTan
h[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(a + b*x) - ((4*a*c*d + 4
*b*c*e - a*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c
]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)*(a + b*x))
```


Rule 1000

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{\left(2ab+2b^2x\right)\sqrt{c+ex+dx^2}}{x^3}\,dx}{2ab+2b^2x}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}}{4c\left(2\right.}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{\left(2b^2d\sqrt{a^2+2abx+\right.}{2ab}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{\left(4b^2d\sqrt{a^2+2abx+\right.}{}$$

$$= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b}}{}$$

Mathematica [A] time = 0.231853, size = 161, normalized size = 0.75

$$-\frac{\sqrt{(a+bx)^2}\left(x^2\left(4acd-ae^2+4bce\right)\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+x(dx+e)}}\right)+2\sqrt{c}\sqrt{c+x(dx+e)}(2ac+aex+4bcx)-8bc^{3/2}\sqrt{d}x^2\tanh}{8c^{3/2}x^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] -(Sqrt[(a + b*x)^2]*(2*Sqrt[c]*(2*a*c + 4*b*c*x + a*e*x)*Sqrt[c + x*(e + d*
x)] - 8*b*c^(3/2)*Sqrt[d]*x^2*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + x*(e
+ d*x)])] + (4*a*c*d + 4*b*c*e - a*e^2)*x^2*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*
Sqrt[c + x*(e + d*x)])]))/(8*c^(3/2)*x^2*(a + b*x))
```

Maple [C] time = 0.206, size = 358, normalized size = 1.7

$$\frac{\mathrm{csgn}\,(bx+a)}{8\,c^2x^2}\left(-4\,d^{5/2}c^{3/2}\ln\left(\frac{2\,c+ex+2\,\sqrt{c}\sqrt{dx^2+ex+c}}{x}\right)x^2a-2\,d^{5/2}\sqrt{dx^2+ex+c}x^3ae+8\,d^{5/2}\sqrt{dx^2+ex+c}x^3bc-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x)
```

```
[Out] 1/8*csgn(b*x+a)*(-4*d^(5/2)*c^(3/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/
2))/x)*x^2*a-2*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^3*a*e+8*d^(5/2)*(d*x^2+e*x+c)^
(1/2)*x^3*b*c-4*d^(3/2)*c^(3/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/
x)*x^2*b*e+4*d^(5/2)*(d*x^2+e*x+c)^(1/2)*x^2*a*c+d^(3/2)*c^(1/2)*ln((2*c+e*
x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*x^2*a*e^2+2*d^(3/2)*(d*x^2+e*x+c)^(3/2)
*x*a*e-8*d^(3/2)*(d*x^2+e*x+c)^(3/2)*x*b*c-2*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^
2*a*e^2+8*d^(3/2)*(d*x^2+e*x+c)^(1/2)*x^2*b*c*e+8*ln(1/2*(2*(d*x^2+e*x+c)^(
1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*x^2*b*c^2*d^2-4*d^(3/2)*(d*x^2+e*x+c)^(3/2)*
a*c)/x^2/c^2/d^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2 + ex + c}\sqrt{(bx + a)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3, x)
```

Fricas [A] time = 4.57283, size = 1670, normalized size = 7.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c
)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (4*a*c*d + 4*b*c*e - a*e^2)*sqrt(c)*
x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*
sqrt(c) + 8*c^2)/x^2) - 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 + e*x
+ c))/(c^2*x^2), -1/16*(16*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(d*x^2 + e*x +
 c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + (4*a*c*d + 4*b*c*e - a*
e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)
*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
```

```
(d*x^2 + e*x + c))/(c^2*x^2), 1/8*(4*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*
e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (4*a*c*d
 + 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*
c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sq
rt(d*x^2 + e*x + c))/(c^2*x^2), -1/8*(8*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(
d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (4*a*c*d +
 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)
*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
(d*x^2 + e*x + c))/(c^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c + dx^2 + ex}\sqrt{(a + bx)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**3, x)
```

Giac [B] time = 1.27092, size = 608, normalized size = 2.83

$$-b\sqrt{d}\log\left(\left|-2\left(\sqrt{d}x - \sqrt{dx^2 + xe + c}\right)d - \sqrt{d}e\right|\right)\operatorname{sgn}(bx + a) + \frac{\left(4\,acd\operatorname{sgn}(bx + a) + 4\,bce\operatorname{sgn}(bx + a) - ae^2\operatorname{sgn}(bx + a\right.}{4\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -b*sqrt(d)*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*d - sqrt(d)*e))*s
gn(b*x + a) + 1/4*(4*a*c*d*sgn(b*x + a) + 4*b*c*e*sgn(b*x + a) - a*e^2*sgn(
b*x + a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + x*e + c))/sqrt(-c))/(sqrt(-c)*c
) + 1/4*(4*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*c*d*sgn(b*x + a) + 4*(sq
rt(d)*x - sqrt(d*x^2 + x*e + c))^3*b*c*e*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt
(d*x^2 + x*e + c))^2*b*c^2*sqrt(d)*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt(d*x^2
 + x*e + c))^2*a*c*sqrt(d)*e*sgn(b*x + a) + 4*(sqrt(d)*x - sqrt(d*x^2 + x*e
```

```
 + c))*a*c^2*d*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))^3*a*e^2*s
gn(b*x + a) - 4*(sqrt(d)*x - sqrt(d*x^2 + x*e + c))*b*c^2*e*sgn(b*x + a) -
8*b*c^3*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + x*e + c))*a*c*e^2*
sgn(b*x + a))/(((sqrt(d)*x - sqrt(d*x^2 + x*e + c))^2 - c)^2*c)
```

3.52 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=452

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c\left(e^2-df\right)\right)}{2\sqrt{c}f^3}-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e\right.}}$$

```
[Out] -((2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((a*f^2 + 2*c*(e^2 - d*f))*ArcTanh
[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) - ((e*(e - Sqrt[e^2 - 4*d*f]
)*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f
 - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((e*(e + Sqrt[e^2 - 4*d
*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*
a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 1.96739, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1069, 1080, 217, 206, 1034, 725}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c\left(e^2-df\right)\right)}{2\sqrt{c}f^3}-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -((2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + ((a*f^2 + 2*c*(e^2 - d*f))*ArcTanh
[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) - ((e*(e - Sqrt[e^2 - 4*d*f]
)*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f
 - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((e*(e + Sqrt[e^2 - 4*d
*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*
a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
```

```
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 1069

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*
(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !
IGtQ[q, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
```

```
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf-ce(2cd-af)x-c\left(af^2+2c\left(e^2-df\right)\right)x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2cf^2}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf^2+cd\left(af^2+2c\left(e^2-df\right)\right)+\left(-cef(2cd-af)+ce\left(af^2+2c\left(e^2-df\right)\right)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2cf^3} + \frac{\left(af^2+2c\left(e^2-\ldots\right.\right.}{2\ldots}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(af^2+2c\left(e^2-df\right)\right)\operatorname{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3} + \frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(a\ldots\right.\right.}{\ldots}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3} - \frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-\ldots\right.\right.\right.}{\ldots}$$

$$= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3} - \frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-\ldots\right.\right.\right.}{\sqrt{2}f^3\ldots}$$

Mathematica [A] time = 2.59567, size = 516, normalized size = 1.14

$$\frac{\frac{2f\left(\frac{a^{3/2}\sqrt{\frac{cx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}}+ax+cx^3\right)}{\sqrt{a+cx^2}} + \frac{\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)-\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\ldots\right.}{f}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (-2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - 2*(e + (-e^2 +
2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + (2*f*(a*x + c*x^3 + (a^(3/2)*Sq
rt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/Sqrt[c]))/Sqrt[a + c*x^2] +
 ((e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(-(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f]
)*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]) + Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f
+ e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f
 + ((e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sq
rt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f
^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(f*Sqrt[e
^2 - 4*d*f]))/(4*f^2)
```

Maple [B] time = 0.319, size = 7739, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.53 $\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=395

$$-\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef-\left(\sqrt{e^2-4df}+e\right)\left(\right.\right.}{\sqrt{2}f^2\sqrt{e^2}}$$

```
[Out] Sqrt[a + c*x^2]/f - (Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 -
((2*c*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a
*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f -
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*c*d*e*f - (e + Sq
rt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
- 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 0.931406, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1020, 1080, 217, 206, 1034, 725}

$$-\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef-\left(\sqrt{e^2-4df}+e\right)\left(\right.\right.}{\sqrt{2}f^2\sqrt{e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + c*x^2]/f - (Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 -
((2*c*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a
*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f -
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*c*d*e*f - (e + Sq
rt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
- 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
```

```
2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx &= \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{-(cd-af)x-cex^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f} \\
&= \frac{\sqrt{a+cx^2}}{f} + \frac{\int \frac{cde+\left(ce^2+f(-cd+af)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f^2} \\
&= \frac{\sqrt{a+cx^2}}{f} - \frac{(ce)\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f^2} + \frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\int \frac{}{(e}}{f^2\sqrt{e^2-4df}} \\
&= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\mathrm{Subst}\left(\int \frac{}{4af^2+c}\right.}{f^2\sqrt{e^2-4df}} \\
&= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{2af}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2}\right.}}
\end{aligned}
$$

Mathematica [A] time = 1.58469, size = 422, normalized size = 1.07

$$
-\frac{\frac{\left(\sqrt{e^2-4df}-e\right)\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}} + \frac{e\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4d}\right.}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-}\right.}}\right)}{\sqrt{e^2-4df}}}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -(-4*f*Sqrt[a + c*x^2] + 4*Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] +
 ((-e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 -
4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c
*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/Sqrt[e^2 - 4*d*f]
 + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f -
 c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f])]*Sqrt[a + c*x^2])] + (e*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2
 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/Sqrt[e^2 - 4
```

```
*d*f])/(4*f^2)
```

Maple [B] time = 0.258, size = 5581, normalized size = 14.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a + cx^2}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.54 $\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=298

$$-\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e \right.}\ \ldots}{\sqrt{2}\ \ldots}$$

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f - (Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(
Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

Rubi [A] time = 0.375888, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {991, 217, 206, 1034, 725}

$$-\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\frac{\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e \right.}\ \ldots}{\sqrt{2}\ \ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f - (Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(
Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

Rule 991

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
```

```
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+cex}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{f} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f}$$

$$= \frac{c\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{\left(2f(cd-af)-ce\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f\sqrt{e^2-4df}} +$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-ce\left(e+\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2a}{}\right.}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

Mathematica [A] time = 0.413816, size = 282, normalized size = 0.95

$$\frac{-\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}{2f\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[e^2 - 4*d*f]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*
a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e +
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*
f])]*Sqrt[a + c*x^2])] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*f*Sqrt[e^2 - 4*d*f]
)
```

Maple [B] time = 0.256, size = 3249, normalized size = 10.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/2/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*
d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2*c^(1/2)/f*ln((-1/2*c*(e-(-4*d*f+e
^2)^(1/2))/f+(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/2
/(-4*d*f+e^2)^(1/2)*c^(1/2)/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+1/2/f^2*2^(1/2)/((-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^
2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e-1/(
-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4
*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f))*a+1/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-1/2/(-4*d*f+
e^2)^(1/2)/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f))*c*e^2-1/2/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2/(-4*d*f
+e^2)^(1/2)*c^(1/2)/f*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)*c)/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d
*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+1/2*c^(1/2)/f*ln((-1/2*c*(e+(-4*d*f
```

```
+e^2)^(1/2))/f+(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))+1/2/f
^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/
f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*
(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
))*c*e+1/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))*a-1/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/(((-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*d+1/2/(-4*d*f+e^2)^(
1/2)/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(
1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*
c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f))*c*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.55 $\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)} dx$

Optimal. Leaf size=358

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)+2aef\right)\text{ta}}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+}}$$

```
[Out] ((2*a*e*f + (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*e*f + (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) -
 (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rubi [A] time = 1.31351, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 266, 50, 63, 208, 1020, 1034, 725, 206}

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)+2aef\right)\text{ta}}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*a*e*f + (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*e*f + (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) -
 (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+cx^2}}{dx}+\frac{(-e-fx)\sqrt{a+cx^2}}{d\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d}+\frac{\int \frac{(-e-fx)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d}$$

$$= -\frac{\sqrt{a+cx^2}}{d}+\frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx,x,x^2\right)}{2d}+\frac{\int \frac{-aef+f(cd-af)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{df}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d}-\frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d\sqrt{e^2-4df}}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd}+\frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{\cdots}\right.}\cdots\right.}{d\sqrt{e^2-4df}}$$

$$= \frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}-\frac{\left(2aef+(cd-\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.733813, size = 314, normalized size = 0.88

$$\frac{\left(\sqrt{e^2-4df}+e\right)\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\left(\sqrt{e^2-4df}-e\right)\sqrt{4af^2}}{4df\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] ((e + Sqrt[e^2 - 4*d*f])*Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*
d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] + (-e + Sqrt[e^2 - 4
*d*f])*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] - 4*Sqrt[a]*f*Sqrt[e^2 - 4*d*f]*ArcTanh[S
qrt[a + c*x^2]/Sqrt[a]])/(4*d*f*Sqrt[e^2 - 4*d*f])
```

Maple [B] time = 0.266, size = 3544, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2
*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+1/(-e+(-4*d*f+e
^2)^(1/2))*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)*c)/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^
2)^(1/2)*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)*c)/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+
e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+1/f/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/
((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4
```

```
*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*
c*e-2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*a+2/(-e+(-4*d*f+
e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/
2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-1/f/(-e+(-4*d*f+e^2)^(1/2))/(-4*
d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f))*c*e^2+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1
/2))*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-4*f/(-e+(-4*d*f+e^2)^(1/
2))/(e+(-4*d*f+e^2)^(1/2))*(c*x^2+a)^(1/2)+f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f
+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/
2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)-1/(e+(-4*d*f+e^2)^(1/2))*c^(1/2)*ln((-1/2*c*(e+(-4*
d*f+e^2)^(1/2))/f+(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/
(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2
)^(1/2))/f+(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e-1/f/(e+
(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4
*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f))*c*e-2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/
2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
```

```
*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d
*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*a+2/(
e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/
2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*d-1/f/(e+(-4*d*f+e^2)^(1/2))/(-4
*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f
+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f))*c*e^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.56 $\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=382

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)+2c\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2a}}$$

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^2*Sqrt
[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(
2*c*d^2 + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/
Sqrt[a]])/d^2
```

Rubi [A] time = 1.41705, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6728, 277, 217, 206, 266, 50, 63, 208, 1020, 1080, 1034, 725}

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)+2c\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```


```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^2*Sqrt
[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(
2*c*d^2 + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/
```

```
Sqrt[a]])/d^2
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{\sqrt{a+cx^2}}{dx^2}-\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{\left(e^2-df+efx\right)\sqrt{a+cx^2}}{d^2\left(d+ex+fx^2\right)}\right)dx$$
$$=\frac{\int\frac{\left(e^2-df+efx\right)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d^2}+\frac{\int\frac{\sqrt{a+cx^2}}{x^2}\,dx}{d}-\frac{e\int\frac{\sqrt{a+cx^2}}{x}\,dx}{d^2}$$
$$=\frac{e\sqrt{a+cx^2}}{d^2}-\frac{\sqrt{a+cx^2}}{dx}+\frac{c\int\frac{1}{\sqrt{a+cx^2}}\,dx}{d}-\frac{e\,\text{Subst}\left(\int\frac{\sqrt{a+cx}}{x}\,dx,x,x^2\right)}{2d^2}+\frac{\int\frac{af\left(e^2-df\right)-ef(cd-af)x-}{\sqrt{a+cx^2}\left(d+ex+fx\right.}}{d^2f}$$
$$=-\frac{\sqrt{a+cx^2}}{dx}-\frac{c\int\frac{1}{\sqrt{a+cx^2}}\,dx}{d}+\frac{c\,\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{d}-\frac{(ae)\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right.}{2d^2}$$
$$=-\frac{\sqrt{a+cx^2}}{dx}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d}-\frac{c\,\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{d}-\frac{(ae)\,\text{Subst}\left(\int\frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,d\right.}{cd^2}$$
$$=-\frac{\sqrt{a+cx^2}}{dx}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}+\frac{\left(f\left(2cd^2+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int\frac{}{4af^2-}\right.}{d^2\sqrt{e^2-4df}}$$
$$=-\frac{\sqrt{a+cx^2}}{dx}-\frac{f\left(2cd^2+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

Mathematica [A] time = 3.20202, size = 569, normalized size = 1.49

$$\frac{\frac{\left(e\sqrt{e^2-4df}-2df+e^2\right)\left(\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)-\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)\right)}{f\sqrt{e^2-4df}}+\frac{\left(-e\sqrt{e^2-4df}-2df+\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + 2*(e + (-e^2 + 2
*d*f)/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - (4*d*(a + c*x^2 - Sqrt[a]*Sqrt[c
]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/(x*Sqrt[a + c*x^2])
+ ((e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*Ar
```

```
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*S
qrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(f*Sqr
t[e^2 - 4*d*f]) + ((e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e
^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])
*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])]))/(f*Sqrt[e^2 - 4*d*f]) - 4*e*(Sqrt[a + c*x^2] - Sqrt[a]*ArcTanh[Sqrt[a
 + c*x^2]/Sqrt[a]]))/(4*d^2)
```

Maple [B] time = 0.311, size = 3703, normalized size = 9.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] 2*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*f/(-e+(
-4*d*f+e^2)^(1/2))^2*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c
*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-2*f/(-e+(-4*d*f+e^2)^(1/2)
)^2/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+2/(-e+(-4*d*f+e^2)^(1/
2))^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*l
n(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1
/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^
2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f))*c*e-4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2
)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(
-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
```

```
)*a+4*f/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-2/(-e+(-4*
d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2
*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e^2+16*f^2*e/(-e+(-4*d*f+e^2
)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/
2))/x)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2*(c*x^2+a
)^(1/2)-2*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*f/(
e+(-4*d*f+e^2)^(1/2))^2*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))+2*f/(e+(-4*d*f+e^2)^(1/2))^
2/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(
e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+2/(e+(-4*d*f+e^2)^(1/2))^2*2
^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c
*e+4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*a-4*f/(e+(-4*d*f+e^2)^
(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
```

```
/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))*c*d+2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/
2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/
f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*
(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
))*c*e^2+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+a)^(
3/2)-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a*x*(c*x^2+a)^(1/
2)-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c^(1/2)*ln(x*c^(1/2)+
(c*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.57 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)}\,dx$

Optimal. Leaf size=507

$$\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f(a$$

```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (f*(c*d^2*(e + S
qrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2
 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 - (f*(c*d^2*(e - Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*
d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f])]) - (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) -
(Sqrt[a]*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rubi [A] time = 1.87982, antiderivative size = 507, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {6728, 266, 47, 63, 208, 277, 217, 206, 50, 1020, 1080, 1034, 725}

$$\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f(a$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```


```
[Out] -Sqrt[a + c*x^2]/(2*d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (f*(c*d^2*(e + S
qrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2
 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 - (f*(c*d^2*(e - Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*
```

```
d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f])]) - (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d) -
(Sqrt[a]*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
```

```
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^2}}{x^3\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{\sqrt{a+cx^2}}{dx^3}-\frac{e\sqrt{a+cx^2}}{d^2x^2}+\frac{\left(e^2-df\right)\sqrt{a+cx^2}}{d^3x}+\frac{\left(-e\left(e^2-2df\right)-f\left(e^2-df\right)x\right)\sqrt{a+cx^2}}{d^3\left(d+ex+fx^2\right)}\cdots\right.$$

$$=\frac{\int\frac{\left(-e\left(e^2-2df\right)-f\left(e^2-df\right)x\right)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d^3}+\frac{\int\frac{\sqrt{a+cx^2}}{x^3}\,dx}{d}-\frac{e\int\frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^2}+\frac{\left(e^2-df\right)\int\frac{\sqrt{a+cx^2}}{x}\,dx}{d^3}$$

$$=-\frac{\left(e^2-df\right)\sqrt{a+cx^2}}{d^3}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{\text{Subst}\left(\int\frac{\sqrt{a+cx}}{x^2}\,dx,x,x^2\right)}{2d}-\frac{(ce)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{d^2}+\frac{\int\frac{-aef\left(e^{\cdots}\right.}{\cdots}}{\cdots}$$

$$=-\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{c\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{4d}+\frac{(ce)\int\frac{1}{\sqrt{a+cx^2}}\,dx}{d^2}-\frac{(ce)\,\text{Subst}\left(\int\frac{1}{\cdots}\right.}{\cdots}$$

$$=-\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^2}+\frac{\text{Subst}\left(\int\frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{2d}+\frac{(ce)\,\text{S}\cdots}{\cdots}$$

$$=-\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}-\frac{\sqrt{a}\left(e^2-df\right)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+\frac{\left(f\left(cd^2\left(e+\cdots\right.\right.\right.}{\cdots}$$

$$=-\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{f\left(cd^2\left(e+\sqrt{e^2-4df}\right)+a\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-\cdots}\right.\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2\cdots\right.}}$$

Mathematica [A] time = 2.75061, size = 642, normalized size = 1.27

$$-\frac{-\frac{2d^2\left(cx^2\sqrt{\frac{cx^2}{a}+1}\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)+a+cx^2\right)}{x^2\sqrt{a+cx^2}}+\frac{\left(\frac{e(e^2-3df)}{\sqrt{e^2-4df}}-df+e^2\right)\left(\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)-\sqrt{c}\left(\sqrt{e^2-4d\cdots}\right.\right.}{f}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(e^2 - d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] - 2*(
e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2] + (4*d*e*(
a + c*x^2 - Sqrt[a]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/a]*ArcSinh[(Sqrt[c]*x)/Sqrt[
a]]))/(x*Sqrt[a + c*x^2]) + ((e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*
f])*(-(Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
]) + Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*
f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqr
t[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f + ((e^2 - d*f - (e*(e^2 - 3*d*f))/Sq
rt[e^2 - 4*d*f])*(Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[
a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTa
nh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f + 4*(e^2 - d*f)*(Sqrt[a + c*x^
2] - Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]) - (2*d^2*(a + c*x^2 + c*x^2*
Sqrt[1 + (c*x^2)/a]*ArcTanh[Sqrt[1 + (c*x^2)/a]]))/(x^2*Sqrt[a + c*x^2]))/(
4*d^3)
```

Maple [B] time = 0.268, size = 3993, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+4*f^2/(-e
+(-4*d*f+e^2)^(1/2))^3*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f
```

+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-4*f^2/(-e+(-4*d*f+e^2)^(
1/2))^3/(-4*d*f+e^2)^(1/2)*c^(1/2)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)*c)/c^(1/2)+((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))*e+4*f/(-e+(-4*d*f+e^
2)^(1/2))^3*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e
^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*
(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f))*c*e-8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*
2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f
(x-1/2(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f))*a+8*f^2/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e
^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*d-4*
f/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e^2-64*f^4/(-e+(-4
*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^
2+a)^(1/2))/x)*d+64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*
a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)*e^2+64*f^4/(-e+(-4*d*f+e^2)^(
1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*(c*x^2+a)^(1/2)*d-64*f^3/(-e+(-4*d*f+e^2)^
(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3*(c*x^2+a)^(1/2)*e^2+4*f^3/(e+(-4*d*f+e^2)
^(1/2))^3/(-4*d*f+e^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e
+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-4*f^2/(e+(-4*d*f+e^2)^(1/2))^3*c^(
1/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c
)/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2))-4*f^2/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*c^(1
/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)*c)

```
/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2))*e-4*f/(e+(-4*d*f+e^2)^(1/2))^3*2^(1/2)/(((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e-8*f^3/(e+(-4*d*f+e^2
)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f))*a+8*f^2/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)
^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*l
n((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/
2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f))*c*d-4*f/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/
2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*e^2+2*f/(-e+(-
4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+a)^(3/2)+2*f/(-e+(-4*
d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a
)^(1/2))/x)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/a*(c*x^2+a
)^(1/2)+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/x*(c*
x^2+a)^(3/2)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2*c/
a*x*(c*x^2+a)^(1/2)-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2
))^2*c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{\left(fx^2 + ex + d\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{x^3 \left(d + ex + fx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**3/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.58 $\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=795

$$-\frac{(4e-3fx)\left(cx^2+a\right)^{3/2}}{12f^2}-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{cx^2+a}}{8f^4}+\frac{\left(3a^2f^4+12ac\left(e^2-df\right)f^2+\right.}{}$$

```
[Out] -((8*e*(a*f^2 + c*(e^2 - 2*d*f)) - f*(3*a*f^2 + 4*c*(e^2 - d*f))*x)*Sqrt[a
+ c*x^2])/(8*f^4) - ((4*e - 3*f*x)*(a + c*x^2)^(3/2))/(12*f^2) + ((3*a^2*f^
4 + 12*a*c*f^2*(e^2 - d*f) + 8*c^2*(e^4 - 3*d*e^2*f + d^2*f^2))*ArcTanh[(Sq
rt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c]*f^5) - ((a^2*f^4*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4
*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 -
 2*d^3*f^3 - e^5*Sqrt[e^2 - 4*d*f] + 4*d*e^3*f*Sqrt[e^2 - 4*d*f] - 3*d^2*e*
f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
 - 4*d*f])]) + ((a^2*f^4*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e
^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d
*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 + e^5*Sqrt[e^2 - 4*
d*f] - 4*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*
f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 4.26402, antiderivative size = 795, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1069, 1068, 1080, 217, 206, 1034, 725}

$$-\frac{(4e-3fx)\left(cx^2+a\right)^{3/2}}{12f^2}-\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{cx^2+a}}{8f^4}+\frac{\left(3a^2f^4+12ac\left(e^2-df\right)f^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -((8*e*(a*f^2 + c*(e^2 - 2*d*f)) - f*(3*a*f^2 + 4*c*(e^2 - d*f))*x)*Sqrt[a
+ c*x^2])/(8*f^4) - ((4*e - 3*f*x)*(a + c*x^2)^(3/2))/(12*f^2) + ((3*a^2*f^
4 + 12*a*c*f^2*(e^2 - d*f) + 8*c^2*(e^4 - 3*d*e^2*f + d^2*f^2))*ArcTanh[(Sq
rt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c]*f^5) - ((a^2*f^4*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4
*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 -
 2*d^3*f^3 - e^5*Sqrt[e^2 - 4*d*f] + 4*d*e^3*f*Sqrt[e^2 - 4*d*f] - 3*d^2*e*
f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
 - 4*d*f])]) + ((a^2*f^4*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e
^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d
*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 + e^5*Sqrt[e^2 - 4*
d*f] - 4*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*
f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 1069

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*
(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !
IGtQ[q, 0]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
```

```
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^2\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2} - \frac{\int \frac{\sqrt{a+cx^2}\left(3acdf-3ce(4cd-af)x-3c\left(3af^2+4c\left(e^2-df\right)\right)x^2\right)}{d+ex+fx^2}\,dx}{12cf^2}$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2} + \cdots$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2} + \cdots$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2} + \cdots$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2} + \cdots$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2} + \cdots$$

Mathematica [A] time = 3.7611, size = 793, normalized size = 1.

$$\frac{3f\sqrt{a+cx^2}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{a}+1}}+5ax+2cx^3\right) - \frac{3\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(\frac{2\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\left(-\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}\tanh^{-1}\left(\frac{2af+cx\left(\cdots\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2\cdots}}\right)\cdots\right)}{f^2}\cdots\right)}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (-4*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) - 4*(e + (-e^2
+ 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) + 3*f*Sqrt[a + c*x^2]*(5*a*x
+ 2*c*x^3 + (3*a^(3/2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])/(Sqrt[c]*Sqrt[1 + (c*x
^2)/a])) - (3*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*(-e + Sqrt
```

```
[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*Arc
Sinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 -
 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*
a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x
^2])]))/f^2))/(2*f) + (3*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*
(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sq
rt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sqrt[c]*(e + S
qrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*
d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
c*x^2])]))/f^2))/(2*f))/(24*f^2)
```

Maple [B] time = 0.275, size = 19148, normalized size = 24.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(a + cx^2\right)^{\frac{3}{2}}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.59 $\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=553

$$-\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2}}\right.}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] ((2*(a*f^2 + c*(e^2 - d*f)) - c*e*f*x)*Sqrt[a + c*x^2])/(2*f^3) + (a + c*x^
2)^(3/2)/(3*f) - (Sqrt[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*ArcTanh[(Sqrt[c]*
x)/Sqrt[a + c*x^2]])/(2*f^4) - ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e
 - Sqrt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2
*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[
2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a
^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[
(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 2.43059, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1020, 1068, 1080, 217, 206, 1034, 725}

$$-\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\tanh^{-1}\left(\frac{2af}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2}}\right.}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] ((2*(a*f^2 + c*(e^2 - d*f)) - c*e*f*x)*Sqrt[a + c*x^2])/(2*f^3) + (a + c*x^
2)^(3/2)/(3*f) - (Sqrt[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*ArcTanh[(Sqrt[c]*
x)/Sqrt[a + c*x^2]])/(2*f^4) - ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e
 - Sqrt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2
*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
```

```
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a
^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[
(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \frac{\left(a+cx^2\right)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+cx^2}\left(-3(cd-af)x-3cex^2\right)}{d+ex+fx^2}\,dx}{3f}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def+3c\left(ace^2f+2(cd-af)\left(ce^2-cdf+af^2\right)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}}{6cf^3}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def^2-3c^2de\left(3af^2+2c\left(e^2-2df\right)\right)+\left(-3c^2e^2\right.}{\sqrt{a+}}}{}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\left(ce\left(3af^2+2c\left(e^2-2df\right)\right)\right)\text{Subst}\left(\int \right.}{2f^4}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\sqrt{c}e\left(3af^2+2c\left(e^2-2df\right)\right)\tanh^{-1}\left(\right.}{2f^4}$$

$$= \frac{\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)\sqrt{a+cx^2}}{2f^3} + \frac{\left(a+cx^2\right)^{3/2}}{3f} - \frac{\sqrt{c}e\left(3af^2+2c\left(e^2-2df\right)\right)\tanh^{-1}\left(\right.}{2f^4}$$

Mathematica [A] time = 2.16838, size = 755, normalized size = 1.37

$$\frac{8f^3\left(a+cx^2\right)^{5/2}\sqrt{\frac{cx^2}{a}+1}\left(\sqrt{e^2-4df}-e\right)+8f^3\left(a+cx^2\right)^{5/2}\sqrt{\frac{cx^2}{a}+1}\left(\sqrt{e^2-4df}+e\right)+3\left(e-\sqrt{e^2-4df}\right)\left(2\sqrt{c}f^2\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (8*f^3*(-e + Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(5/2)*Sqrt[1 + (c*x^2)/a] + 8*f
^3*(e + Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(5/2)*Sqrt[1 + (c*x^2)/a] + 3*(e - S
qrt[e^2 - 4*d*f])*(2*Sqrt[c]*f^2*(e - Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(a
*Sqrt[c]*x*(1 + (c*x^2)/a)^(3/2) + Sqrt[a]*(a + c*x^2)*ArcSinh[(Sqrt[c]*x)/
Sqrt[a]]) - a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*(1 + (c*x^2)/a)^(3/2)
*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x
)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*
d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-
```

```
e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])) - 3*(e + Sqrt[e^2 -
 4*d*f])*(2*Sqrt[c]*f^2*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(a*Sqrt[c]*
x*(1 + (c*x^2)/a)^(3/2) + Sqrt[a]*(a + c*x^2)*ArcSinh[(Sqrt[c]*x)/Sqrt[a]])
 - a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*(1 + (c*x^2)/a)^(3/2)*(2*f*Sqr
t[a + c*x^2] - Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a +
 c*x^2]] - Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[
(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])))/(48*a*f^4*Sqrt[e^2 - 4*d*f]*(1 + (
c*x^2)/a)^(3/2))
```

Maple [B] time = 0.26, size = 14709, normalized size = 26.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+cx^{2}\right)^{\frac{3}{2}}}{d+ex+fx^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.60 $\int \frac{\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=484

$$-\frac{\left(ce\left(e-\sqrt{e^2-4df}\right)\left(2af^2+c\left(e^2-2df\right)\right)-2f\left(-a^2f^3+2acdf^2+c^2d\left(e^2-df\right)\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] -(c*(2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + (Sqrt[c]*(3*a*f^2 + 2*c*(e^2 - d
*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^3) - ((c*e*(e - Sqrt[e^2 -
4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)) - 2*f*(2*a*c*d*f^2 - a^2*f^3 + c^2*d*(e
^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*f
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 + ((c*e*(e + Sqrt[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)) - 2*f*(2*a*c*d
*f^2 - a^2*f^3 + c^2*d*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])]])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 4.23987, antiderivative size = 482, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {979, 1080, 217, 206, 1034, 725}

$$\frac{\left(-2a^2f^4-ce\left(e-\sqrt{e^2-4df}\right)\left(2af^2+c\left(e^2-2df\right)\right)+4acdf^3+2c^2df\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -(c*(2*e - f*x)*Sqrt[a + c*x^2])/(2*f^2) + (Sqrt[c]*(3*a*f^2 + 2*c*(e^2 - d
*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^3) + ((4*a*c*d*f^3 - 2*a^2*
f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e - Sqrt[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2
 - 2*d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*f
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
```

```
 - ((4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e + Sqrt[e^2 -
4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rule 979

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(c*(e*(2*p + q) - 2*f*(p + q)*x)*(a + c*x^2)^(p - 1)*(d +
 e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*
(p + q)*(2*p + 2*q + 1)), Int[(a + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[
-(a*c*e^2*(1 - p)*(2*p + q)) + a*(p + q)*(-2*a*f^2*(2*p + 2*q + 1) + c*(2*d
*f - e^2*(2*p + q))) + (2*(c*d - a*f)*(c*e)*(1 - p)*(2*p + q) + 4*a*c*e*f*(
1 - p)*(p + q))*x + (p*c^2*e^2*(1 - p) - c*(p + q)*(2*a*f^2*(4*p + 2*q - 1)
 + c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1))))*x^2, x], x], x] /; FreeQ[{a, c
, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && N
eQ[2*p + 2*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
```

```
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{af(cd-2af)-ce(2cd-af)x-c\left(3af^2+2c\left(e^2-df\right)\right)x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2f^2}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{af^2(cd-2af)+cd\left(3af^2+2c\left(e^2-df\right)\right)+\left(-cef(2cd-af)+ce\left(3af^2+2c\left(e^2-df\right)\right)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2f^3} + \frac{\left(c\left(3af^2\right.\right.}{}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\left(c\left(3af^2+2c\left(e^2-df\right)\right)\right)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3} - \frac{\left(2f\left(af^2(cd-2a\right.\right.}{}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}\left(3af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^3} + \frac{\left(2f\left(af^2(cd-2af)+cd\left(3af\right.\right.\right.}{}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}\left(3af^2+2c\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^3} - \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)\left(2af^2+c\right.\right.}{}$$

Mathematica [A] time = 1.15791, size = 603, normalized size = 1.25

$$\frac{\frac{2\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\left(-\sqrt{4af^2-2ce\sqrt{e^2-4df}-4cdf+2ce^2}\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)+\sqrt{c}\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)+2f\sqrt{a+cx^2}\right.}{f^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] ((2*Sqrt[c]*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c
*x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*
Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)
/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*
f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]
*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e
^2 - 4*c*d*f + 4*a*f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e +
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*
f])]*Sqrt[a + c*x^2])]))/f^2 + (2*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(S
qrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/f^2)/(8*f*Sqrt[e^
2 - 4*d*f])
```

Maple [B] time = 0.269, size = 8954, normalized size = 18.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.61 $\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)} dx$

Optimal. Leaf size=496

$$-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2ef\left(c^2d^2\right.\right.}{}$$

```
[Out] (a*Sqrt[a + c*x^2])/d + ((c*d - a*f)*Sqrt[a + c*x^2])/(d*f) - (c^(3/2)*e*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*e*f*(c^2*d^2 - a^2*f^2) - (c^
2*d*e^2 - f*(c*d - a*f)^2)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*e*f*(c^2*d^2 - a^2*f^2) -
(c^2*d*e^2 - f*(c*d - a*f)^2)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*
f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (a^(3/2)*ArcTanh[Sqrt[a + c
*x^2]/Sqrt[a]])/d
```

Rubi [A] time = 2.56885, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6728, 266, 50, 63, 208, 1020, 1080, 217, 206, 1034, 725}

$$-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2ef\left(c^2d^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```


```
[Out] (a*Sqrt[a + c*x^2])/d + ((c*d - a*f)*Sqrt[a + c*x^2])/(d*f) - (c^(3/2)*e*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*e*f*(c^2*d^2 - a^2*f^2) - (c^
2*d*e^2 - f*(c*d - a*f)^2)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*e*f*(c^2*d^2 - a^2*f^2) -
```

```
(c^2*d*e^2 - f*(c*d - a*f)^2)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*
f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (a^(3/2)*ArcTanh[Sqrt[a + c
*x^2]/Sqrt[a]])/d
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
```

```
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{x\left(d+ex+fx^2\right)}\,dx=\int\left(\frac{\left(a+cx^2\right)^{3/2}}{dx}+\frac{(-e-fx)\left(a+cx^2\right)^{3/2}}{d\left(d+ex+fx^2\right)}\right)dx$$

$$=\frac{\int\frac{\left(a+cx^2\right)^{3/2}}{x}\,dx}{d}+\frac{\int\frac{(-e-fx)\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx}{d}$$

$$=-\frac{\left(a+cx^2\right)^{3/2}}{3d}+\frac{\text{Subst}\left(\int\frac{(a+cx)^{3/2}}{x}\,dx,x,x^2\right)}{2d}+\frac{\int\frac{\left(-3aef+3f(cd-af)x\right)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{3df}$$

$$=\frac{(cd-af)\sqrt{a+cx^2}}{df}+\frac{a\,\text{Subst}\left(\int\frac{\sqrt{a+cx}}{x}\,dx,x,x^2\right)}{2d}+\frac{\int\frac{-3a^2ef^2-3f(cd-af)^2x-3c^2defx^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{3df^2}$$

$$=\frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}+\frac{a^2\,\text{Subst}\left(\int\frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d}+\frac{\int\frac{3c^2d^2ef-3a^2ef^3+\left(3c^2de^2f-3f^2(c\right.}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}}{3df^3}$$

$$=\frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}+\frac{a^2\,\text{Subst}\left(\int\frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd}-\frac{\left(c^2e\right)\text{Subst}\left(\int\frac{1}{1-cx^2}\right.}{f^2}$$

$$=\frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{\left(2ef\left(c^2d^2-\right.\right.}{}$$

$$=\frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(c^2de^2-f(cd\right.\right.}{\sqrt{2}df^2\sqrt{e^2}}$$

Mathematica [A] time = 1.62658, size = 746, normalized size = 1.5

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{a\sqrt{af^2+\frac{1}{2}c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2\right.}}\right.}{2df}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (c*Sqrt[a + c*x^2])/f - (c^(3/2)*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^
2 - ((c*d*(-e + Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[4*a*
```

```
f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])])/(4*d*f^2*Sqrt[e^2 - 4*d*f]) - (c*Sqrt[a*f^2 + (c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))/2]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*
x^2])])/(2*f^2) + (a*Sqrt[a*f^2 + (c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))/2
]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*d*f) - (c*e*Sqrt[4*a*f^
2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e
^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])])/(4*f^2*Sqrt[e^2 - 4*d*f]) - (a*e*Sqrt[4*a*f^2 + 2*c*(e^2 -
 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(4*d*f*Sqrt[e^2 - 4*d*f]) - (a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Maple [B] time = 0.262, size = 9728, normalized size = 19.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^{2}\right)^{\frac{3}{2}}}{x\left(d + ex + fx^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(x*(d + e*x + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.62 $\int \frac{\left(a+cx^2\right)^{3/2}}{x^2\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=604

$$-\frac{\left(a^2 f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] -((a*e*Sqrt[a + c*x^2])/d^2) + (3*c*x*Sqrt[a + c*x^2])/(2*d) + ((2*a*e - c*
d*x)*Sqrt[a + c*x^2])/(2*d^2) - (a + c*x^2)^(3/2)/(d*x) + (3*a*Sqrt[c]*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d) + (Sqrt[c]*(2*c*d - 3*a*f)*ArcTanh[
(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d*f) - ((4*a*c*d^2*f^2 + c^2*d^2*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]) + a^2*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*A
rcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((4*a*c*
d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sq
rt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)])/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (a^(3/2)*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

Rubi [A] time = 2.80936, antiderivative size = 604, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {6728, 277, 195, 217, 206, 266, 50, 63, 208, 1020, 1068, 1080, 1034, 725}

$$-\frac{\left(a^2 f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*e*Sqrt[a + c*x^2])/d^2) + (3*c*x*Sqrt[a + c*x^2])/(2*d) + ((2*a*e - c*
d*x)*Sqrt[a + c*x^2])/(2*d^2) - (a + c*x^2)^(3/2)/(d*x) + (3*a*Sqrt[c]*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d) + (Sqrt[c]*(2*c*d - 3*a*f)*ArcTanh[
(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d*f) - ((4*a*c*d^2*f^2 + c^2*d^2*(e^2 - 2*
```

```
d*f - e*Sqrt[e^2 - 4*d*f]) + a^2*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*A
rcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((4*a*c*
d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sq
rt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)])/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (a^(3/2)*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
```

```
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{\left(a+cx^2\right)^{3/2}}{dx^2} - \frac{e\left(a+cx^2\right)^{3/2}}{d^2x} + \frac{\left(e^2-df+efx\right)\left(a+cx^2\right)^{3/2}}{d^2\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(e^2-df+efx\right)\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx}{d^2} + \frac{\int \frac{\left(a+cx^2\right)^{3/2}}{x^2}\,dx}{d} - \frac{e\int \frac{\left(a+cx^2\right)^{3/2}}{x}\,dx}{d^2}$$

$$= \frac{e\left(a+cx^2\right)^{3/2}}{3d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{(3c)\int\sqrt{a+cx^2}\,dx}{d} - \frac{e\,\mathrm{Subst}\left(\int \frac{(a+cx)^{3/2}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{\int \frac{\sqrt{a+cx}}{\ldots}}{\ldots}$$

$$= \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{(3ac)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2d} - \frac{(ae)\,\mathrm{Subst}\left(\int \frac{\sqrt{a+\ldots}}{x}\ldots\right)}{2d^2}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{(3ac)\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\ldots\right)}{2d}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{\left(a+cx^2\right)^{3/2}}{dx} + \frac{3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d}$$

Mathematica [C] time = 4.63904, size = 885, normalized size = 1.47

$$\frac{-x\left(2\sqrt{a}\sqrt{c}df\sqrt{e^2-4df}\sqrt{cx^2+a}\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right) + \sqrt{\frac{cx^2}{a}+1}\left(-2c\sqrt{4af^2+2c\left(e^2+\sqrt{e^2-4df}e-2df\right)}\tanh^{-1}\left(\frac{2a\ldots}{\sqrt{4af^2+2c(\ldots}}\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] (-(x*(2*Sqrt[a]*Sqrt[c]*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + c*x^2]*ArcSinh[(Sqrt
[c]*x)/Sqrt[a]] + Sqrt[1 + (c*x^2)/a]*(2*c*d*f*Sqrt[e^2 - 4*d*f]*x*Sqrt[a +
```

```
c*x^2] - 4*Sqrt[c]*d*(c*d - a*f)*Sqrt[e^2 - 4*d*f]*ArcTanh[(Sqrt[c]*x)/Sqr
t[a + c*x^2]] + (2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*Sqrt[4*a*
f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f + c*(-e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])] + Sqrt[2]*a*e*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2
- 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*
x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)] - 2*c*d^2*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] - a*e^2*Sqrt[4*a*f^2 + 2*c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/
(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
+ 2*a*d*f*Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(
2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*S
qrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])] - 4*a^(3/2)*e*f*Sqrt[e^2 - 4*d*f]*ArcT
anh[Sqrt[a + c*x^2]/Sqrt[a]]))) - 4*a*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + c*x^2]
*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/a)])/(4*d^2*f*Sqrt[e^2 - 4*d*
f]*x*Sqrt[1 + (c*x^2)/a])
```

Maple [B] time = 0.288, size = 9912, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + cx^2\right)^{\frac{3}{2}}}{x^2 \left(d + ex + fx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(x**2*(d + e*x + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.63 $\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx$

Optimal. Leaf size=668

$$\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)+c^2d^3\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\ldots}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2\ldots}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] (3*c*Sqrt[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*Sqrt[a + c*x^2])/d^3 - (3*c*e*
x*Sqrt[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)) - c*d*e*x)*Sqrt[a
+ c*x^2])/(2*d^3) - (a + c*x^2)^(3/2)/(2*d*x^2) + (e*(a + c*x^2)^(3/2))/(d^
2*x) + ((c^2*d^3*(e - Sqrt[e^2 - 4*d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*
f]) + a^2*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])
)*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^3*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*c
*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e
^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[a]*c*ArcTanh[
Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x
^2]/Sqrt[a]])/d^3
```

Rubi [A] time = 3.46473, antiderivative size = 668, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 15, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6728, 266, 47, 50, 63, 208, 277, 195, 217, 206, 1020, 1068, 1080, 1034, 725}

$$\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)+c^2d^3\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\ldots}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2\ldots}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (3*c*Sqrt[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*Sqrt[a + c*x^2])/d^3 - (3*c*e*
x*Sqrt[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)) - c*d*e*x)*Sqrt[a
```

```
+ c*x^2])/(2*d^3) - (a + c*x^2)^(3/2)/(2*d*x^2) + (e*(a + c*x^2)^(3/2))/(d^
2*x) + ((c^2*d^3*(e - Sqrt[e^2 - 4*d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*
f]) + a^2*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])
)*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*c
*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e
^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[a]*c*ArcTanh[
Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x
^2]/Sqrt[a]])/d^3
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
```

```
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1068

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(-(c*e*(2*p + q + 2))) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*(d + e*x
+ f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*(-(a*e))*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1080

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{\left(a+cx^2\right)^{3/2}}{x^3\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{\left(a+cx^2\right)^{3/2}}{dx^3} - \frac{e\left(a+cx^2\right)^{3/2}}{d^2x^2} + \frac{\left(e^2-df\right)\left(a+cx^2\right)^{3/2}}{d^3x} + \frac{\left(-e\left(e^2-2df\right)-f\left(e^2-df\right)x\right)\left(a\right.}{d^3\left(d+ex+fx^2\right)}\right.$$

$$= \frac{\int \frac{\left(-e\left(e^2-2df\right)-f\left(e^2-df\right)x\right)\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx}{d^3} + \frac{\int \frac{\left(a+cx^2\right)^{3/2}}{x^3}\,dx}{d} - \frac{e\int \frac{\left(a+cx^2\right)^{3/2}}{x^2}\,dx}{d^2} + \frac{\left(e^2-df\right)\int \frac{\left(a+cx^2\right)^{3/2}}{x}\,dx}{d^3}$$

$$= -\frac{\left(e^2-df\right)\left(a+cx^2\right)^{3/2}}{3d^3} + \frac{e\left(a+cx^2\right)^{3/2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x^2}\,dx,x,x^2\right)}{2d} - \frac{(3ce)\int\sqrt{a+cx^2}\,dx}{d^2}$$

$$= -\frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)\sqrt{a+cx^2}}{2d^3} - \frac{\left(a+cx^2\right)^{3/2}}{2dx^2} + \frac{e\left(a+cx^2\right)^{3/2}}{d^2x} + \frac{(3}{}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a\left(e^2-df\right)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)\sqrt{a+cx^2}}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a\left(e^2-df\right)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)\sqrt{a+cx^2}}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a\left(e^2-df\right)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)\sqrt{a+cx^2}}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a\left(e^2-df\right)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{\left(2\left(cd^2+a\left(e^2-df\right)\right)-cdex\right)\sqrt{a+cx^2}}{2d^3}$$

Mathematica [C] time = 3.54652, size = 904, normalized size = 1.35

$$\frac{6cd^2\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{a}+1\right)\left(cx^2+a\right)^{5/2}}{a^2} - 5\left(e^2-\frac{\left(e^2-3df\right)e}{\sqrt{e^2-4df}}-df\right)\left(cx^2+a\right)^{3/2} - 5\left(e^2+\frac{\left(e^2-3df\right)e}{\sqrt{e^2-4df}}-df\right)\left(cx^2+a\right)^{3/2} + \frac{30ade\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx}{a}\right.}{x\sqrt{\frac{cx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (-5*(e^2 - d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) - 5
*(e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(a + c*x^2)^(3/2) + (15*
(-e^2 + d*f - (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*((2*Sqrt[c]*(-e + Sqrt[e
^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/a] + Sqrt[a]*ArcSi
nh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]))*(2*f*Sqrt[a + c*x^2] + Sqrt[c]*(-e + Sqrt[e^2 - 4
*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] - Sqrt[2*c*e^2 - 4*c*d*f + 4*a*
f^2 - 2*c*e*Sqrt[e^2 - 4*d*f]]*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*
x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])]))/f^2))/(8*f) - (15*(-e^2 + d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*
((2*Sqrt[c]*(e + Sqrt[e^2 - 4*d*f])*Sqrt[a + c*x^2]*(Sqrt[c]*x*Sqrt[1 + (c*
x^2)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]*x)/Sqrt[a]]))/Sqrt[1 + (c*x^2)/a] + (2*(
2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*(-2*f*Sqrt[a + c*x^2] + Sq
rt[c]*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]] + Sqrt[4
*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + S
qrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])]))/f^2))/(8*f) + 10*(e^2 - d*f)*(Sqrt[a + c*x^2]*(4*a +
 c*x^2) - 3*a^(3/2)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]) + (30*a*d*e*Sqrt[a +
c*x^2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((c*x^2)/a)])/(x*Sqrt[1 + (c*x^2
)/a]) + (6*c*d^2*(a + c*x^2)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (c*x^
2)/a])/a^2)/(30*d^3)
```

Maple [B] time = 0.312, size = 10298, normalized size = 15.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2+a\right)^{\frac{3}{2}}}{\left(fx^2+ex+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(3/2)/x**3/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.64 $\int \frac{x^3}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=380

$$-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2def-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2}}$$

```
[Out] Sqrt[a + c*x^2]/(c*f) - (e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*f
^2) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*d*e*f - (e^2 - d*f)*(e +
Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])])
```

Rubi [A] time = 1.16951, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6728, 217, 206, 261, 1034, 725}

$$-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2def-\left(e^2-df\right)\left(\sqrt{e^2-4df}+e\right)\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] Sqrt[a + c*x^2]/(c*f) - (e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*f
^2) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*d*e*f - (e^2 - d*f)*(e +
Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
```

```
*d*f])])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \int\left(-\frac{e}{f^2\sqrt{a+cx^2}}+\frac{x}{f\sqrt{a+cx^2}}+\frac{de+\left(e^2-df\right)x}{f^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$=\frac{\int \frac{de+\left(e^2-df\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2}-\frac{e\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f^2}+\frac{\int \frac{x}{\sqrt{a+cx^2}}\,dx}{f}$$

$$=\frac{\sqrt{a+cx^2}}{cf}-\frac{e\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{}{(e}}{f^2\sqrt{e^2-4df}}$$

$$=\frac{\sqrt{a+cx^2}}{cf}-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\mathrm{Subst}\left(\int \frac{}{4af^2+c(}\right.}{f^2\sqrt{e^2-4df}}$$

$$=\frac{\sqrt{a+cx^2}}{cf}-\frac{e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{2af^2}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-\right.}}$$

Mathematica [A] time = 1.37714, size = 378, normalized size = 0.99

$$-\frac{\frac{\sqrt{2}\left(\left(e^2-df\right)\left(\sqrt{e^2-4df}-e\right)+2def\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-\right.}}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((-2*f*Sqrt[a + c*x^2])/c + (2*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqr
t[c] + (Sqrt[2]*(2*d*e*f + (e^2 - d*f)*(-e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2
*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*(e^3 - 3*d*e*f + e^2*Sqrt[
e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*
d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])]))/(2*f^2)
```

Maple [B] time = 0.291, size = 2397, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] (c*x^2+a)^(1/2)/c/f-1/f^2*e*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2
^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c
*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f))*d-1/2/f^3*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*
d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f))*e^2-3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(
1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d*e+1/2/f^3
/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(
-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f))*e^3+1/2/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2
*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d-1/2/f^3*2^(1/2)/(((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
```

```
2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2+3/2/f^2/(-4*d*f+e^2)^
(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln
((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2
))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4
*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f))*d*e-1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^
(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2
))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a + cx^2} \left(d + ex + fx^2\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.65 $\int \frac{x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=344

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2a\cdots}{\sqrt{2}\sqrt{a+cx^2}\cdots}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4d\cdots}\right.}}$$

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - ((e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*
f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
- ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4
*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])]])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 0.540944, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1081, 217, 206, 1034, 725}

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\frac{2a\cdots}{\sqrt{2}\sqrt{a+cx^2}\cdots}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4d\cdots}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - ((e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*
f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
- ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4
*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])]])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rule 1081

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dis
t[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]
/; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f\sqrt{e^2-4df}} -$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c}{}\right.}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

Mathematica [A] time = 0.732037, size = 334, normalized size = 0.97

$$\frac{\frac{\sqrt{2}\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{2\tanh^{-1}\left(\right.}{\sqrt{c}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/Sqrt[c] + (Sqrt[2]*(-e^2 + 2*d*f
+ e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sq
rt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (S
qrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]))/(2*f)
```

Maple [B] time = 0.269, size = 1796, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] 1/f*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)+1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/
f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/f/(-4*d*f+e^2
)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^
(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f))*d-1/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2+1/2/f^2*2^(1/2)/((
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^
2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/f/(-4*
d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f))*d+1/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.66 $\int \frac{x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=294

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
4*d*f])]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 0.235271, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1034, 725, 206}

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
4*d*f])]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
```

```
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = -\left(\left(-1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx\right)+\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{}{(e}$$

$$=\left(-1+\frac{e}{\sqrt{e^2-4df}}\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx,x,\frac{2af-c\left(e-\sqrt{e^2-4\right.}}{\sqrt{a+cx^2}}\right.$$

$$=-\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}-\frac{\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{2}}\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-\right.}}$$

Mathematica [A] time = 0.384988, size = 275, normalized size = 0.94

$$\frac{\sqrt{2}\left(-\frac{\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{2\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{2\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*(-((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*
d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
 c*x^2])])/(2*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((e
+ Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*
a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(2*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])))/Sqrt[e^2 - 4*d*f]
```

Maple [B] time = 0.261, size = 1172, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/2/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*
ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(
1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f))+1/2/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+
1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e-1/2/(-4*d*f+e^2)^(1/2)/f*
2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+
(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*
e-1/2/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*
ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1
/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c
-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/
```

```
2))/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.07096, size = 10211, normalized size = 34.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a
^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(
a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^
3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*
d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 +
2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a
^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a^2*d*
e^2 + sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 +
 a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^
4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^
2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4
*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2
)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 +
 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*
e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2
)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*
d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*
a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*
c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*
e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*
```

a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*
d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c
^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 +
a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a
^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*
d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2
 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c
*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^
4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f
^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*
a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3
+ (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*a*c*d^
2*e*x - 2*a^2*d*e^2 - sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f - (2*c^3*d^4*e^2 + 3
*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^
3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^
2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 +
 a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d
^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^
2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sq
rt((2*c*d^2 + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a
*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^
4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e
^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^
2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4
)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2
*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) + 2*(a*c^2*d^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2
*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)
*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 +
(16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*
a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*
e^2 + 2*a^2*c^2*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*
f - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2
*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^
2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3
+ a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 -
 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^
4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f
))*log((4*a*c*d^2*e*x - 2*a^2*d*e^2 + sqrt(2)*(a^2*e^4 - 4*a^2*d*e^2*f + (2
*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4 - 6*(4*a^2*c*d^3
 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*e^4)*f^2 - 2*(4*
c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2
*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 -
 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2
 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sq
rt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*

```
a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt
(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a
^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3
*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 +
 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 +
a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(a*c^2*d^3*e^2 + a^2*c*d
*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2*a*c^2*d^4 + 3*a
^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6
- 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*
e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^
5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^2
+ a*e^2 - 2*a*d*f - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2
*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*
c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12
*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 +
a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*
d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3
+ 3*a*c*d*e^2)*f))*log((4*a*c*d^2*e*x - 2*a^2*d*e^2 - sqrt(2)*(a^2*e^4 - 4*
a^2*d*e^2*f + (2*c^3*d^4*e^2 + 3*a*c^2*d^2*e^4 + a^2*c*e^6 + 8*a^3*d^2*f^4
- 6*(4*a^2*c*d^3 + a^3*d*e^2)*f^3 + (24*a*c^2*d^4 + 22*a^2*c*d^2*e^2 + a^3*
e^4)*f^2 - 2*(4*c^3*d^5 + 9*a*c^2*d^3*e^2 + 4*a^2*c*d*e^4)*f)*sqrt(a^2*e^2/
(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2
+ a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11
*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^
2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (c^2*d^2*e^
2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*
c*d*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*
f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3
*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3
 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(a*c^2*d
^3*e^2 + a^2*c*d*e^4 - 4*a^3*d^2*f^3 + (8*a^2*c*d^3 + a^3*d*e^2)*f^2 - 2*(2
*a*c^2*d^4 + 3*a^2*c*d^2*e^2)*f)*sqrt(a^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^
4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^
2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.67 $\int \frac{1}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] -((Sqrt[2]*f*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + (Sqrt[2
]*f*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 0.150652, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {985, 725, 206}

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + (Sqrt[2
]*f*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 985

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f
```

```
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}} + \frac{(2f)\,\mathrm{Subst}\left(\int \frac{}{4af^2+c(e\cdots}\right.}{}$$

$$= -\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4\cdots}\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2\cdots}\right.}}\right.}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+\cdots\right.}}$$

Mathematica [A] time = 0.336369, size = 247, normalized size = 0.93

$$\frac{2\sqrt{2}f\left(\frac{\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{2\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{2\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (2*Sqrt[2]*f*(-ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2
 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]/(2*Sqrt[2*a*
f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ArcTanh[(2*a*f - c*(e + Sqr
t[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2])]/(2*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
)))/Sqrt[e^2 - 4*d*f]
```

Maple [B] time = 0.263, size = 589, normalized size = 2.2

$$-\sqrt{2}\ln\left(\left(\frac{1}{f^2}\left(-\sqrt{-4\,df+e^2}ce+2\,af^2-2\,cdf+ce^2\right)-\frac{c}{f}\left(e-\sqrt{-4\,df+e^2}\right)\left(x-\frac{1}{2\,f}\left(-e+\sqrt{-4\,df+e^2}\right)\right)+\frac{\sqrt{2}}{2}\sqrt{\frac{1}{f^2}\left(\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e
-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f))+1/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.90328, size = 10168, normalized size = 38.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a
^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(
c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^
3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*
d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 +
2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a
^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x - 2*a*c*e^
2*f + sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*c*e^3)*f - (c^3
*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e + a^3*e^3)*f^3 + (4
*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e + 5*a*c^2*d^2*e^3 - a^2*c*
e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d
*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3
+ 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c
^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f +
2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2
- 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^
4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^
2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 +
a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*
e^2)*f)) + 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*(2*a*c^2*d^3 +
3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^2*e^2/(c^4*d^4*e
^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)
*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d
^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f
)))/x) + 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e
^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*
f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5
+ (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(
8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^
```

3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c
*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*e*f*x -
2*a*c*e^2*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a*c*e^3)*
f - (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e + a^3*e^3)*
f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e + 5*a*c^2*d^2*e^3
- a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 -
4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e
^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5
+ 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*
c*d*f + 2*a*f^2 + (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e
^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^
3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(
2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^
3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^
2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 +
3*a*c*d*e^2)*f)) + 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2*(2*a*c^
2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^2*e^2/(c
^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 +
a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a
^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*
d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2
+ a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c
*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^
4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f
^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*
a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3
+ (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((4*c^2*d*
e*f*x - 2*a*c*e^2*f + sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d^2*e + a
*c*e^3)*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d^2*e + a
^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e + 5*a*c^2*
d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c
^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a
^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*
(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*sqrt((c*
e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2
+ a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2
+ 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^
4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*
e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))
/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^
2*d^3 + 3*a*c*d*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)*f^3 + 2
*(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)*sqrt(c^
2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*
c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^
4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*

```
a^2*c^2*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2
*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3
 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^
6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*
d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*
d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^
2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f))*log((
4*c^2*d*e*f*x - 2*a*c*e^2*f - sqrt(2)*(c^2*d*e^3 + 4*a*c*d*e*f^2 - (4*c^2*d
^2*e + a*c*e^3)*f + (c^3*d^3*e^3 + a*c^2*d*e^5 - 4*a^3*d*e*f^4 + (4*a^2*c*d
^2*e + a^3*e^3)*f^3 + (4*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^2 - (4*c^3*d^4*e +
5*a*c^2*d^2*e^3 - a^2*c*e^5)*f)*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4
 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2
*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*
f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e^4)*f)))*sqrt(c*x^2 + a)*
sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8
*a*c*d^2 + a^2*e^2)*f^2 - 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)*sqrt(c^2*e^2/(c^4*
d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (16*a^3*c*d^2 + a^4
*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*a*c^3*d^4 + 11*a^2*
c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*e^2 + 2*a^2*c^2*d*e
^4)*f)))/(c^2*d^2*e^2 + a*c*e^4 - 4*a^2*d*f^3 + (8*a*c*d^2 + a^2*e^2)*f^2 -
 2*(2*c^2*d^3 + 3*a*c*d*e^2)*f)) - 2*(4*a^3*d*f^4 - (8*a^2*c*d^2 + a^3*e^2)
*f^3 + 2*(2*a*c^2*d^3 + 3*a^2*c*d*e^2)*f^2 - (a*c^2*d^2*e^2 + a^2*c*e^4)*f)
*sqrt(c^2*e^2/(c^4*d^4*e^2 + 2*a*c^3*d^2*e^4 + a^2*c^2*e^6 - 4*a^4*d*f^5 +
(16*a^3*c*d^2 + a^4*e^2)*f^4 - 12*(2*a^2*c^2*d^3 + a^3*c*d*e^2)*f^3 + 2*(8*
a*c^3*d^4 + 11*a^2*c^2*d^2*e^2 + a^3*c*e^4)*f^2 - 4*(c^4*d^5 + 3*a*c^3*d^3*
e^2 + 2*a^2*c^2*d*e^4)*f)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.68 $\int \frac{1}{x\sqrt{a+cx^2}\left(d+ex+fx^2\right)} \, dx$

Optimal. Leaf size=330

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[Out] (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(S
qrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f])]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d
)
```

Rubi [A] time = 0.824597, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6728, 266, 63, 208, 1034, 725, 206}

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(S
qrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f])]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d
)
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx\sqrt{a+cx^2}} + \frac{-e-fx}{d\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} - \frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d} - \frac{\left(f\left(1+\right.\right.}{}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\right.}{d}$$

$$= \frac{f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\right.}{\sqrt{2}d\sqrt{2af^2+c}}$$

Mathematica [A] time = 0.788217, size = 319, normalized size = 0.97

$$\frac{\frac{\sqrt{2}f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}f\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((Sqrt[2]*f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]) + (Sqrt[2]*f*(-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f])]) - (2*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a])/(2*d)
```

Maple [B] time = 0.319, size = 681, normalized size = 2.1

$$-2\frac{f\sqrt{2}}{\left(-e+\sqrt{-4df+e^2}\right)\sqrt{-4df+e^2}}\ln\left(\left(\frac{-\sqrt{-4df+e^2}ce+2af^2-2cdf+ce^2}{f^2}-\frac{c\left(e-\sqrt{-4df+e^2}\right)}{f}\left(x-1/2\frac{-e+\sqrt{-4}}{f}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2
)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/
x)-2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.69 $\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=367

$$-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2a\cdots}{\sqrt{2}\sqrt{a+cx^2}\cdots}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4\cdots}\right)}}$$

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh
[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[
2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])
/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rubi [A] time = 1.19924, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6728, 264, 266, 63, 208, 1034, 725, 206}

$$-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2a\cdots}{\sqrt{2}\sqrt{a+cx^2}\cdots}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4\cdots}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh
[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]))/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[
2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])
/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\sqrt{a+cx^2}} - \frac{e}{d^2x\sqrt{a+cx^2}} + \frac{e^2-df+efx}{d^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{e^2-df+efx}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{d^2} + \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} - \frac{\left(f\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\int \frac{\quad}{(e+\sqrt{e^2-4df}\ldots}}{d^2\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2} + \frac{\left(f\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\mathrm{Subs}\ldots}{\ldots}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{f\left(e^2-2df+e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \ldots$$

Mathematica [A] time = 0.902893, size = 356, normalized size = 0.97

$$-\frac{\frac{\sqrt{2}f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}f\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{a+cx^2}\sqrt{4af^2+2c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{2d\sqrt{a+cx^2}}{ax}\ldots}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((2*d*Sqrt[a + c*x^2])/(a*x) + (Sqrt[2]*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^
2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
```

```
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[2]*f*(-e^2 + 2*
d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(S
qrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(
Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) -
(2*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a])/(2*d^2)
```

Maple [B] time = 0.273, size = 736, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2)
)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-
4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x
^2+a)^(1/2))/x)+4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e
^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(
-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x*(c*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.70 $\int \frac{1}{x^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)} \, dx$

Optimal. Leaf size=457

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(-\left(e\right.\right.}{}$$

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f*(2*e^3 -
4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e
+ Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/
(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - ((e^2 - d
*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rubi [A] time = 1.86333, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 266, 51, 63, 208, 264, 1034, 725, 206}

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(-\left(e\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -Sqrt[a + c*x^2]/(2*a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f*(2*e^3 -
4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e
+ Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/
(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 -
```

```
 4*d*f])]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - ((e^2 - d
*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx^3\sqrt{a+cx^2}} - \frac{e}{d^2x^2\sqrt{a+cx^2}} + \frac{e^2-df}{d^3x\sqrt{a+cx^2}} + \frac{-e\left(e^2-2df\right)-f\left(e^2-df\right)x}{d^3\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\right.$$

$$= \frac{\int \frac{-e\left(e^2-2df\right)-f\left(e^2-df\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{d^3} + \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} + \frac{\left(e^2-df\right)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx,x,x^2\right)}{2d} + \frac{\left(e^2-df\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{2d^3} +$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx,x,x^2\right)}{4ad} + \frac{\left(e^2-df\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\right.}{cd^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f\left(2e^3-4def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-\right.}}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{f\left(2e^3-4def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-\right.}}$$

Mathematica [A] time = 1.67879, size = 460, normalized size = 1.01

$$-\frac{\frac{cd^2\sqrt{a+cx^2}\left(\frac{a}{cx^2}-\frac{\tanh^{-1}\left(\sqrt{\frac{cx^2}{a}+1}\right)}{\sqrt{\frac{cx^2}{a}+1}}\right)}{a^2}-\frac{\sqrt{2}f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{2}f\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4\cdots\right.}{\sqrt{e^2-\cdots}}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((-2*d*e*Sqrt[a + c*x^2])/(a*x) - (Sqrt[2]*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2
 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*
d*f])]) + (Sqrt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2
- 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (2*(e^2 - d*f)*Ar
cTanh[Sqrt[a + c*x^2]/Sqrt[a]])/Sqrt[a] + (c*d^2*Sqrt[a + c*x^2]*(a/(c*x^2)
 - ArcTanh[Sqrt[1 + (c*x^2)/a]]/Sqrt[1 + (c*x^2)/a]))/a^2)/(2*d^3)
```

Maple [B] time = 0.278, size = 911, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x)
```

```
[Out] -8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2)
)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-64*f^4/(-e+(-4*
d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2
+a)^(1/2))/x)*d+64*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a
^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)*e^2-8*f^3/(e+(-4*d*f+e^2)^(1/2
```

```
))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e
+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/
a/x^2*(c*x^2+a)^(1/2)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*c/
a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+16*f^2*e/(-e+(-4*d*f+e^2)^(1/
2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/x*(c*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}\left(fx^2 + ex + d\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a + cx^2}\left(d + ex + fx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.71 $\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=499

$$\frac{cex\left(a\left(e^2-2df\right)+cd^2\right)+af\left(a\left(e^2-df\right)+cd^2\right)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{2af-}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2a}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\right.}}$$

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + (a*f*(c*d^2 +
a*(e^2 - d*f)) + c*e*(c*d^2 + a*(e^2 - 2*d*f))*x)/(a*f^2*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[a + c*x^2]) - ((2*a*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(c*d^2 + a
*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2
]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f])]) + ((2*a*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(c*d^2 +
a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 2.1068, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6728, 191, 261, 1017, 1034, 725, 206}

$$\frac{cex\left(a\left(e^2-2df\right)+cd^2\right)+af\left(a\left(e^2-df\right)+cd^2\right)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\tanh^{-1}\left(\frac{2af-}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2a}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + (a*f*(c*d^2 +
a*(e^2 - d*f)) + c*e*(c*d^2 + a*(e^2 - 2*d*f))*x)/(a*f^2*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[a + c*x^2]) - ((2*a*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(c*d^2 + a
*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2
]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
```

```
 - e*Sqrt[e^2 - 4*d*f])]) + ((2*a*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(c*d^2 +
a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
```

```
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \left(-\frac{e}{f^2\left(a+cx^2\right)^{3/2}} + \frac{x}{f\left(a+cx^2\right)^{3/2}} + \frac{de+\left(e^2-df\right)x}{f^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{de+\left(e^2-df\right)x}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{e\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{f^2} + \frac{\int \frac{x}{\left(a+cx^2\right)^{3/2}}\,dx}{f}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2df\right)\right)x}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} +$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2df\right)\right)x}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} +$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2df\right)\right)x}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} -$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af\left(cd^2+a\left(e^2-df\right)\right)+ce\left(cd^2+a\left(e^2-2df\right)\right)x}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} -$$

Mathematica [A] time = 2.90962, size = 577, normalized size = 1.16

$$\frac{\left(-\frac{e(e^2-3df)}{\sqrt{e^2-4df}}-df+e^2\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{af^2\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)}+\frac{\left(\frac{e(e^2-3df)}{\sqrt{e^2-4df}}-df+e^2\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{af^2\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{\sqrt{2}\left(-e^2\sqrt{e^2-4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + ((e^2 - d*f -
(e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))
/(a*f^2*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + ((e^2 -
d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f
])*x))/(a*f^2*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (S
qrt[2]*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*ArcT
anh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*
f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqrt[2]*(e^3 - 3*d*e*f + e
^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[
e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*S
qrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]))^(3/2))
```

Maple [B] time = 0.276, size = 6124, normalized size = 12.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.72 $\int \frac{x^2}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=410

$$-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-4df}+e\right)\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace\right.}$$

```
[Out] -((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) - (f*(
2*d*(c*d - a*f) + a*e*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^
2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*d*(c*d -
a*f) + a*e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 0.708933, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1063, 1034, 725, 206}

$$-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-4df}+e\right)\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) - (f*(
2*d*(c*d - a*f) + a*e*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^
2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*d*(c*d -
a*f) + a*e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a
```

```
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 1063

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2
)^(q + 1)*((A*c - a*C)*(2*a*c*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*
d - a*f)))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((
-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*
x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p + 1)
 + (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) -
e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q
 + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(-(c*e*(2*p + q + 4))))*x
 - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x
], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p
, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]
) &&  !IGtQ[q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{2acd(cd-af)-2a^2cefx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{}{\left(e-\sqrt{e^2-4df\right.}}}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{}{4a}\right.}{\sqrt{e^2-4df}\left(ace^2+(\right.}$$

$$= -\frac{ae+(cd-af)x}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{2}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e\right.}}$$

Mathematica [A] time = 2.49389, size = 509, normalized size = 1.24

$$\frac{-\frac{\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)} - \frac{\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)} + \frac{\sqrt{2}f^2\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}} + -\sqrt{}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (x/(a*Sqrt[a + c*x^2]) - ((e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c
*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqr
t[a + c*x^2]) - ((e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt
[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^
2]) + (Sqrt[2]*f^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f + c*
(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2)))/f
```

Maple [B] time = 0.296, size = 4752, normalized size = 11.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/f*x/a/(c*x^2+a)^(1/2)-1/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)*e-2*f/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f
*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2)^(1/2)*d+1/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)
^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*e^2+2/f*(-4*d*f+e^2)^(1/2)*c^2/(-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*
d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e+4*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*x*d-4/f*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f
*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2
*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e^2-4/(-4*d*f+e^2)^(1
/2)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^
2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*
(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e*d+2/f/(-4*d*f+e^2)^(1/2)
*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f
^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-
(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e^3+1/(-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c
*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*
d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x
```

-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+2*f/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/
2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-
e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d-1/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2
^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2-1/((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2
)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*e+2*f/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*
d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*d-1/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e
+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*e^2-2/f*(-4*d*f+e^2)^(1/2)*c^2/(
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-
1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e+4*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*x*d-4/f*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^
2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e^2+4/(-4*d*f+e^2)^(1/
2)*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/
f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+
(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e*d-2/f/(-4*d*f+e^2)^(1/2)*c^2/
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2
-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f
+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e^3+1/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+

```
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f))*e-2*f/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2
)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f))*d+1/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*l
n((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/
2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f))*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.73 $\int \frac{x}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=411

$$\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right.}$$

```
[Out] -((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) + (f*(2*
c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e
^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*c*d*e - (c*
d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
+ c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f
^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 0.825514, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1017, 1034, 725, 206}

$$\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) + (f*(2*
c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e
^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*c*d*e - (c*
d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
+ c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f
```

```
^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx = -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{-2ac^2de-2acf(cd-af)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{}{\left(e-\sqrt{e^2-4d\right.}}}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int\right.}{\sqrt{e^2-4df}\left(ace^2+\right.}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c}}$$

Mathematica [A] time = 0.911636, size = 457, normalized size = 1.11

$$\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)} + \frac{\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)} + \frac{\sqrt{2}f^2\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((1 - e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f
^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + ((1 + e/Sqrt[e^2 - 4*d
*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 -
 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^2*(e - Sqrt[e^2 - 4*d*f])*ArcTanh
[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) - (Sqrt[2]*f^2*(e + Sqrt[e^2 - 4
*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 + 2*c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f]*(2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))
```

Maple [B] time = 0.29, size = 3000, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/((x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+
1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2*(-4*d*f+e^
2)^(1/2)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f
*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2
*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x+4*c^2/(-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f
+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/
f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*x*e-f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((
(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2)
)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*f*e/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*
(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2)-2/(-4*d*f+e^2)^(1/2)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2)*x*e^2+1/(-4*d*f+e^2)^(1/2)*f*e/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(
-4*d*f+e^2)^(1/2))/f))+1/(-4*d*f+e^2)^(1/2)*f*e/((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)
^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+4*e*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(
```

```
-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x+
2/(-4*d*f+e^2)^(1/2)*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*
a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/
2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e^2-1/(-4*d*f
+e^2)^(1/2)*f*e/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*2^(1/2)/(((-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)
^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+f/((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*(-4*d*f+e^2)^(1/2)*c^2/((-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/
f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^
2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x-f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*
ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1
/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c
-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.74 $\int \frac{1}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=416

$$-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df \cdots\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ \cdots\right.}$$

```
[Out] (c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) - (
f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*S
qrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rubi [A] time = 0.61705, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {976, 1034, 725, 206}

$$-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\tanh^{-1}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df \cdots\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) - (
f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*S
```

```
qrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{-2ac\left(af^2+c\left(e^2-df\right)\right)-2ac^2efx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\int \frac{}{\left(e+\sqrt{e^2-4\right.}}}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int\right.}{\sqrt{e^2-4df}\left(ace^2+\right.}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{f\left(2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{}{\sqrt{2}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c}}$$

Mathematica [A] time = 2.33532, size = 320, normalized size = 0.77

$$\frac{c(a(e-fx)+cdx)}{a\sqrt{a+cx^2}\left(a^2f^2+ac\left(e^2-2df\right)+c^2d^2\right)} - \frac{2\sqrt{2}f^3\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}} + \frac{2\sqrt{2}f^3\tanh^{-1}\left(\frac{}{\sqrt{a+cx}}\right.}{\sqrt{e^2-4df}\left(2af^2+c\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c*(c*d*x + a*(e - f*x)))/(a*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[a
 + c*x^2]) - (2*Sqrt[2]*f^3*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/
(Sqrt[4*a*f^2 - 2*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2
)) + (2*Sqrt[2]*f^3*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a
*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^
2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2))
```

Maple [B] time = 0.304, size = 1713, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 2/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2)-4*c^2*f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4
*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x+4/(-4*d*f+e^2)^
(1/2)*c^2*f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*
d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*
c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e-2/(-4*d*f+e^2)^(1/2)
/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^
2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-2/(-4*d*f+e^2)
^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-4*c^2*f
/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^
2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*
f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x-4/(-4*d*f+e^2)^(1/2)*c^2*f/((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4
*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2
))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2)*x*e+2/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*
(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.75 $\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=526

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c}}$$

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)/(a*d*(a*c
*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f))
- (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e -
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
 a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e*(
a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))
*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 -
 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rubi [A] time = 2.18307, antiderivative size = 526, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)/(a*d*(a*c
*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f))
- (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e -
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
 a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e*(
```

```
a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))
*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 -
 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
```

```
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx\left(a+cx^2\right)^{3/2}} + \frac{-e-fx}{d\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d} + \frac{\int \frac{-e-fx}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{d}$$

$$= -\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx, x, x^2\right)}{2d} + \frac{\int \frac{-2ace\left(af^2+c\left(e^2-\right.\right.}{\sqrt{a-}}}{2acd\left(a\right.}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2ad} - \frac{\left(f\left(\right.\right.}{}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{acd}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\right.\right.}{\sqrt{2}d\sqrt{e^2-4df}}$$

Mathematica [C] time = 4.02099, size = 497, normalized size = 0.94

$$\frac{-\frac{f\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)} - \frac{f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)} + \frac{\sqrt{2}f^3\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4df}-e\right)}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2-4df}+2df-e^2\right)}}\right)}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}} + \frac{\sqrt{2}f^3\left(\right.}{}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-((f*(1 + e/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*d*f])*x))/(a*(
4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2])) - (f*(1 - e/Sqrt[e
^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*x))/(a*(4*a*f^2 + c*(e + Sq
rt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (Sqrt[2]*f^3*(e + Sqrt[e^2 - 4*d*f])
*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c*(-e^2 +
 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4*d*f]*(2*a*f
^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^3*(-e + Sqr
```

```
t[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2
 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 -
4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))^(3/2)) + Hypergeom
etric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/a]/(a*Sqrt[a + c*x^2]))/d
```

Maple [B] time = 0.273, size = 1945, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e
^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-8*f^2/(-e+(-4*d*f+e^2)^(1/2))*c^2/(-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^
2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f+e^2
)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x+8*f^2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^
2)^(1/2)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f
*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2
*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e-4*f^3/(-e+(-4*d*f+e
^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((
(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2)
)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c
-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f))-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/(c*x^2+a)^(
1/2)+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(3/2)*ln((2*a+2*a
^(1/2)*(c*x^2+a)^(1/2))/x)+4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+8*f^2/(e+(-4*d*f+e
^2)^(1/2))*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/
f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2
*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x+8*f^2/(e+(-4*d*f+e^2)^(
```

```
1/2))/(-4*d*f+e^2)^(1/2)*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e-4*f^3/(
e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^
2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.76 $\int \frac{1}{x^2(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=618

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{cdx\left(af^2+c\left(e^2-df\right)\right)+ae\left(af^2+c\left(e^2-2df\right)\right)}{ad^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2\right.\right.\right.}{\sqrt{2}a}$$

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*(a*f^2 + c*(e^2 - d*
f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(e*(e - Sqrt[
e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3
*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/
(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e*(e + Sqrt[e^2 - 4*d*f])*(a*f^2
 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2))
)*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^
2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)
```

Rubi [A] time = 2.2803, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6728, 271, 191, 266, 51, 63, 208, 1017, 1034, 725, 206}

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{cdx\left(af^2+c\left(e^2-df\right)\right)+ae\left(af^2+c\left(e^2-2df\right)\right)}{ad^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2\right.\right.\right.}{\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*(a*f^2 + c*(e^2 - d*
f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(e*(e - Sqrt[
e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3
```

```
*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/
(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e*(e + Sqrt[e^2 - 4*d*f])*(a*f^2
 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2))
)*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^
2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1017

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^
(q + 1)*(g*c*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c
*(2*a*f)) - h*(-2*a*c*e))*x))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - (-(a*e))*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + (-(a*h))*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q +
2) - (2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(-(c*e*(2*p + q + 4))))*x - c*f*(
2*(g*c*(c*d - a*f) - a*(-(h*c*e))))*(2*p + 2*q + 5)*x^2, x], x], x] /; Free
Q[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[
a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{1}{dx^2\left(a+cx^2\right)^{3/2}}-\frac{e}{d^2x\left(a+cx^2\right)^{3/2}}+\frac{e^2-df+efx}{d^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$=\frac{\int\frac{e^2-df+efx}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{d^2}+\frac{\int\frac{1}{x^2\left(a+cx^2\right)^{3/2}}\,dx}{d}-\frac{e\int\frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d^2}$$

$$=-\frac{1}{adx\sqrt{a+cx^2}}+\frac{ae\left(af^2+c\left(e^2-2df\right)\right)+cd\left(af^2+c\left(e^2-df\right)\right)x}{ad^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}-\frac{(2c)\int\frac{}{(a+c}}{ad}$$

$$=-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{ae\left(af^2+c\left(e^2-2df\right)\right)+cd\left(a\right.}{ad^2\left(ace^2+(cd-af)^2\right)}$$

$$=-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{ae\left(af^2+c\left(e^2-2df\right)\right)+cd\left(a\right.}{ad^2\left(ace^2+(cd-af)^2\right)}$$

$$=-\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{ae\left(af^2+c\left(e^2-2df\right)\right)+cd\left(a\right.}{ad^2\left(ace^2+(cd-af)^2\right)}$$

Mathematica [C] time = 4.9239, size = 557, normalized size = 0.9

$$-\frac{\frac{d\left(a+2cx^2\right)}{a^2x\sqrt{a+cx^2}}-\frac{f\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(e-\sqrt{e^2-4df}\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(e-\sqrt{e^2-4df}\right)^2\right)}-\frac{f\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)}{a\sqrt{a+cx^2}\left(4af^2+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{\sqrt{2}f^3\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{2af+cx\left(\sqrt{e^2-4}\right.}{\sqrt{a+cx^2}\sqrt{4af^2-2c\left(e\sqrt{e^2}\right.}}\right.}{\sqrt{e^2-4df}\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)^{3/2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((-((f*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e - Sqrt[e^2 - 4*
d*f])*x))/(a*(4*a*f^2 + c*(e - Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2])) - (f
```

```
*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*a*f + c*(e + Sqrt[e^2 - 4*d*f])*
x))/(a*(4*a*f^2 + c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + c*x^2]) + (d*(a + 2
*c*x^2))/(a^2*x*Sqrt[a + c*x^2]) + (Sqrt[2]*f^3*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f])*ArcTanh[(2*a*f + c*(-e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[4*a*f^2 - 2*c
*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f
]*(2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))^(3/2)) + (Sqrt[2]*f^3*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[4*a*f^2 + 2*c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*
x^2])]])/(Sqrt[e^2 - 4*d*f]*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
)^(3/2)) + (e*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/a])/(a*Sqrt[a + c
*x^2]))/d^2)
```

Maple [B] time = 0.273, size = 2046, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 8*f^4/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*f
+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-16*f^3/(-e+(-4*d*f+e^2)^(1/2))^2*c^2/(
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2
-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x+16*f^3/(-e+(-4*d*f+e^2)^(1/2))^2/(-
4*d*f+e^2)^(1/2)*c^2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c
-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2
*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*x*e-8*f^4/(-e+(
-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+
e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2
*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f))-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/
2))^2/a/(c*x^2+a)^(1/2)+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^
(1/2))^2/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-8*f^4/(e+(-4*d*f+e^2
```

```
)^(1/2))^2/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2*c^2/((-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2)/((
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*x-16*f^3/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*c^2/((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4
*d*f+e^2)*c^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2
))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2)*x*e+8*f^4/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1
/2)/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*2^(1/2)/(((-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/
2))/(e+(-4*d*f+e^2)^(1/2))/a/x/(c*x^2+a)^(1/2)+8*f/(-e+(-4*d*f+e^2)^(1/2))/
(e+(-4*d*f+e^2)^(1/2))*c/a^2*x/(c*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}\left(fx^2+ex+d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.77 $\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=392

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}-\frac{d\sqrt{a\ldots}}{}$$

```
[Out] -((d*Sqrt[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8
*c^2*f) - (a + b*x + c*x^2)^(3/2)/(3*c*f) - (b*d*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - (b*(b^2 - 4*a*c)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*f) - (d*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])])/(2*f^(5/2)) + (d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rubi [A] time = 0.952051, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6725, 640, 612, 621, 206, 1021, 1078, 1033, 724}

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}-\frac{d\sqrt{a\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((d*Sqrt[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8
*c^2*f) - (a + b*x + c*x^2)^(3/2)/(3*c*f) - (b*d*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - (b*(b^2 - 4*a*c)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*f) - (d*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])])/(2*f^(5/2)) + (d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
```

```
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \left(-\frac{x\sqrt{a+bx+cx^2}}{f} + \frac{dx\sqrt{a+bx+cx^2}}{f\left(d-fx^2\right)}\right)dx$$

$$= -\frac{\int x\sqrt{a+bx+cx^2}\,dx}{f} + \frac{d\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{f}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} + \frac{d\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^2} + \frac{b\int \sqrt{a+bx+cx^2}\,dx}{2cf}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} - \frac{d\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^3} - \frac{(}{}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} - \frac{(bd)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \right.}{f^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} - \frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} -$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{\left(a+bx+cx^2\right)^{3/2}}{3cf} - \frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} -$$

Mathematica [A] time = 1.02982, size = 327, normalized size = 0.83

$$\frac{-\frac{2\sqrt{f}\sqrt{a+x(b+cx)}\left(2cf(4a+bx)-3b^2f+8c^2\left(3d+fx^2\right)\right)}{c^2} - \frac{3b\sqrt{f}\left(-4acf+b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}} + 24d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a}{2\sqrt{a}}\right.}{48f^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] ((-2*Sqrt[f]*Sqrt[a + x*(b + c*x)]*(-3*b^2*f + 2*c*f*(4*a + b*x) + 8*c^2*(3
*d + f*x^2)))/c^2 - (3*b*Sqrt[f]*(8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2) + 24*d*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt
[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 24*
d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x
 + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
```

```
x*(b + c*x)])])/(48*f^(5/2))
```

Maple [B] time = 0.266, size = 1817, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/c/f+1/4/f*b/c*x*(c*x^2+b*x+a)^(1/2)+1/8/f*b^2/c^2*
(c*x^2+b*x+a)^(1/2)+1/4/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*a-1/16/f*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2/f
^2*d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)+1/2/f^3*d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f
)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*(d*f)^
(1/2)-1/4/f^2*d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/
2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b-1/2/f^3*d/(1/f*(-b*(d*f)^(1/2)+a
*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)
^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c
*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)+1/2/f^2*d/(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f
)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f
+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/f^3*d^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d
))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(
1/2))/(x+(d*f)^(1/2)/f))*c-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)-1/2/f^3*d*ln((1
/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1
/2))*c^(1/2)*(d*f)^(1/2)-1/4/f^2*d*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)
^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b+1/2/f^3*d/((b*(d*f)^
(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*
f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2))/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)+1/2/f^2*d/((b*(d*f)^(1/2)+a*f+c*d)
```

```
/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*
f)^(1/2)/f))*a+1/2/f^3*d^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)
^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/
2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3\sqrt{a + bx + cx^2}}{-d + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.78 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=316

$$-\frac{\left(4acf+b^2(-f)+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2}+\frac{\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2}+\ldots$$

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c*f) - ((8*c^2*d - b^2*f + 4*a*c*f)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*f^2) +
(Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[
f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sq
rt[a + b*x + c*x^2])]])/(2*f^2) + (Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[
c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*f^2)
```

Rubi [A] time = 0.488628, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1071, 1078, 621, 206, 1033, 724}

$$-\frac{\left(4acf+b^2(-f)+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2}+\frac{\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```


```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c*f) - ((8*c^2*d - b^2*f + 4*a*c*f)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*f^2) +
(Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[
f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sq
rt[a + b*x + c*x^2])]])/(2*f^2) + (Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[
c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*f^2)
```


Rule 1071

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*
```

```
(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*
f)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*
(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\int \frac{-\frac{1}{4}(b^2+4ac)df-2bcdfx-\frac{1}{4}f(8c^2d-b^2f+4acf)x^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{2cf^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} + \frac{\int \frac{\frac{1}{4}(b^2+4ac)df^2+\frac{1}{4}df(8c^2d-b^2f+4acf)+2bcdf^2x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{2cf^3} - \frac{(8c^2d-b^2f+4acf)\int}{8cf^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{(-\sqrt{d}\sqrt{f}-fx)\sqrt{a+bx+cx^2}}\,dx}{2f^{3/2}} + \frac{\left(\sqrt{d}\left(cd+b\right.\right.}{}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(8c^2d-b^2f+4acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\left(\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+\right.\right.}{}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(8c^2d-b^2f+4acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+}}{}$$

Mathematica [A] time = 0.522703, size = 302, normalized size = 0.96

$$\frac{\left(-4acf+b^2f-8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\left(-2c\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-2c\right.}{8c^{3/2}f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] ((-8*c^2*d + b^2*f - 4*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)])] - 2*Sqrt[c]*(f*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 2*c*Sqrt[d]*Sq
rt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sq
rt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b
 + c*x)])] - 2*c*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*
Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]))/(8*c^(3/2)*f^2)
```

Maple [B] time = 0.263, size = 1810, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*x*(c*x^2+b*x+a)^(1/2)-1/4/f/c*(c*x^2+b*x+a)^(1/2)*b-1/2/f/c^(1/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/8/f/c^(3/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/2*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(
1/2)-1/2*d/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2
)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)+1/4*d/(d*f)^(1/2)/f*ln((1/2/f*(-2*c
*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)
)/c^(1/2)*b+1/2*d/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f
)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d
*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*
b-1/2*d/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f
)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d
*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*
a-1/2*d^2/(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*
(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-
b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/
f))*c-1/2*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)-1/2*d/f^2*ln((1/2*(2*c*(d*
f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2
)-1/4*d/(d*f)^(1/2)/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/
c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b
*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b+1/2*d/f^2/((b*(d*f)^(1/2)+a*f+c*d
)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(
d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d
*f)^(1/2)/f))*b+1/2*d/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2
*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/
f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+
1/2*d^2/(d*f)^(1/2)/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1
```

```
/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+
a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.79 $\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=282

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/f) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(2*Sqrt[c]*f) - (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)) + (Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*f^(3/2))
```

Rubi [A] time = 0.294989, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1021, 1078, 621, 206, 1033, 724}

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```


```
[Out] -(Sqrt[a + b*x + c*x^2]/f) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(2*Sqrt[c]*f) - (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)) + (Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*f^(3/2))
```


Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
```

```
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{\sqrt{a+bx+cx^2}}{f} + \frac{\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f^2} - \frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+c}}}{2f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\right.}{f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right.}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+}}\right.}{2f^{3/2}}$$

Mathematica [A] time = 0.333127, size = 272, normalized size = 0.96

$$\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right) - \sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right) -}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[f]*Sqrt[a + x*(b + c*x)] - (b*Sqrt[f]*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*A
rcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d
+ b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - Sqrt[c*d - b*Sqrt[d]*S
qrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*
x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*f^(3
/2))
```

Maple [B] time = 0.258, size = 1667, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)+1/2/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f
)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*(d*f)^
(1/2)-1/4/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b-1/2/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d
))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(
1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f
*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/
2))/(x+(d*f)^(1/2)/f))*a+1/2/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2
/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*
(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)
^(1/2)/f))*c*d-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)-1/2/f^2*ln((1/2*(2*c*(d*f)^(1/2
)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*(d*f)
^(1/2)-1/4/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+(
(x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1
/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b+1/2/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*
ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2
*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+
b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f
))*b*(d*f)^(1/2)+1/2/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/
2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a
*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/f^2/((b*(
d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(
1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*
d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.80 $\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f}-$$

```
[Out] -((Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f) + (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*Sqrt[d]*f) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f)
```

Rubi [A] time = 0.226448, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {990, 621, 206, 1033, 724}

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f) + (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*Sqrt[d]*f) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f)
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
```

```
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \frac{\int \frac{cd+af+bfx}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f} - \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= -\frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{1}{2}\left(b-\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx + $$

$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \left(-b-\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\mathrm{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, -b\right.$$

$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f} + \frac{\sqrt{cd+b\sqrt{}}}{}$$

Mathematica [A] time = 0.18554, size = 253, normalized size = 0.95

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)+\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)-}{2\sqrt{d}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[d]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
+ Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*
c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
x*(b + c*x)])] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[d]*f)
```

Maple [B] time = 0.255, size = 1669, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)-1/2/f*ln((1/2/f*(-2*c*(d*f)^(1
/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f
)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)
+1/4/(d*f)^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1
/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+
c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+
(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d)
)^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d)
)^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f
)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1
/2))/(x+(d*f)^(1/2)/f))*a-1/2/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^
(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2
))/(x+(d*f)^(1/2)/f))*c*d-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)-1/2/f*ln((
1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)
^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(
1/2))*c^(1/2)-1/4/(d*f)^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2
)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b+1/2/f/((b*(d*f)^(1/2)+a*f
+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2
*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(
x-(d*f)^(1/2)/f))*b+1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2
*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/
f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+
1/2/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*
f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*
d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a + bx + cx^2}}{-d + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.81 $\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=267

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{f}}$$

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*d*Sqrt[f]) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f])
```

Rubi [A] time = 0.778078, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6725, 734, 843, 621, 206, 724, 1021, 1078, 1033}

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*d*Sqrt[f]) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
```

```
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{fx\sqrt{a+bx+cx^2}}{d\left(-d+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d} - \frac{f\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx}{d}$$

$$= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{\int \frac{-\frac{bd}{2}-(cd+af)x-\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\,dx}{d}$$

$$= \frac{a\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{\int \frac{-bdf+f(-cd-af)x}{\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\,dx}{df}$$

$$= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d} - \frac{(cd+b\cdots}{\cdots}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}+\cdots}{\cdots}\right.}{d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{f}} + \frac{\sqrt{cd+b\cdots}}{\cdots}$$

Mathematica [A] time = 0.312997, size = 255, normalized size = 0.96

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)\cdots}{2d\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] -(2*Sqrt[a]*Sqrt[f]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]
+ Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*
c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
x*(b + c*x)])] - Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f]*Sqrt[a + x*(b + c*x)])])/(2*d*Sqrt[f])
```

Maple [B] time = 0.253, size = 1764, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x)
```

```
[Out] 1/d*(c*x^2+b*x+a)^(1/2)+1/2/d*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
/c^(1/2)-1/d*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-1/2/d*((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(
d*f)^(1/2)+a*f+c*d))^(1/2)+1/2/d/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f
)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+
(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4
/d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^
(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/
2)+a*f+c*d))^(1/2))/c^(1/2)*b-1/2/d/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*
ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+
(d*f)^(1/2)/f))*b*(d*f)^(1/2)+1/2/d/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln
((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d
*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d
```

```
*f)^(1/2)/f))*a+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f
)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d
*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*
c-1/2/d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b
*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)-1/2/d/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(
d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-
(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)-1/4/d*
ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2
)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/
f)^(1/2))/c^(1/2)*b+1/2/d/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f
)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1
/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*(d*f)^(1
/2)+1/2/d/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1
/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/2/f/((b*(d*f)^(1/2)+a*f+
c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x
-(d*f)^(1/2)/f))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 - d\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x), x)
```

Fricas [B] time = 116.964, size = 2695, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log((2*sqrt(c*x
^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d +
 a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x)
- d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*x^2
+ b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*
f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d
*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b
*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)
/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + d*s
qrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*
x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/
(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + 2*sq
rt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2))/d, 1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a
*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2
*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a
*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/
(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*
sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*
f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d
^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sq
rt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)
*sqrt(b^2/(d^3*f)))/x) + d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2
*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqr
t(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*
sqrt(b^2/(d^3*f)))/x) + 4*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x +
2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a + bx + cx^2}}{-dx + fx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.82 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx$

Optimal. Leaf size=286

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}}-$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTa
nh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)) + (Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d^(3/2))
```

Rubi [A] time = 0.705713, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6725, 732, 843, 621, 206, 724, 990, 1033}

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTa
nh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)) + (Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d^(3/2))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
```

```
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d-fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2}+\frac{f\sqrt{a+bx+cx^2}}{d\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx}{d}+\frac{f\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{\int \frac{cd+af+bfx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d}-\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d}-\frac{(2c)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d}-$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d}+\frac{(2c)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}+\frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+c}}\right.}{2d^{3/2}}$$

Mathematica [A] time = 0.455615, size = 275, normalized size = 0.96

$$\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] ((-2*Sqrt[d]*Sqrt[a + x*(b + c*x)])/x - (b*Sqrt[d]*ArcTanh[(2*a + b*x)/(2*S
qrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[
c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt
[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*
d^(3/2))
```

Maple [B] time = 0.293, size = 1819, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x)
```

```
[Out] 1/2*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d
*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)-1/2/d*ln((1/2/f*(-2*c*(d*f
)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(
1/2)+1/4*f/d/(d*f)^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)
*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b+1/2/d/(1/f*(-b*(d*f)^(1
/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/
2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b-1/2*f/d/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1
/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/
2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+
a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f
)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+
c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c-1/d/a/x*(c*x^2+b*x+a)^(3/2)+1/d*b/a*(c*x^
2+b*x+a)^(1/2)-1/2/d*b/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
)+1/d*c/a*(c*x^2+b*x+a)^(1/2)*x+1/d*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))-1/2*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*
f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)-1/2/d*ln((1/2*(2*c*
(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(
1/2)-1/4*f/d/(d*f)^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*
```

```
c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)
+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b+1/2/d/((b*(d*f)^(1/2)+a*f+c*d)
/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*
f)^(1/2)/f))*b+1/2*f/d/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*
(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(
d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1
/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c
*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/
f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(
b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{cx^2+bx+a}}{\left(fx^2-d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2), x)
```

Fricas [B] time = 141.882, size = 2422, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*sq
rt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 +
 (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5)
+ c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt
(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/
x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*sq
```

```
rt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2
- (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5
) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*s
qrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5
))/x) + sqrt(a)*b*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
+ a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*sqrt(c*x^2 + b*x + a)*a)/(a*d*x)
, 1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*s
qrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2
+ (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5)
 + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqr
t(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))
/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*s
qrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2
 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^
5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*
sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^
5))/x) + 2*sqrt(-a)*b*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-
a)/(a*c*x^2 + a*b*x + a^2)) - 4*sqrt(c*x^2 + b*x + a)*a)/(a*d*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.83 $\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=353

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\cdots\right.}{2\sqrt{a+bx+cx^2}\sqrt{af+b(\cdots}}\right.}{2d^2}$$

```
[Out] -((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2 - 4*a*c)*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2)*d) - (Sqrt[a]*f*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 - (Sqrt[f]*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*d^2) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)
```

Rubi [A] time = 0.878815, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6725, 720, 724, 206, 734, 843, 621, 1021, 1078, 1033}

$$\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\cdots\right.}{2\sqrt{a+bx+cx^2}\sqrt{af+b(\cdots}}\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] -((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(4*a*d*x^2) + ((b^2 - 4*a*c)*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2)*d) - (Sqrt[a]*f*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 - (Sqrt[f]*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*d^2) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^3} + \frac{f\sqrt{a+bx+cx^2}}{d^2x} + \frac{f^2x\sqrt{a+bx+cx^2}}{d^2\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx}{d} + \frac{f\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d^2} + \frac{f^2\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{d^2}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} - \frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{8ad} - \frac{f\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d^2} + \frac{f\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}}{\sqrt{a+bx+cx^2}\left(d-f\right.}}{d^2}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} - \frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d^2} + \frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{4ad}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{(2af)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2}{\sqrt{a+}}\right.}{d^2}$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} -$$

$$= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} -$$

Mathematica [A] time = 0.549134, size = 316, normalized size = 0.9

$$\frac{x^2\left(b^2d-4a(2af+cd)\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right) - 2\sqrt{a}\left(-2a\sqrt{f}x^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right.\right.}{8a^{3/2}d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] ((b^2*d - 4*a*(c*d + 2*a*f))*x^2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*
(b + c*x)])] - 2*Sqrt[a]*(d*(2*a + b*x)*Sqrt[a + x*(b + c*x)] - 2*a*Sqrt[f]
*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*x^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] +
 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
 + x*(b + c*x)])] + 2*a*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*x^2*Arc
Tanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d -
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]))/(8*a^(3/2)*d^2*x^2)
```

Maple [B] time = 0.299, size = 1953, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x)
```

```
[Out] f/d^2*(c*x^2+b*x+a)^(1/2)+1/2*f/d^2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))/c^(1/2)-f/d^2*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-
1/2*f/d^2*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)+1/2/d^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+
b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1
/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*(d*
f)^(1/2)-1/4*f/d^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^
(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1
/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b-1/2/d^2/(1/f*(-b*(d*f)^(1/2)+
a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f
)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+
c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)+1/2*f/d^2/(1/f*(-b*(d*f)^(1/2
)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*
f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)
/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*
f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a+1/2/d/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1
/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-
2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))
/(x+(d*f)^(1/2)/f))*c-1/2/d/a/x^2*(c*x^2+b*x+a)^(3/2)+1/4/d*b/a^2/x*(c*x^2+
b*x+a)^(3/2)-1/4/d*b^2/a^2*(c*x^2+b*x+a)^(1/2)+1/8/d*b^2/a^(3/2)*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-1/4/d*b/a^2*c*(c*x^2+b*x+a)^(1/2)*x+1/2
/d*c/a*(c*x^2+b*x+a)^(1/2)-1/2/d*c/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x
+a)^(1/2))/x)-1/2*f/d^2*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)-1/2/d^2*ln((1/2*(2*c*(d*f)^(
1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*(d
*f)^(1/2)-1/4*f/d^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^
(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(
d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b+1/2/d^2/((b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^
(1/2)/f))*b*(d*f)^(1/2)+1/2*f/d^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(
b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d
```

```
*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*
(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a+1/
2/d/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((
x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/
2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{cx^2+bx+a}}{\left(fx^2-d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**3/(-f*x**2+d),x)
```

```
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.84 $\int \frac{x^3\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

Optimal. Leaf size=501

$$-\frac{d\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^3}-\frac{bd\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3}-\frac{3b\left(b^2-4ac\right)(}{}$$

```
[Out] (-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*f) - (d*(8*
c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^3) - (d*
(a + b*x + c*x^2)^(3/2))/(3*f^2) + (b*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/
(16*c^2*f) - (a + b*x + c*x^2)^(5/2)/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*f) - (b*d*(24
*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(16*c^(3/2)*f^3) - (d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh
[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sq
rt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(7/2)) + (d*(c*d + b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] +
 b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]
)])/(2*f^(7/2))
```

Rubi [A] time = 1.41215, antiderivative size = 501, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6725, 640, 612, 621, 206, 1021, 1070, 1078, 1033, 724}

$$-\frac{d\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^3}-\frac{bd\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3}-\frac{3b\left(b^2-4ac\right)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*f) - (d*(8*
c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^3) - (d*
(a + b*x + c*x^2)^(3/2))/(3*f^2) + (b*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/
(16*c^2*f) - (a + b*x + c*x^2)^(5/2)/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*f) - (b*d*(24
*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(16*c^(3/2)*f^3) - (d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh
```

```
[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sq
rt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(7/2)) + (d*(c*d + b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] +
 b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]
)])/(2*f^(7/2))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
```

```
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3 \left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = \int \left(-\frac{x\left(a+bx+cx^2\right)^{3/2}}{f} + \frac{dx\left(a+bx+cx^2\right)^{3/2}}{f\left(d-fx^2\right)}\right)dx$$

$$= -\frac{\int x\left(a+bx+cx^2\right)^{3/2}\,dx}{f} + \frac{d\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx}{f}$$

$$= -\frac{d\left(a+bx+cx^2\right)^{3/2}}{3f^2} - \frac{\left(a+bx+cx^2\right)^{5/2}}{5cf} + \frac{d\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3f^2} + \frac{b\int \left(a+b\right.}{}$$

$$= -\frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^3} - \frac{d\left(a+bx+cx^2\right)^{3/2}}{3f^2} + \frac{b(b+2cx)\left(a+bx+\right.}{16c^2f}$$

$$= -\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^3} - \frac{d}{}$$

$$= -\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^3} - \frac{d}{}$$

$$= -\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^3} - \frac{d}{}$$

$$= -\frac{3b\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^3} - \frac{d}{}$$

Mathematica [A] time = 1.56267, size = 447, normalized size = 0.89

$$-\frac{\sqrt{f}\sqrt{a+x(b+cx)}\left(24c^2f\left(16a^2f+7abfx+b^2\left(10d+fx^2\right)\right)-30b^2cf^2(10a+bx)+16c^3f\left(160ad+48afx^2+70bdx+33bfx^3\right)+45b^4f^2+128c^4\left(15d^2+5dfx^2+3f^2x^4\right)\right)}{c^3} + 96$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (b*(-384*c^4*d^2 - 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 16*c^2*f*(b
^2*d + 3*a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2
56*c^(7/2)*f^3) + (-((Sqrt[f]*Sqrt[a + x*(b + c*x)]*(45*b^4*f^2 - 30*b^2*c*
```

```
f^2*(10*a + b*x) + 16*c^3*f*(160*a*d + 70*b*d*x + 48*a*f*x^2 + 33*b*f*x^3)
+ 128*c^4*(15*d^2 + 5*d*f*x^2 + 3*f^2*x^4) + 24*c^2*f*(16*a^2*f + 7*a*b*f*x
 + b^2*(10*d + f*x^2))))/c^3) + 960*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt
[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - 960*d*(c*d + b*Sq
rt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[
d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x
)])])/(1920*f^(7/2))
```

Maple [B] time = 0.273, size = 4884, normalized size = 9.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/f^3*d/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c
*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f
+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/
2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)
*a-1/f^4*d^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a
*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(
1/2)*c+1/8/f*b/c*x*(c*x^2+b*x+a)^(3/2)-3/64/f*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x
-1/2/f^3*d^2*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/
2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*c+1/16/f*b^2/c^2*(c*x^2+b*x+a)^(3
/2)+1/2/f^4*d^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/
2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f^3*d^2/(1/f*(-b*(
d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d
*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b^2+1/2/f^2*d/(1/f*(-b*(d*f)^(1/
2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a
*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a^2-3/4/f^3*d^2*ln((1/2*(2*c*(d*f)^(1/2)
+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*b-1/8/
f^2*d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(
```

d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b-5/8/f^3*d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b*(d*f)^(
1/2)-1/16/f^2*d/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2+1/32/f^2*d/c^(3/2)*ln((1/2*(2*
c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2
c(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*b
^3+3/32/f*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a+3/16/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))*a^2-3/32/f*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*a-1/2/f^4*d^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1
/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f^3*d^2/(
(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*
f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2+1/2/f^2*d/((b*(d*f)^(1/2)+a*f+c*d)/
f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f
)^(1/2)/f))*a^2+1/2/f^4*d^3/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f
)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1
/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2-1/8/f^
2*d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
(-b(d*f)^(1/2)+a*f+c*d))^(1/2)*x*b+5/8/f^3*d*((x+(d*f)^(1/2)/f)^2*c+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)
b(d*f)^(1/2)-1/16/f^2*d/c*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*
f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2+1/2/f^4*d^3/(1
/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2
c(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2
)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c^2+1/32/f^2*d/c^(3/2)*l
n((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a
*f+c*d))^(1/2))*b^3-3/4/f^3*d^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(
1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*b-1/6/f^2*d*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a
*f+c*d)/f)^(3/2)-1/6/f^2*d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f
)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(3/2)-3/128/f*b^4/c^3*(c*
x^2+b*x+a)^(1/2)+3/256/f*b^5/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))-1/2/f^3*d^2*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*c+1/f^3*d/((b*(d*f)^(1/2)+a*f+c*d)
/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*

```
f)^(1/2)/f))*b*(d*f)^(1/2)*a+1/f^4*d^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln
((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*(
(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*
f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
*b*(d*f)^(1/2)*c-1/2/f^2*d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*a-1/2/f^2*d*((x+(d*f)^(1/
2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
a*f+c*d))^(1/2)*a-1/5*(c*x^2+b*x+a)^(5/2)/c/f+1/4/f^3*d*((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*
d))^(1/2)*x*c*(d*f)^(1/2)+3/4/f^3*d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*
f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*(d*f)^(1/2)*a-
3/8/f^2*d/c^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(
1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*a*b+3/16/f^3*d*ln((1/2/f*(-2*c*(d*f)^(1/
2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*
b^2*(d*f)^(1/2)+3/16/f*b/c*(c*x^2+b*x+a)^(1/2)*x*a+1/f^3*d^2/(1/f*(-b*(d*f)
^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^
(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/
2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c-3/4/f^3*d*ln((1/2*(2*c*(d*f)^(1/
2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*(d*f
)^(1/2)*a-3/8/f^2*d/c^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/
f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*a*b-3/16/f^3*d*ln((1/2*(2*c*(d*f)^(1/
2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b^2*
(d*f)^(1/2)+1/f^3*d^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2
)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*
f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c-1/4/f^3*d*((
x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/
2)+a*f+c*d)/f)^(1/2)*x*c*(d*f)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx^4\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^5\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**4
*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**5*sqrt(a + b*x +
c*x**2)/(-d + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.85 $\int \frac{x^2\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

Optimal. Leaf size=417

$$-\frac{\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(12acf-\right.\right.}{}$$

```
[Out] -((b*(80*c^2*d - 3*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f + 12*a*c*f)*
x)*Sqrt[a + b*x + c*x^2])/(64*c^2*f^2) - ((b + 2*c*x)*(a + b*x + c*x^2)^(3/
2))/(8*c*f) - ((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 +
48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/(128*c^(5/2)*f^3) + (Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3) + (Sqrt[d]*(c*d
 + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*S
qrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*f^3)
```

Rubi [A] time = 1.0152, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1071, 1070, 1078, 621, 206, 1033, 724}

$$-\frac{\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3}-\frac{\sqrt{a+bx+cx^2}\left(2cx\left(12acf-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -((b*(80*c^2*d - 3*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f + 12*a*c*f)*
x)*Sqrt[a + b*x + c*x^2])/(64*c^2*f^2) - ((b + 2*c*x)*(a + b*x + c*x^2)^(3/
2))/(8*c*f) - ((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 +
48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/(128*c^(5/2)*f^3) + (Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3) + (Sqrt[d]*(c*d
 + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*S
qrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
```

```
+ c*x^2])])/(2*f^3)
```

Rule 1071

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*
(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*
f)*(C*(-(b*f))*(q + 1)) + (p + q + 1)*(-(b*c*(C*(-4*d*f)*(2*p + q + 2) + f*
(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*(-(b*f))*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
```

```
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = -\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(-\frac{3}{4}\left(3b^2+4ac\right)df-12bcdfx-\frac{3}{4}f\left(16c^2d-3\left(b^2-4ac\right)f\right)x^2\right)}{d-fx^2}\,dx}{12cf^2}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)\left(a+\right.}{8c}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)\left(a+\right.}{8c}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)\left(a+\right.}{8c}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)\left(a+\right.}{8c}$$

$$= -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)\left(a+\right.}{8c}$$

Mathematica [A] time = 1.08196, size = 395, normalized size = 0.95

$$-\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\left(f\sqrt{a+x(b+cx)}\left(4bc\left(5\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^
2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) - 2*S
qrt[c]*(f*Sqrt[a + x*(b + c*x)]*(-3*b^3*f + 2*b^2*c*f*x + 8*c^2*x*(4*c*d +
5*a*f + 2*c*f*x^2) + 4*b*c*(20*c*d + 5*a*f + 6*c*f*x^2)) + 32*c^2*Sqrt[d]*(
c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] -
2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
+ x*(b + c*x)])] + 32*c^2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Arc
Tanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])))/(128*c^(5/2)*f^3)
```

Maple [B] time = 0.269, size = 4900, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -3/16*d/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((
x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/
2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b^2-3/4*d/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f
)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*a+3/32
/f/c*(c*x^2+b*x+a)^(1/2)*x*b^2-3/16/f/c*(c*x^2+b*x+a)^(1/2)*b*a+3/16/f/c^(3
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-1/2*d/(d*f)^(1/2)/f*(
(x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1
/2)+a*f+c*d)/f)^(1/2)*a-1/2*d^2/(d*f)^(1/2)/f^2*((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*c-3/
16*d/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+
(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b^2+1/2*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f
)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+
c*d))^(1/2)*a+1/2*d^2/(d*f)^(1/2)/f^2*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f
)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*c-1/4*d/
f^2*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c-d^2/(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/
2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a
*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c+3/8*d/(d*f)^(1/2)/f/c^(1/2)*ln((1/2/
f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d)
)^(1/2))*a*b-3/8*d/(d*f)^(1/2)/f/c^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x
-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*a*b+d^2/(d*f)^(1/2)/f^2/
((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a
*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c-1/4*d/f^2*((x-(d*f)^(1/2)/f)^2*c+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x
*c-3/4*d/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+(
(x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1
/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*a-1/8/f/c*(c*x^2+b*x+a)^(3/2)*b-3/8/f*(c*x^2
```

+b*x+a)^(1/2)*x*a+3/64/f/c^2*(c*x^2+b*x+a)^(1/2)*b^3-3/8/f/c^(1/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-3/128/f/c^(5/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*b^4+1/6*d/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+1/f
*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(3/
2)-5/8*d/f^2*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/
2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b-1/2*d^2/f^3*ln((1/2/f*(-2*c*(d*
f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^
(3/2)-1/6*d/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(3/2)-5/8*d/f^2*((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2)*b-1/2*d^2/f^3*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c
^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(3/2)-1/4/f*x*(c*x^2+b*x+a)^(3/2)+d^2/f^3/
((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a
*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*c+1/2*d/(d*f)^(1/2)/f/((b*(d*f)^(1/2
)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2
*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/
2))/(x-(d*f)^(1/2)/f))*a^2+1/2*d^3/(d*f)^(1/2)/f^3/((b*(d*f)^(1/2)+a*f+c*d)
/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*
f)^(1/2)/f))*c^2+1/32*d/(d*f)^(1/2)/f/c^(3/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)
/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*b^3-3/4*d^2/(d*f)^(
1/2)/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(
d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
a*f+c*d)/f)^(1/2))*c^(1/2)*b+1/2*d^2/(d*f)^(1/2)/f^2/((b*(d*f)^(1/2)+a*f+c*
d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(
d*f)^(1/2)/f))*b^2+d/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(
1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)
+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*a+1/16*d/(d
f)^(1/2)/f/c((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2+d^2/f^3/(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f
)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f
+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*c-1/2*d/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/
2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b

```
*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a
*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a^2-1/2*d^2/(d*f)^(1/2)/f^2/(1/f*(-b*(d*
f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(
1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f
)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(
1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b^2-1/32*d/(d*f)^(1/2)/f/c^(3/2)*l
n((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a
*f+c*d))^(1/2))*b^3+3/4*d^2/(d*f)^(1/2)/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f
)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)
+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*b-1/2*
d^3/(d*f)^(1/2)/f^3/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^
(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f
)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(
x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c^
2+d/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*
d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+
c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*a+1/8*d/(d*f
)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*b-1/8*d/(d*f)^(1/2)/f*((x-(d*f)^(1
/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d
)/f)^(1/2)*x*b-1/16*d/(d*f)^(1/2)/f/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{ax^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^4\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**3
*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**4*sqrt(a + b*x +
c*x**2)/(-d + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.86 $\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2} dx$

Optimal. Leaf size=349

$$-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{}\right.}{}$$

```
[Out] -((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2)
- (a + b*x + c*x^2)^(3/2)/(3*f) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^2) - ((c*d -
b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*f^(5/2)) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rubi [A] time = 0.521544, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1021, 1070, 1078, 621, 206, 1033, 724}

$$-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cf^2}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2)
- (a + b*x + c*x^2)^(3/2)/(3*f) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^2) - ((c*d -
b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*f^(5/2)) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = -\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}+\frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3f}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{\int \frac{-\frac{3}{8}bdf\left(8c^2d+b^2f+20acf\right) \cdots}{\cdots}}{\cdots}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}+\frac{\int \frac{\frac{3}{8}bdf^2\left(24c^2d-b^2f+12acf\right) \cdots}{\cdots}}{\cdots}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}+\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2 \int \cdots}{2 \cdots}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{b\left(24c^2d-b^2f+12ac \cdots\right.}{16c \cdots}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{b\left(24c^2d-b^2f+12ac \cdots\right.}{16c \cdots}$$

Mathematica [A] time = 0.874897, size = 330, normalized size = 0.95

$$\frac{b\left(-12acf+b^2f-24c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{16c^{3/2}f^2}-\frac{\sqrt{f}\sqrt{a+x(b+cx)}\left(2cf(16a+7bx)+3b^2f+8c^2\left(3d+fx^2\right)\right)-12c \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (b*(-24*c^2*d + b^2*f - 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])])/(16*c^(3/2)*f^2) - (Sqrt[f]*Sqrt[a + x*(b + c*x)]*(3*b^2*f +
 2*c*f*(16*a + 7*b*x) + 8*c^2*(3*d + f*x^2)) - 12*c*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt
[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + 12*
c*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*
x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
 + x*(b + c*x)])])/(24*c*f^(5/2))
```

Maple [B] time = 0.263, size = 4567, normalized size = 13.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -3/16/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-
(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)
+a*f+c*d)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)-3/4/f^2*ln((1/2*(2*c*(d*f)^(1/2
)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*d*b-1
/2/f^3*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*
f+c*d)/f)^(1/2))*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^
(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(
1/2)/f))*b^2*d+1/2/f^3/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/
2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a
*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2*d^2+1/4/f^2
*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-
b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c*(d*f)^(1/2)+3/4/f^2*ln((1/2/f*(-2*c*(d*f)
^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1
/2)*(d*f)^(1/2)*a-3/8/f/c^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(
1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
```

f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*a*b+1/2/f^3/(1/f*(-b*(d*f)
^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^
(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/
2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c^2*d^2+3/16/f^2*ln((1/2/f*(-2*c*(d*
f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^
(1/2)*b^2*(d*f)^(1/2)-3/4/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/
2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*d*b+1/2/f^3*ln((1/2/
f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*
c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d)
)^(1/2))*c^(3/2)*(d*f)^(1/2)*d+1/2/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)
ln((2/f(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x
+(d*f)^(1/2)/f))*b^2*d-3/8/f/c^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*
f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
f)^(1/2)/f)+(b(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*a*b-1/4/f^2*((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2)*x*c*(d*f)^(1/2)-3/4/f^2*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/
2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*(d*f)^(1/2)*a+1/f^3/((b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1
/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d
)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)*c*d-1/f^3/(1/f*(-b*(d*f)^(1/2)
+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f
)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f
+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)*c*d-1/8/f*((x+(d*f)^(1/2)/f)
^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c
*d))^(1/2)*x*b-1/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)
^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*
f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b
*(d*f)^(1/2)*a+1/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)
+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f
+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)*a+1
/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*(
(x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1
/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c*d+1/f^2/(1/f*(-b*(d*f)^(1/2)+
a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)

```
*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f
)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+
c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c*d-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*a-1/2/f*
((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b
*(d*f)^(1/2)+a*f+c*d))^(1/2)*a-1/8/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b+1/2/f/(1/f*
(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a^2+5/8/f^2*((x+(d*f)^(1/2)
/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*
f+c*d))^(1/2)*b*(d*f)^(1/2)-1/16/f/c*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2+1/32/
f/c^(3/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x
+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d
*f)^(1/2)+a*f+c*d))^(1/2))*b^3-1/2/f^2*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*c*d-5/8
/f^2*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2)*b*(d*f)^(1/2)-1/16/f/c*((x-(d*f)^(1/2)/f)^2*c+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b
^2+1/32/f/c^(3/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1
/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+a*f+c*d)/f)^(1/2))*b^3-1/2/f^2*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*c*d+1/2/f/((
b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f
)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f
+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2-1/6/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(3/2)-1/6/f*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+a*f+c*d))^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{ax\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int\frac{bx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int\frac{cx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**2*sq
rt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**3*sqrt(a + b*x + c*x
**2)/(-d + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.87 $\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

Optimal. Leaf size=315

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}+\frac{(af}{}$$

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*f) - ((8*c^2*d + 3*b^2*f + 12*a*c
*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*f^2)
 + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f]
+ (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + b*x + c*x^2])])/(2*Sqrt[d]*f^2) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2
)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f^2)
```

Rubi [A] time = 0.517015, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {978, 1078, 621, 206, 1033, 724}

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2}+\frac{(af}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```


```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*f) - ((8*c^2*d + 3*b^2*f + 12*a*c
*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*f^2)
 + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f]
+ (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + b*x + c*x^2])])/(2*Sqrt[d]*f^2) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2
)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f^2)
```


Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
```

```
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} + \frac{\int \frac{\frac{1}{4}\left(5b^2d+4a(cd+2af)\right)+4b(cd+af)x+\frac{1}{4}\left(8c^2d+3b^2f+12acf\right)x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2f}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\int \frac{-\frac{1}{4}d\left(8c^2d+3b^2f+12acf\right)-\frac{1}{4}f\left(5b^2d+4a(cd+2af)\right)-4bf(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{2f^2} - \frac{\left(8c^2d+3\right.}{}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2 \int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}f^{3/2}} + \frac{\left(cd+b\sqrt{d}\sqrt{f}+\right.}{}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\left(8c^2d+3b^2f+12acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+\right.}{}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\left(8c^2d+3b^2f+12acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+\right.}{}$$

Mathematica [A] time = 0.842412, size = 298, normalized size = 0.95

$$-\frac{\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + \frac{4\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{d}} + \frac{4\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\right.}{2\sqrt{a+x(b+cx}}\right.}{\sqrt{d}}}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] -(2*f*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] + ((8*c^2*d + 3*b^2*f + 12*a*c*f)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (4*(c*d
- b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*
Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*
(b + c*x)])])/Sqrt[d] + (4*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-
2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[d])/(8*f^2)
```

Maple [B] time = 0.263, size = 4574, normalized size = 14.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/6/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(3/2)-1/6/(d*f)^(1/2)*((x-(d*f)^(1/2
)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/
f)^(3/2)+3/4/(d*f)^(1/2)/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/
f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*d*b-1/32/(d*f)^(1/2)/c^
(3/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+a*f+c*d))^(1/2))*b^3-1/2/f^2*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f
)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+
(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(3/2)*d-1/2/(d*f)^(1/
2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a^2+1/16/(d*f)^(1/2
)/c*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2-3/4/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)
+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*a-3/16/
f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+a*f+c*d))^(1/2))/c^(1/2)*b^2+1/8/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)
*x*b-1/4/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c+1/(d*f)^(1/2)/f/((b*(d*f)^(1/2)
+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*
(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*
c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2
))/(x-(d*f)^(1/2)/f))*a*c*d-1/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^
(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2
))/(x+(d*f)^(1/2)/f))*a*c*d-3/4/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^
(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*a-3/16/f*ln((1/2*(2*c*(
d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*
```

```
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1
/2)*b^2+1/32/(d*f)^(1/2)/c^(3/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(
1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*b^3-1/2/f^2*ln((1/2*(2*c*(d*f)^(
1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(3/2)*d+
1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+
c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)
/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+3/8/(d*f)^(1/2)/c^
(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+a*f+c*d))^(1/2))*a*b+1/2/(d*f)^(1/2)/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2
*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*c
*d+1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*a-5/8/f*((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1
/2)*b-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*a-5/8/f*((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*b-1/4/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*c-1/8/(d*f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^
(1/2)*x*b-1/16/(d*f)^(1/2)/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2+1/f/(1/f*(-b*(d*f)^
(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*a-3/8/(d*f)^(1/2)/c^(1/2)*ln((1/2*(
2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+
(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))
*a*b-1/2/(d*f)^(1/2)/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*c*d+1/f/((b*(d*f)^(1/2)+a*f+c
*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-
(d*f)^(1/2)/f))*b*a-1/2/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*
ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+
(d*f)^(1/2)/f))*b^2*d+1/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-
b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*
(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2
)/f))*b*c*d-1/2/(d*f)^(1/2)/f^2/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/
```

```
f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(
1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^
(1/2)/f))*c^2*d^2+1/2/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2
*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/
f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^
2*d+1/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1
/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*c*d-3/4/(d*f)^(1/2)/f*ln((
1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)
^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(
1/2))*c^(1/2)*d*b+1/2/(d*f)^(1/2)/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln(
(2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((
b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*
c^2*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x*sqrt(a
+ b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/
(-d + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.88 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=469

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cdf}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{16c^{3/2}df}$$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d) - ((8*c^2*d + b^2*f
 + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d*f) - (a^(3/2)*ArcTanh
[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d - (b*(b^2 - 12*a*c)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d) - (b*(24
*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(16*c^(3/2)*d*f) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2)) + ((c*d + b*Sqrt[
d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d*f^(3/2))
```

Rubi [A] time = 1.27415, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {6725, 734, 814, 843, 621, 206, 724, 1021, 1070, 1078, 1033}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cdf}-\frac{b\left(12acf+b^2(-f)+24c^2d\right)\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{16c^{3/2}df}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d) - ((8*c^2*d + b^2*f
 + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d*f) - (a^(3/2)*ArcTanh
[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d - (b*(b^2 - 12*a*c)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d) - (b*(24
*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(16*c^(3/2)*d*f) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
```

```
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2)) + ((c*d + b*Sqrt[
d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d*f^(3/2))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x\left(d-fx^2\right)}\,dx = \int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx} - \frac{fx\left(a+bx+cx^2\right)^{3/2}}{d\left(-d+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx}{d} - \frac{f\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{-d+fx^2}\,dx}{d}$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}\left(-\frac{3bd}{2}-3(cd+af)x-\frac{3}{2}bfx^2\right)}{-d+fx^2}\,dx}{3d} - \frac{\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx}{2d}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} + \frac{\int \frac{8a^2c-\frac{1}{2}b(}{x\sqrt{a+}}}{8}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} + \frac{a^2\int \frac{}{x\sqrt{a+}}}{d}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} - \frac{\left(2a^2\right)\mathrm{Sub}}{}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} - \frac{a^{3/2}\tanh^{}}{}$$

$$= \frac{\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd} - \frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cdf} - \frac{a^{3/2}\tanh^{}}{}$$

Mathematica [A] time = 0.532428, size = 755, normalized size = 1.61

$$-\frac{2a^{3/2}f^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+2cd\sqrt{f}\sqrt{a+x(b+cx)}-af\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] -(2*c*d*Sqrt[f]*Sqrt[a + x*(b + c*x)] + 2*a^(3/2)*f^(3/2)*ArcTanh[(2*a + b*
x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 3*b*Sqrt[c]*d*Sqrt[f]*ArcTanh[(b +
2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - c*d*Sqrt[c*d - b*Sqrt[d]*Sqrt[f
] + a*f]*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)
```

```
/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + b*Sqrt[d]
*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sq
rt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]
*Sqrt[a + x*(b + c*x)])] - a*f*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[
(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + c*d*Sqrt[c*d + b*Sqrt[d]*S
qrt[f] + a*f]*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*
x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + b*Sqrt
[d]*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(-2*(a*Sqrt[f] + c*
Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + x*(b + c*x)])] + a*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh
[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*d*f^(3/2))
```

Maple [B] time = 0.278, size = 4765, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x)
```

```
[Out] 3/4/d*b/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+5/8/d/f*((x+(
d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f
)^(1/2)+a*f+c*d))^(1/2)*b*(d*f)^(1/2)+1/d/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/
2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2
)/f))*b*(d*f)^(1/2)*a-1/d/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-
b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*
(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2
)/f))*b*(d*f)^(1/2)*a-3/8/d/c^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*
f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*a*b+1/f/(1/f*(-b*(d*f)
^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^
(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/
2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c+1/2*d/f^2/(1/f*(-b*(d*f)^(1/2)+a
*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)
^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c
```

```
*d))^(1/2))/(x+(d*f)^(1/2)/f))*c^2-5/8/d/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b*(d*f)^(
1/2)-3/8/d/c^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(
1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2))*a*b+1/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((
2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b
*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a
*c+1/2*d/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d
)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2-1/6/d*((x-(d*f)^(1/2)
/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f
)^(3/2)-1/6/d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(3/2)+1/3/d*(c*x^2+b*x+a)^(3/2)+1/d*a*
(c*x^2+b*x+a)^(1/2)-1/d*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)-1/2/d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(
b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*a-1/2/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*c-1/2/d*((x+
(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+a*f+c*d))^(1/2)*a-1/2/f*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*c-1/2/f^2*ln(
(1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^
(1/2))*c^(3/2)*(d*f)^(1/2)+1/2/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b
*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*
f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2-1
/8/d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b-1/8/d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f
)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*b-1/16
/d/c*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2+1/4/d*b*(c*x^2+b*x+a)^(1/2)*x+1/8/d/c
*(c*x^2+b*x+a)^(1/2)*b^2-1/16/d/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))*b^3+1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(
1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)
^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b^2
+1/2/d/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d
)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c
*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a^2+1/32/d/c^(3
/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)
^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+a*f+c*d))^(1/2))*b^3-3/4/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1
```

```
/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f
)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*b+1/2/f^2*ln((1/2/f
*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2))*c^(3/2)*(d*f)^(1/2)-1/16/d/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2+1/32/d/c^(3
/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^
(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c
*d)/f)^(1/2))*b^3-3/4/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c
)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*b+1/2/d/((b*(d*f)^(1/2)+a*f+c*d)/
f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f
)^(1/2)/f))*a^2+3/16/d/f*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)
*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)-1/f^2/(1/
f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)
*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-
b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*(d*f)^(1/2)*c+1/4/d/f*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c*(d*f)^(1/2)+3/4/d/f*ln((1/2/f*(-2*c*(d*f)^(
1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2
)*(d*f)^(1/2)*a+1/f^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2
)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*
f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*(d*f)^(1/2)*c-
1/4/d/f*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b
*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*c*(d*f)^(1/2)-3/4/d/f*ln((1/2*(2*c*(d*f)^(
1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*(d
*f)^(1/2)*a-3/16/d/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c
^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/c^(1/2)*b^2*(d*f)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(fx^2 - d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(b*x*sqrt(
a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(c*x**2*sqrt(a + b*x + c*x*
*2)/(-d*x + f*x**3), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.89 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=463

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df}+\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh}{2d^3}$$

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*Sqrt[a + b*x
 + c*x^2])/(4*d) - (a + b*x + c*x^2)^(3/2)/(d*x) - (3*Sqrt[a]*b*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d) + (3*(b^2 + 4*a*c)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*d) - ((8*c^2*
d + 3*b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(8*Sqrt[c]*d*f) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f) + ((c*d + b*Sqrt[d]*S
qrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*d^(3/2)*f)
```

Rubi [A] time = 1.20114, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6725, 732, 814, 843, 621, 206, 724, 978, 1078, 1033}

$$-\frac{\left(12acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df}+\frac{3\left(4ac+b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*Sqrt[a + b*x
 + c*x^2])/(4*d) - (a + b*x + c*x^2)^(3/2)/(d*x) - (3*Sqrt[a]*b*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d) + (3*(b^2 + 4*a*c)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*d) - ((8*c^2*
d + 3*b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(8*Sqrt[c]*d*f) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*
```

```
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f) + ((c*d + b*Sqrt[d]*S
qrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*d^(3/2)*f)
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
```

```
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2\left(d-fx^2\right)}dx=\int\left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx^2}+\frac{f\left(a+bx+cx^2\right)^{3/2}}{d\left(d-fx^2\right)}\right)dx$$

$$=\frac{\int\frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}dx}{d}+\frac{f\int\frac{\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}dx}{d}$$

$$=-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}+\frac{\int\frac{\frac{1}{4}\left(5b^2d+4a(cd+2af)\right)+4b(cd+af)x+\frac{1}{4}\left(8c^2d+3b^2f+12a\right.}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}}{2d}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{3\int\frac{-4abc-c\left(b^2+4a\right.}{x\sqrt{a+bx+cx^2}}}{8cd}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}+\frac{(3ab)\int\frac{1}{x\sqrt{a+bx+c}}}{2d}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{\left(8c^2d+3b^2f+\right.}{}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{3\sqrt{a}b\tanh^{-1}\left(\frac{}{2}\right.}{2d}$$

Mathematica [A] time = 0.669464, size = 765, normalized size = 1.65

$$-\frac{2c^{3/2}d^{3/2}x\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)+2a\sqrt{d}f\sqrt{a+x(b+cx)}+cdx\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right.}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] -(2*a*Sqrt[d]*f*Sqrt[a + x*(b + c*x)] + 3*Sqrt[a]*b*Sqrt[d]*f*x*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])] + 2*c^(3/2)*d^(3/2)*x*ArcTanh[(
```

```
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c*d*Sqrt[c*d - b*Sqrt[d]*Sq
rt[f] + a*f]*x*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt
[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] - b*S
qrt[d]*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*x*ArcTanh[(-(b*Sqrt[d])
+ 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f
] + a*f]*Sqrt[a + x*(b + c*x)])] + a*f*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*
x*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqr
t[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + c*d*Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*x*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d
] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)
])] + b*Sqrt[d]*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*x*ArcTanh[(-2*(
a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]
*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + a*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
+ a*f]*x*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(2*d^(3/2)*f
*x)
```

Maple [B] time = 0.268, size = 4799, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x)
```

```
[Out] -3/8*f/d/(d*f)^(1/2)/c^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)
/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*a*b+3/8*f/d/(d*f)^(1/2)/c^(1/2)*ln((
1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f
)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+
c*d))^(1/2))*a*b+9/4/d*b*(c*x^2+b*x+a)^(1/2)-5/8/d*((x+(d*f)^(1/2)/f)^2*c+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(
1/2)*b+1/2/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*c-1/2/f*ln((1/2/f*(-2*c
*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)
)*c^(3/2)-5/8/d*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b-1/2/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)
^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(
1/2)*c-1/2/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+(
(x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1
/2)+a*f+c*d)/f)^(1/2))*c^(3/2)-3/16/d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(
```

d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/c^(1/2)*b^2+3/4/(d*f
)^(1/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(
d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f
)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*b-1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f
+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2
c+1/f(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d
))^(1/2))/(x+(d*f)^(1/2)/f))*b^2-1/6*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(3/2)
-1/4/d*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c-3/4/d*ln((1/2/f*(-2*c*(d*f)^(1/2)+b
*f)+(x+(d*f)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*c^(1/2)*a+3/
2/d*c^(1/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/d/a/x*(c*x^2+b*
x+a)^(5/2)+1/d*b/a*(c*x^2+b*x+a)^(3/2)+3/8/d*b^2/c^(1/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))-3/2/d*b*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+
a)^(1/2))/x)+3/2/d*c*(c*x^2+b*x+a)^(1/2)*x-3/4/d*ln((1/2*(2*c*(d*f)^(1/2)+b
*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+
b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*a-3/16/d
ln((1/2(2*c*(d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/
2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)
/f)^(1/2))/c^(1/2)*b^2-3/4/(d*f)^(1/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+(x-(
d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-
(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*c^(1/2)*b+1/2/(d*f)^(1/2)/
((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a
*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b^2-1/4/d*((x-(d*f)^(1/2)/f)^2*c+(2*c*
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*c+1
/6*f/d/(d*f)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(3/2)+1/2*f/d/(d*f)^(1/2)*((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+a*f+c*d))^(1/2)*a+1/d/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-
b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*
(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2
)/f))*b*a-1/8*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/
f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b-1/16*f/d/(d*f)^(1/
2)/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2+1/32*f/d/(d*f)^(1/2)/c^(3/2)*ln((1/2*(2*c*(
d*f)^(1/2)+b*f)/f+(x-(d*f)^(1/2)/f)*c)/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*
(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))*b^3+
1/2*f/d/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+
a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+

```
c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a^2+1/2/f*d/(d*f)^
(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*
((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(
1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*c^2+1/8*f/d/(d*f)^(1/2)*((x+(d*f
)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(
1/2)+a*f+c*d))^(1/2)*x*b+1/16*f/d/(d*f)^(1/2)/c*((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2
)*b^2-1/32*f/d/(d*f)^(1/2)/c^(3/2)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+(x+(d*f
)^(1/2)/f)*c)/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+
(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))*b^3-1/2*f/d/(d*f)^(1/2)
/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(
1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a^2-1/2/f*d/(d*f)^(1/
2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*c^2+1/f/(1/f*(-b*(d
*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*b*c-1/(d*f)^(1/2)/(1/f*(-b*(d*f)^
(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))*a*c+1/d*c/a*(c*x^2+b*x+a)^(3/2)*x-1/2
*f/d/(d*f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*a+1/d/((b*(d*f)^(1/2)+a*f+c*d)/f)^(
1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1
/2)/f))*b*a+1/f/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+
c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)
/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*b*c+1/(d*f)^(1/2)/((b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^
(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c
*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))*a*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(cx^2+bx+a\right)^{\frac{3}{2}}}{\left(fx^2-d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx-\int \frac{bx\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx-\int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x**2/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(b*x*sq
rt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(c*x**2*sqrt(a + b*x
+ c*x**2)/(-d*x**2 + f*x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.90 $\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx$

Optimal. Leaf size=614

$$-\frac{a^{3/2}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{a+bx+cx^2}\,(8acf+b^2f+2bcfx+8c^2d)}{8cd^2}-\frac{bf(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}-$$

```
[Out] (-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) + (f*(b^2 + 8*a*c + 2*b*c*x)
*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - ((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x
)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - (a + b*x + c*x^2)^(3/2)/(2*d*x^2) - (3
*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*S
qrt[a]*d) - (a^(3/2)*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/d^2 + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(2*d) - (b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(16*c^(3/2)*d^2) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^2) - ((c*d - b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d
] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x
^2])])/(2*d^2*Sqrt[f]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f])
```

Rubi [A] time = 1.436, antiderivative size = 614, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {6725, 732, 812, 843, 621, 206, 724, 734, 814, 1021, 1070, 1078, 1033}

$$-\frac{a^{3/2}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{a+bx+cx^2}\,(8acf+b^2f+2bcfx+8c^2d)}{8cd^2}-\frac{bf(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```



```
[Out] (-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) + (f*(b^2 + 8*a*c + 2*b*c*x)
*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - ((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x
)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - (a + b*x + c*x^2)^(3/2)/(2*d*x^2) - (3
*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*S
```

```
qrt[a]*d) - (a^(3/2)*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/d^2 + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(2*d) - (b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(16*c^(3/2)*d^2) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^2) - ((c*d - b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d
] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x
^2])])/(2*d^2*Sqrt[f]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1070

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*p + 2*q + 3)
+ C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + f*x^2)^(q + 1
))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(-(
b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(-(b*c*(C*(-4*d*f
)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3)))))*x + (p*(-(b*f))*(C*
(-(b*f))*(q + 1) - c*(-(B*f))*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3\left(d-fx^2\right)}\,dx = \int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx^3}+\frac{f\left(a+bx+cx^2\right)^{3/2}}{d^2x}+\frac{f^2x\left(a+bx+cx^2\right)^{3/2}}{d^2\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3}\,dx}{d}+\frac{f\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}\,dx}{d^2}+\frac{f^2\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx}{d^2}$$

$$= -\frac{\left(a+bx+cx^2\right)^{3/2}}{2dx^2}+\frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx}{4d}+\frac{f\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3d^2}-\frac{f\int \frac{(-2}{}}{}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8acf+2bc\right.}{8cd^2}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8acf+2bc\right.}{8cd^2}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8acf+2bc\right.}{8cd^2}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8acf+2bc\right.}{8cd^2}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}+\frac{f\left(b^2+8ac+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(8c^2d+b^2f+8acf+2bc\right.}{8cd^2}$$

Mathematica [A] time = 1.01125, size = 303, normalized size = 0.49

$$-\frac{\frac{\left(4a(2af+3cd)+3b^2d\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}-\frac{4\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}-b\sqrt{d}+b\sqrt{f}x-2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{f}}+\frac{4\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}\right.}{2\sqrt{a+x(b}}\right)}{\sqrt{f}}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] -((2*d*(2*a + 5*b*x)*Sqrt[a + x*(b + c*x)])/x^2 + ((3*b^2*d + 4*a*(3*c*d +
2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] - (
```

```
4*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f]
 - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt
[a + x*(b + c*x)])])/Sqrt[f] + (4*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Arc
Tanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[f])/(8*d^2)
```

Maple [B] time = 0.276, size = 5056, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(fx^2 - d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x**3/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(b*x*sq
rt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(c*x**2*sqrt(a + b*x
+ c*x**2)/(-d*x**3 + f*x**5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.91 $\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(12ac+3b^2+8c^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b-2c)-b}{2\sqrt{a-b+c}\sqrt{a+bx+\cdots}}\right.$$

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/4 - ((a - b + c)^(3/2)*ArcTanh[(2*a
- b + (b - 2*c)*x)/(2*Sqrt[a - b + c]*Sqrt[a + b*x + c*x^2])])/2 - ((3*b^2
+ 12*a*c + 8*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*Sqrt[c]) + ((a + b + c)^(3/2)*ArcTanh[(2*a + b + (b + 2*c)*x)/(2*Sqrt[a +
 b + c]*Sqrt[a + b*x + c*x^2])])/2
```

Rubi [A] time = 0.284912, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {978, 1078, 621, 206, 1033, 724}

$$-\frac{\left(12ac+3b^2+8c^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+x(b-2c)-b}{2\sqrt{a-b+c}\sqrt{a+bx+\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] -((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/4 - ((a - b + c)^(3/2)*ArcTanh[(2*a
- b + (b - 2*c)*x)/(2*Sqrt[a - b + c]*Sqrt[a + b*x + c*x^2])])/2 - ((3*b^2
+ 12*a*c + 8*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*Sqrt[c]) + ((a + b + c)^(3/2)*ArcTanh[(2*a + b + (b + 2*c)*x)/(2*Sqrt[a +
 b + c]*Sqrt[a + b*x + c*x^2])])/2
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
```

```
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{1-x^2}\,dx = -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}+\frac{1}{2}\int \frac{\frac{1}{4}\left(8a^2+5b^2+4ac\right)+4b(a+c)x+\frac{1}{4}\left(3b^2+12ac+8c^2\right)x^2}{\left(1-x^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}\int \frac{\frac{1}{4}\left(-8a^2-5b^2-4ac\right)+\frac{1}{4}\left(-3b^2-12ac-8c^2\right)-4b(a+c)x}{\left(1-x^2\right)\sqrt{a+bx+cx^2}}$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^2\int \frac{1}{(-1-x)\sqrt{a+bx+cx^2}}\,dx+\frac{1}{2}(a+b+c)^2\int \frac{}{(1}$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{\left(3b^2+12ac+8c^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}+(a-b+c)^2\,\mathrm{Subst}\Big($$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a-b+(b-2c)x}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)-\frac{\left(3b^2+12\right.}{}$$

Mathematica [A] time = 0.621409, size = 181, normalized size = 0.96

$$\frac{1}{8}\left(-\frac{\left(4c(3a+2c)+3b^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}-2(5b+2cx)\sqrt{a+x(b+cx)}-4(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a+b(x-1)}{2\sqrt{a-b+c}\sqrt{a+}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] (-2*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)] - 4*(a - b + c)^(3/2)*ArcTanh[(2*a
+ b*(-1 + x) - 2*c*x)/(2*Sqrt[a - b + c]*Sqrt[a + x*(b + c*x)])] - ((3*b^2
+ 4*c*(3*a + 2*c))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/
Sqrt[c] + 4*(a + b + c)^(3/2)*ArcTanh[(2*a + b + b*x + 2*c*x)/(2*Sqrt[a + b
 + c]*Sqrt[a + x*(b + c*x)])])/8
```

Maple [B] time = 0.2, size = 1346, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(-x^2+1),x)
```

```
[Out] 1/2*c*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2)-1/2*c^(3/2)*ln((1/2*b-c+(1+x)*c
)/c^(1/2)+((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))-5/8*((1+x)^2*c+(b-2*c)*(1+
x)+a-b+c)^(1/2)*b-5/8*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2)*b+1/2*a*((1+x
)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2)-1/2*a*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1
/2)-1/2*c*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2)-1/2*c^(3/2)*ln((1/2*b+c+c
*(-1+x))/c^(1/2)+((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))-1/6*((-1+x)^2*c+(
b+2*c)*(-1+x)+a+b+c)^(3/2)+1/6*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(3/2)+3/8*b/
c^(1/2)*ln((1/2*b-c+(1+x)*c)/c^(1/2)+((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))
*a-3/8*b/c^(1/2)*ln((1/2*b+c+c*(-1+x))/c^(1/2)+((-1+x)^2*c+(b+2*c)*(-1+x)+a
+b+c)^(1/2))*a-3/16/c^(1/2)*ln((1/2*b+c+c*(-1+x))/c^(1/2)+((-1+x)^2*c+(b+2*
c)*(-1+x)+a+b+c)^(1/2))*b^2+1/16/c*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2)*b^
2-3/4*c^(1/2)*ln((1/2*b-c+(1+x)*c)/c^(1/2)+((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^
(1/2))*a-3/16/c^(1/2)*ln((1/2*b-c+(1+x)*c)/c^(1/2)+((1+x)^2*c+(b-2*c)*(1+x)
+a-b+c)^(1/2))*b^2-3/4*c^(1/2)*ln((1/2*b+c+c*(-1+x))/c^(1/2)+((-1+x)^2*c+(b
+2*c)*(-1+x)+a+b+c)^(1/2))*a+3/4*b*ln((1/2*b-c+(1+x)*c)/c^(1/2)+((1+x)^2*c+
(b-2*c)*(1+x)+a-b+c)^(1/2))*c^(1/2)+1/2*b*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-
2*c)*(1+x)+2*(a-b+c)^(1/2)*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))/(1+x))-1/
2*a*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(1+x)+2*(a-b+c)^(1/2)*((1+x)^2*c+
(b-2*c)*(1+x)+a-b+c)^(1/2))/(1+x))-1/32/c^(3/2)*ln((1/2*b-c+(1+x)*c)/c^(1/2
)+((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))*b^3-1/4*c*((1+x)^2*c+(b-2*c)*(1+x)
+a-b+c)^(1/2)*x+1/8*b*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2)*x-1/4*c*((-1+x)
^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2)*x-1/2*c*(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2
*c)*(1+x)+2*(a-b+c)^(1/2)*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))/(1+x))+1/2
*b*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(-1+x)+2*(a+b+c)^(1/2)*((-1+x)^2*c
+(b+2*c)*(-1+x)+a+b+c)^(1/2))/(-1+x))+1/2*a*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(
b+2*c)*(-1+x)+2*(a+b+c)^(1/2)*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))/(-1+
x))-1/8*b*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2)*x-1/16/c*((-1+x)^2*c+(b+2
*c)*(-1+x)+a+b+c)^(1/2)*b^2+1/32/c^(3/2)*ln((1/2*b+c+c*(-1+x))/c^(1/2)+((-1
+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))*b^3+1/2*c*(a+b+c)^(1/2)*ln((2*a+2*b+2*
c+(b+2*c)*(-1+x)+2*(a+b+c)^(1/2)*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))/(
-1+x))-3/4*b*ln((1/2*b+c+c*(-1+x))/c^(1/2)+((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c
)^(1/2))*c^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a+bx+cx^2}}{x^2-1}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{x^2-1}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-x**2+1),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(b*x*sqrt(a + b
*x + c*x**2)/(x**2 - 1), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(x**2
- 1), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.92 $\int \frac{\sqrt{-1-x+x^2}}{1-x^2} \, dx$

Optimal. Leaf size=75

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

```
[Out] -ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])]/2 + ArcTanh[(1 - 2*x)/(2*Sqrt[-1 -
x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2
```

Rubi [A] time = 0.0582723, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {990, 621, 206, 1033, 724, 204}

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

```
[Out] -ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])]/2 + ArcTanh[(1 - 2*x)/(2*Sqrt[-1 -
x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx &= -\int \frac{1}{\sqrt{-1-x+x^2}}\,dx - \int \frac{x}{\left(1-x^2\right)\sqrt{-1-x+x^2}}\,dx \\
&= -\left(\frac{1}{2}\int \frac{1}{(-1-x)\sqrt{-1-x+x^2}}\,dx\right) - \frac{1}{2}\int \frac{1}{(1-x)\sqrt{-1-x+x^2}}\,dx - 2\,\mathrm{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-\cdots}{\sqrt{-1\cdots}}\right. \\
&= \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \mathrm{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) + \mathrm{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{\cdots}{\sqrt{-\cdots}}\right. \\
&= -\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{-1-x+x^2}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{1+3x}{2\sqrt{-1-x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0389001, size = 75, normalized size = 1.

$$-\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

```
[Out] -ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])]/2 + ArcTanh[(1 - 2*x)/(2*Sqrt[-1 -
x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2
```

Maple [A] time = 0.055, size = 102, normalized size = 1.4

$$\frac{1}{2}\sqrt{(1+x)^2-2-3x}-\frac{3}{4}\ln\left(-\frac{1}{2}+x+\sqrt{(1+x)^2-2-3x}\right)-\frac{1}{2}\mathrm{Artanh}\left(\frac{-1-3x}{2}\frac{1}{\sqrt{(1+x)^2-2-3x}}\right)-\frac{1}{2}\sqrt{(-1+x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x-1)^(1/2)/(-x^2+1),x)
```

```
[Out] 1/2*((1+x)^2-2-3*x)^(1/2)-3/4*ln(-1/2+x+((1+x)^2-2-3*x)^(1/2))-1/2*arctanh(
1/2*(-1-3*x)/((1+x)^2-2-3*x)^(1/2))-1/2*((-1+x)^2-2+x)^(1/2)-1/4*ln(-1/2+x+
((-1+x)^2-2+x)^(1/2))+1/2*arctan(1/2*(-3+x)/((-1+x)^2-2+x)^(1/2))
```

Maxima [A] time = 1.71787, size = 112, normalized size = 1.49

$$\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5|2x-2|}-\frac{6\sqrt{5}}{5|2x-2|}\right)-\log\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right)-\frac{1}{2}\log\left(\frac{2\sqrt{x^2-x-1}}{|2x+2|}+\frac{2}{|2x+2|}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arcsin(2/5*sqrt(5)*x/abs(2*x - 2) - 6/5*sqrt(5)/abs(2*x - 2)) - log(x +
 sqrt(x^2 - x - 1) - 1/2) - 1/2*log(2*sqrt(x^2 - x - 1)/abs(2*x + 2) + 2/ab
s(2*x + 2) - 3/2)
```

Fricas [A] time = 1.80182, size = 197, normalized size = 2.63

$$\arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}\right)+\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}-2\right)+\log\left(-2x+2\sqrt{x^2-x-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(-x + sqrt(x^2 - x - 1)) + 1/2*
log(-x + sqrt(x^2 - x - 1) - 2) + log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x^2-x-1}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x-1)**(1/2)/(-x**2+1),x)
```

```
[Out] -Integral(sqrt(x**2 - x - 1)/(x**2 - 1), x)
```

Giac [A] time = 1.27606, size = 99, normalized size = 1.32

$$\arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2-x-1}\right|\right)+\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2-x-1}-2\right|\right)+\log\left(\left|-2\,x+2\,\sqrt{x^2-x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(abs(-x + sqrt(x^2 - x - 1))) +
 1/2*log(abs(-x + sqrt(x^2 - x - 1) - 2)) + log(abs(-2*x + 2*sqrt(x^2 - x -
 1) + 1))
```

3.93 $\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx$

Optimal. Leaf size=130

$$\frac{1}{4}\sqrt{x^2+x}(2x+5)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x^2+x}}\right)-\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x^2+x}}\right)-\frac{5}{4}\tanh^{-1}\left(\ldots\right.$$

```
[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[1 + Sqrt[2]]*ArcTan[(1 + Sqrt[2] - x)/(S
qrt[2*(1 + Sqrt[2])]*Sqrt[x + x^2])] - Sqrt[-1 + Sqrt[2]]*ArcTanh[(1 - Sqrt
[2] - x)/(Sqrt[2*(-1 + Sqrt[2])]*Sqrt[x + x^2])] - (5*ArcTanh[x/Sqrt[x + x^
2]])/4
```

Rubi [A] time = 0.160066, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {978, 1078, 620, 206, 12, 1036, 1030, 207, 203}

$$\frac{1}{4}\sqrt{x^2+x}(2x+5)+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x^2+x}}\right)-\sqrt{\sqrt{2}-1}\tanh^{-1}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x^2+x}}\right)-\frac{5}{4}\tanh^{-1}\left(\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[1 + Sqrt[2]]*ArcTan[(1 + Sqrt[2] - x)/(S
qrt[2*(1 + Sqrt[2])]*Sqrt[x + x^2])] - Sqrt[-1 + Sqrt[2]]*ArcTanh[(1 - Sqrt
[2] - x)/(Sqrt[2*(-1 + Sqrt[2])]*Sqrt[x + x^2])] - (5*ArcTanh[x/Sqrt[x + x^
2]])/4
```

Rule 978

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)
*(d + f*x^2)^(q + 1))/(2*f*(p + q)*(2*p + 2*q + 1)), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(x+x^2\right)^{3/2}}{1+x^2}\,dx &= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{1}{2}\int \frac{\frac{5}{4}+4x+\frac{5x^2}{4}}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{1}{2}\int \frac{4x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx-\frac{5}{8}\int \frac{1}{\sqrt{x+x^2}}\,dx \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{x+x^2}}\right)-2\int \frac{x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)+\frac{\int \frac{-1+\left(-1-\sqrt{2}\right)x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx}{\sqrt{2}}-\frac{\int \frac{-1+\left(-1+\sqrt{2}\right)x}{\left(1+x^2\right)\sqrt{x+x^2}}\,dx}{\sqrt{2}} \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)+\left(-2+\sqrt{2}\right)\mathrm{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)+x^2}\,dx,x,\frac{-1+\sqrt{2}+}{\sqrt{x+x^2}}\right. \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}+\sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{1+\sqrt{2}-x}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+x^2}}\right)-\sqrt{-1+\sqrt{2}}\tanh^{-1}\left(\frac{1-\sqrt{2}-}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{}}\right.
\end{aligned}
$$

Mathematica [C] time = 0.171591, size = 120, normalized size = 0.92

$$
\frac{\sqrt{x}\sqrt{x+1}\left(2\sqrt{x+1}x^{3/2}+5\sqrt{x+1}\sqrt{x}+4(-1+i)^{3/2}\tan^{-1}\left(\sqrt{-1+i}\sqrt{\frac{x}{x+1}}\right)-5\sinh^{-1}\left(\sqrt{x}\right)+4(1+i)^{3/2}\tanh^{-1}\left(\sqrt{1+}\right.\right.}{4\sqrt{x(x+1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(5*Sqrt[x]*Sqrt[1 + x] + 2*x^(3/2)*Sqrt[1 + x] - 5*Arc
Sinh[Sqrt[x]] + 4*(-1 + I)^(3/2)*ArcTan[Sqrt[-1 + I]*Sqrt[x/(1 + x)]] + 4*(
1 + I)^(3/2)*ArcTanh[Sqrt[1 + I]*Sqrt[x/(1 + x)]]))/(4*Sqrt[x*(1 + x)])
```

Maple [B] time = 0.127, size = 789, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)^(3/2)/(x^2+1),x)
```

```
[Out] 1/2*x*(x^2+x)^(1/2)+5/4*(x^2+x)^(1/2)-5/8*ln(x+1/2+(x^2+x)^(1/2))+1/2*(4*(-
2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2*2^(1/2)+4
+3*2^(1/2))^(1/2)*2^(1/2)*((-2+2*2^(1/2))^(1/2)*arctan(1/2*((3*2^(1/2)-4)*(
-(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+12*2^(1/2)+17))^(1/2)*(-2+2*2^(1/2))^(1/2
)*(24*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+17*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2*
2^(1/2)-2^(1/2))*(3*2^(1/2)-4)*(-2^(1/2)-1+x)/(1-x-2^(1/2))/((-2^(1/2)-1+x)
^4/(1-x-2^(1/2))^4-34*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+1))*(1+2^(1/2))^(1/2
)*2^(1/2)-2*(-2+2*2^(1/2))^(1/2)*arctan(1/2*((3*2^(1/2)-4)*(-(-2^(1/2)-1+x)
^2/(1-x-2^(1/2))^2+12*2^(1/2)+17))^(1/2)*(-2+2*2^(1/2))^(1/2)*(24*(-2^(1/2)
-1+x)^2/(1-x-2^(1/2))^2+17*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2*2^(1/2)-2^(1/2)
)*(3*2^(1/2)-4)*(-2^(1/2)-1+x)/(1-x-2^(1/2))/((-2^(1/2)-1+x)^4/(1-x-2^(1/2)
)^4-34*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2+1))*(1+2^(1/2))^(1/2)-4*arctanh(1/2
*(4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2*2^(
1/2)+4+3*2^(1/2))^(1/2)/(1+2^(1/2))^(1/2))*2^(1/2)+6*arctanh(1/2*(4*(-2^(1/
2)-1+x)^2/(1-x-2^(1/2))^2-3*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2*2^(1/2)+4+3*2^
(1/2))^(1/2)/(1+2^(1/2))^(1/2)))/(-(3*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2*2^(1
/2)-4*(-2^(1/2)-1+x)^2/(1-x-2^(1/2))^2-3*2^(1/2)-4)/(1+(-2^(1/2)-1+x)/(1-x-
2^(1/2)))^2)^(1/2)/(1+(-2^(1/2)-1+x)/(1-x-2^(1/2)))/(3*2^(1/2)-4)/(1+2^(1/2
))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+x\right)^{\frac{3}{2}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + x)^(3/2)/(x^2 + 1), x)
```

Fricas [B] time = 2.03948, size = 2356, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*
x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*s
qrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)
*(sqrt(2) - 2)*log(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/4)*sqrt(x^2 + x)*(sq
rt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqr
t(2) + 4) + 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/7*sqrt(2)*(sqr
t(2)*(5*x + 1) + 6*x + 4) + 1/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x - 2*(8^(1/
4)*sqrt(x^2 + x)*(sqrt(2) - 1) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2
) + 4) + 4*x + 4*sqrt(2) + 4)*(8*sqrt(2)*(5*sqrt(2) + 6) + (8^(3/4)*(5*sqrt
(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32
) - 1/7*sqrt(x^2 + x)*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) + 1/7*sqrt(
2)*(8*x + 3) + 1/56*(8^(3/4)*(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*
(8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1)) + 8*8^(1/4)*(sqrt(2)*
(2*x - 1) + x + 3))*sqrt(2*sqrt(2) + 4) + 4/7*x + 5/7) + 1/2*8^(1/4)*sqrt(2
)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*sqrt(2)*(sqrt(2)*(5*x + 1) + 6*x + 4) - 1
/112*sqrt(8*x^2 - 8*sqrt(x^2 + x)*x + 2*(8^(1/4)*sqrt(x^2 + x)*(sqrt(2) - 1
) - 8^(1/4)*(sqrt(2)*x - x - 1))*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)
*(8*sqrt(2)*(5*sqrt(2) + 6) - (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(
2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32) + 1/7*sqrt(x^2 + x)*(sqrt(2
)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4) - 1/7*sqrt(2)*(8*x + 3) + 1/56*(8^(3/4)*
(sqrt(2)*(5*x + 1) + 6*x + 4) - sqrt(x^2 + x)*(8^(3/4)*(5*sqrt(2) + 6) + 8*
8^(1/4)*(2*sqrt(2) + 1)) + 8*8^(1/4)*(sqrt(2)*(2*x - 1) + x + 3))*sqrt(2*sq
rt(2) + 4) - 4/7*x - 5/7) + 1/4*sqrt(x^2 + x)*(2*x + 5) + 5/8*log(-2*x + 2*
sqrt(x^2 + x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x(x+1))^{\frac{3}{2}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)**(3/2)/(x**2+1),x)
```

```
[Out] Integral((x*(x + 1))**(3/2)/(x**2 + 1), x)
```

Giac [C] time = 1.41808, size = 369, normalized size = 2.84

$$\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2\sqrt{2}-2}\left(\frac{i}{\sqrt{2}-1}+1\right)\log\left(2\sqrt{10\sqrt{2}-14}\left(-\frac{i}{5\sqrt{2}-7}+1\right)-(4i+8)\,x+(4i+8)\,\sqrt{x^2+x}+8i-4\right)-\left(\frac{1}{4}i+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] (1/4*I + 1/4)*sqrt(2*sqrt(2) - 2)*(I/(sqrt(2) - 1) + 1)*log(2*sqrt(10*sqrt(
2) - 14)*(-I/(5*sqrt(2) - 7) + 1) - (4*I + 8)*x + (4*I + 8)*sqrt(x^2 + x) +
 8*I - 4) - (1/4*I + 1/4)*sqrt(2*sqrt(2) - 2)*(I/(sqrt(2) - 1) + 1)*log(-2*
sqrt(10*sqrt(2) - 14)*(-I/(5*sqrt(2) - 7) + 1) - (4*I + 8)*x + (4*I + 8)*sq
rt(x^2 + x) + 8*I - 4) + (1/4*I + 1/4)*sqrt(2*sqrt(2) + 2)*(I/(sqrt(2) + 1)
 + 1)*log(2*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1) - 4*x + 4*sqrt(x^2 +
 x) - 4*I) - (1/4*I + 1/4)*sqrt(2*sqrt(2) + 2)*(I/(sqrt(2) + 1) + 1)*log(-2
*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1) - 4*x + 4*sqrt(x^2 + x) - 4*I)
+ 1/4*sqrt(x^2 + x)*(2*x + 5) + 5/8*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

3.94 $\int \frac{x^4}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=369

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{2a\cdots}{2\sqrt{a\cdots}}\right)}{2f^2\sqrt{af\cdots}}$$

```
[Out] (3*b*Sqrt[a + b*x + c*x^2])/(4*c^2*f) - (x*Sqrt[a + b*x + c*x^2])/(2*c*f) -
 (d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f^2) -
 ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(5/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b
*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*f^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d^(3/2)*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f])
```

Rubi [A] time = 0.809408, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6725, 621, 206, 742, 640, 984, 724}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}+\frac{d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{2a\cdots}{2\sqrt{a\cdots}}\right)}{2f^2\sqrt{af\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (3*b*Sqrt[a + b*x + c*x^2])/(4*c^2*f) - (x*Sqrt[a + b*x + c*x^2])/(2*c*f) -
 (d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f^2) -
 ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(5/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b
*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*f^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d^(3/2)*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
```

```
f])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/((((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int \left(-\frac{d}{f^2\sqrt{a+bx+cx^2}} - \frac{x^2}{f\sqrt{a+bx+cx^2}} + \frac{d^2}{f^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right)dx$$

$$= -\frac{d\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2} + \frac{d^2\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f^2} - \frac{\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= -\frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} + \frac{d^2\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,d}{2f^2}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{d^2\,\text{Subst}\left(\int \frac{}{4cd^2-4b}\right.}{}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} + \frac{d^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2}{2\sqrt{cd-b}}\right.}{2f^2\sqrt{cd-b}}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{\left(3b^2-4ac\right)\tanh^{-1}}{8c^{5/2}}$$

Mathematica [A] time = 1.92268, size = 300, normalized size = 0.81

$$\frac{-\frac{\left(-4acf+3b^2f+8c^2d\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{5/2}} - \frac{2f(2cx-3b)\sqrt{a+x(b+cx)}}{c^2} + \frac{4d^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} + \frac{4d^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right.}{2\sqrt{a+x(b+cx)}\sqrt{af+b(}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*f*(-3*b + 2*c*x)*Sqrt[a + x*(b + c*x)])/c^2 - ((8*c^2*d + 3*b^2*f - 4*
a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(5/2) + (4
*d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2
*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f] + (4*d^(3/2)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x +
 b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x
*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f])/(8*f^2)
```

Maple [A] time = 0.28, size = 516, normalized size = 1.4

$$-\frac{x}{2\,cf}\sqrt{cx^2+bx+a}+\frac{3\,b}{4\,c^2f}\sqrt{cx^2+bx+a}-\frac{3\,b^2}{8\,f}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{5}{2}}+\frac{a}{2\,f}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2*x*(c*x^2+b*x+a)^(1/2)/c/f+3/4*b*(c*x^2+b*x+a)^(1/2)/c^2/f-3/8/f*b^2/c^
(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/f*a/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/f^2*d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))/c^(1/2)-1/2/f^2*d^2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^
(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2
))/(x+(d*f)^(1/2)/f))+1/2/f^2*d^2/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(
1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1
/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**4/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.95 $\int \frac{x^3}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=287

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{cf}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(2*c^(3/2)*f) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c
*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d*ArcTanh[
(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.632543, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 640, 621, 206, 1033, 724}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```


```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(2*c^(3/2)*f) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c
*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d*ArcTanh[
(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f])
```


Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int \left(-\frac{x}{f\sqrt{a+bx+cx^2}} + \frac{dx}{f\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right)dx$$
$$= -\frac{\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx}{f} + \frac{d\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2cf} + \frac{d\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f} + \frac{d\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx}}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf} - \frac{d\,\mathrm{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\right.}{}$$
$$= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{d\,\mathrm{t}}{}$$

Mathematica [A] time = 1.20162, size = 325, normalized size = 1.13

$$\frac{\frac{b\sqrt{f}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{d\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} - \frac{2\sqrt{f}x^2}{\sqrt{a+x(b+cx)}} - \frac{2b\sqrt{f}x}{c\sqrt{a+x(b+cx)}} - \frac{2a}{c\sqrt{a+x}}}{2f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*a*Sqrt[f])/(c*Sqrt[a + x*(b + c*x)]) - (2*b*Sqrt[f]*x)/(c*Sqrt[a + x*(
b + c*x)]) - (2*Sqrt[f]*x^2)/Sqrt[a + x*(b + c*x)] + (b*Sqrt[f]*ArcTanh[(b
+ 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + (d*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] - (
d*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f])/(2*f^(3/2))
```

Maple [A] time = 0.265, size = 410, normalized size = 1.4

$$-\frac{1}{cf}\sqrt{cx^2+bx+a}+\frac{b}{2f}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{3}{2}}+\frac{d}{2f^2}\ln\left(\left(\left(2\frac{-b\sqrt{df}+af+cd}{f}+\frac{1}{f}\left(-2c\sqrt{df}+bf\right)\right(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -(c*x^2+b*x+a)^(1/2)/c/f+1/2/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))+1/2/f^2*d/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^
(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f
)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(
x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))+1/
2/f^2*d/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2
)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)
^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^3}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.96 $\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=266

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

```
[Out] -(ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f)) + (Sq
rt[d]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.216425, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1079, 621, 206, 984, 724}

$$\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```


```
[Out] -(ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f)) + (Sq
rt[d]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```


Rule 1079

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x]
```

```
 + Dist[(A*c - a*C)/c, Int[1/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /;
 FreeQ[{a, c, d, e, f, A, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = -\frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f} + \frac{d\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{f}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{d\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2f} + \frac{d\int \frac{1}{\left(d+\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx}}}{2f}$$

$$= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{d\,\text{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\right.}{\sqrt{a+bx+cx^2}}\right.}{f}$$

$$= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}+}{2\sqrt{cd+}}\right.}{2f\sqrt{cd+b}}$$

Mathematica [A] time = 0.456077, size = 250, normalized size = 0.94

$$\frac{\sqrt{d}\left(\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right) - \frac{2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + Sqrt
[d]*(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*Sqr
t[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f] + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sq
rt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*f)
```

Maple [A] time = 0.279, size = 399, normalized size = 1.5

$$-\frac{1}{f}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)\frac{1}{\sqrt{c}} - \frac{d}{2f}\ln\left(\left(2\frac{-b\sqrt{df}+af+cd}{f}+\frac{1}{f}\left(-2c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{-b\sqrt{}}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d/(d*f)^(1/2)/
f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f
*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1
/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))+1/2*d/(d*f)^(1/2)/f/
((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a
*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**2/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.97 $\int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

```
[Out] -ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[f]*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
 + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])]/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.12962, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1033, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[f]*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
 + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])]/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
```

```
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx &= \frac{1}{2}\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx+\frac{1}{2}\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}-\frac{\tanh^{-1}\left(\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}\end{aligned}$$

Mathematica [A] time = 0.184004, size = 211, normalized size = 0.96

$$-\frac{\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{-2\left(a\sqrt{f}+c\sqrt{d}x\right)-b\left(\sqrt{d}+\sqrt{f}x\right)}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}}{2\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqrt[d]
```

```
*Sqrt[f] + a*f] + ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt
[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt
[c*d + b*Sqrt[d]*Sqrt[f] + a*f])/(2*Sqrt[f])
```

Maple [B] time = 0.265, size = 354, normalized size = 1.6

$$\frac{1}{2f}\ln\left(\left(2\frac{-b\sqrt{df}+af+cd}{f}+\frac{1}{f}\left(-2c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{-b\sqrt{df}+af+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{-2c\sqrt{df}+bf}{f}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/f/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*
d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))+1/2/f/((b*(d*f)^
(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*
f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)
^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.22874, size = 5501, normalized size = 25.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*
d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 -
 (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f - (c^3*d^3*f + a
^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c
^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a
^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt
((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d
^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c
^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 -
2*a*c)*d*f^2)) - (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (
b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f +
 a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^
2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^2*
f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2
*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^
2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b
*c*d*x + b^2*d - 2*(b^2*d*f - (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*
f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2
 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 -
2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^2*f + a^2*
f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2
*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^
3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) - (2*a*c^2*d^2*f
 + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3
- 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3
)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d
*f^4)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f
^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4
- 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*
f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f + (
c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*
sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 +
b*x + a)*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt
(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*
f^3 - (b^2 - 2*a*c)*d*f^2)) + (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2
*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d
/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
```

```
6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a*
f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4
*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^
3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f
^2))*log((2*b*c*d*x + b^2*d - 2*(b^2*d*f + (c^3*d^3*f + a^3*f^4 - (b^2*c -
3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5
 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 -
2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f - (c^2
*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2
*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a
^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) + (
2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*
b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*
c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2
 - 2*a^3*c)*d*f^4)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.98 $\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=220

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

```
[Out] ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
+ b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])]/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.117588, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {984, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\sqrt{f}+2c\sqrt{d})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
+ b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])]/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx &= \frac{1}{2}\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx+\frac{1}{2}\int \frac{1}{\left(d+\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right) -\\
&= -\frac{\tanh^{-1}\left(\frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}-\frac{\tanh^{-1}\left(\frac{-bd-2a\sqrt{d}\sqrt{f}-\left(2cd+b\sqrt{d}\sqrt{f}\right)x}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}
\end{aligned}
$$

Mathematica [A] time = 0.0973999, size = 209, normalized size = 0.95

$$
\frac{\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}+\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}}{2\sqrt{d}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*
d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d + b*Sqrt[d]*S
qrt[f] + a*f] + ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f
]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c
```

```
*d - b*Sqrt[d]*Sqrt[f] + a*f])/(2*Sqrt[d])
```

Maple [B] time = 0.26, size = 358, normalized size = 1.6

$$-\frac{1}{2}\ln\left(\left(\left(2\,\frac{-b\sqrt{df}+af+cd}{f}+\frac{1}{f}\left(-2\,c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{\frac{-b\sqrt{df}+af+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{-2\,c\sqrt{df}+b}{f}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/
2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(
1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))+1/2/(
d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/
f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(
1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.68168, size = 5328, normalized size = 24.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*
f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6
*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 -
 (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*d + a*b*f - (b*c^2*d^3 +
 a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(
b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b
^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^3 +
 a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 -
 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) - (2*a*c^
2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3
 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)
*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*
f^3)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*
f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4
*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 +
 a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 - 2*(b*c*d + a*b*f -
(b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4
*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^
2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f
+ (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f
^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 -
2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f
)) - (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*
b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^
2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2
*a^3*c)*d^2*f^3)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 -
 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*
f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3))
)/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*
d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c
^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2
*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt
((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d
^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2
)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 -
2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^
2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4
 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*
(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x) - 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*
f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2
*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*
```

```
c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b
^2 - 2*(b*c*d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*s
qrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b
^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b
*x + a)*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(
b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f
^2 - (b^2 - 2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)
*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5
 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.99 $\int \frac{1}{x\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=267

$$-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) - (Sq
rt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.663489, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6725, 724, 206, 1033}

$$-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) - (Sq
rt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

```
[n, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{fx}{d\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\right) dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{f\int \frac{x}{\sqrt{a+bx+cx^2}\left(-d+fx^2\right)}\,dx}{d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{f\int \frac{1}{\left(-\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d} - \frac{f\int \frac{1}{\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{a}}}{2d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{f\,\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}+2af-\left(2c\sqrt{d}\sqrt{}\right.}{\sqrt{a+bx+cx^2}}\right.}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}}{2\sqrt{cd}}\right.}{2d\sqrt{cd+}}$$

Mathematica [A] time = 0.472324, size = 252, normalized size = 0.94

$$\frac{\sqrt{f}\left(\frac{\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\sqrt{d}-b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-\frac{2\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + Sqrt
[f]*(-(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + 2*c*Sqrt[d]*x - b*Sqrt[f]*x)/(2*S
qrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b
*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]/
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*d)
```

Maple [A] time = 0.28, size = 391, normalized size = 1.5

$$-\frac{1}{d}\ln\left(\frac{1}{x}\left(2\,a+bx+2\,\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{1}{2\,d}\ln\left(\left(\left(2\,\frac{-b\sqrt{df}+af+cd}{f}+\frac{1}{f}\left(-2\,c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\,\sqrt{-\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/2/d/(1/f*(-b*(
d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d
*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))+1/2/d/((b*(d*f)^(1/2)+a*f+c*d)/f
)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)
^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2-d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-dx\sqrt{a+bx+cx^2}+fx^3\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x*sqrt(a + b*x + c*x**2) + f*x**3*sqrt(a + b*x + c*x**2)),
x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.100 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=291

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+c}}{adx}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f*ArcTan
h[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f])
```

Rubi [A] time = 0.65675, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {6725, 730, 724, 206, 984}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+c}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```


```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f*ArcTan
h[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f])
```


Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}} + \frac{f}{d\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right) dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{f\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} - \frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad} + \frac{f\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{f\int \frac{1}{\left(d+\sqrt{d}\sqrt{f}x\right)\sqrt{a}}}{2d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad} - \frac{f\,\mathrm{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4ad}\right.}{}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [A] time = 1.06665, size = 325, normalized size = 1.12

$$\frac{\frac{b\sqrt{d}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} + \frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} + \frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} - \frac{2b\sqrt{d}}{a\sqrt{a+x(b+cx)}} - \frac{2c\sqrt{d}x}{a\sqrt{a+x(b+cx)}} - \frac{}{x\sqrt{}}}{2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-2*b*Sqrt[d])/(a*Sqrt[a + x*(b + c*x)]) - (2*Sqrt[d])/(x*Sqrt[a + x*(b +
c*x)]) - (2*c*Sqrt[d]*x)/(a*Sqrt[a + x*(b + c*x)]) + (b*Sqrt[d]*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/a^(3/2) + (f*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f] + (
f*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f])/(2*d^(3/2))
```

Maple [A] time = 0.265, size = 427, normalized size = 1.5

$$-\frac{f}{2d}\ln\left(\left(\left(2\frac{-b\sqrt{df}+af+cd}{f}+\frac{1}{f}\left(-2c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\sqrt{\frac{-b\sqrt{df}+af+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{-2c\sqrt{df}+b}{f}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -1/2*f/d/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)
^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*
f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-(
c*x^2+b*x+a)^(1/2)/a/d/x+1/2/d*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a
)^(1/2))/x)+1/2*f/d/(d*f)^(1/2)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*
(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f
)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{\sqrt{cx^{2}+bx+a}\left(fx^{2}-d\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-dx^2\sqrt{a+bx+cx^2}+fx^4\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x**2*sqrt(a + b*x + c*x**2) + f*x**4*sqrt(a + b*x + c*x**2)
), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.101 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal. Leaf size=376

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}+\frac{f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{\ }}{2\sqrt{a+}}\right)}{2d^2\sqrt{af+}}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x
) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(8*a^(5/2)*d) - (f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[a]*d^2) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])])/(2*d^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f^(3/2)*ArcTanh[(b*Sqrt
[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
 a*f])
```

Rubi [A] time = 0.734105, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 744, 806, 724, 206, 1033}

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x(2c\sqrt{d}-b\sqrt{f})+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b(-\sqrt{d})\sqrt{f}+cd}}+\frac{f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{\ }}{2\sqrt{a+}}\right)}{2d^2\sqrt{af+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x
) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(8*a^(5/2)*d) - (f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[a]*d^2) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])])/(2*d^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f^(3/2)*ArcTanh[(b*Sqrt
[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
```

```
a*f])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx = \int\left(\frac{1}{dx^3\sqrt{a+bx+cx^2}}+\frac{f}{d^2x\sqrt{a+bx+cx^2}}+\frac{f^2x}{d^2\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx}{d}+\frac{f\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2}+\frac{f^2\int \frac{x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2}-\frac{\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx}{2ad}-\frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d^2}+\frac{f^2\int \frac{}{(}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}+\frac{\left(3b^2-4ac\right)\int \frac{}{x\sqrt{a-}}}{8a^2d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{f^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d-}}{2\sqrt{cd-}}\right.}{2d^2\sqrt{cd-b}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{f\tanh^{-}}{}$$

Mathematica [A] time = 2.15525, size = 314, normalized size = 0.84

$$\frac{2\sqrt{a}\left(\frac{2a^2f^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{2a^2f^{3/2}\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{d(2a-3bx)\sqrt{a+x(b+cx)}}{x^2}\right)+\left(4a(cd-2af)-3b^2\right.}{8a^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((-3*b^2*d + 4*a*(c*d - 2*a*f))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(
b + c*x)])] + 2*Sqrt[a]*(-((d*(2*a - 3*b*x)*Sqrt[a + x*(b + c*x)])/x^2) + (
2*a^2*f^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*
x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d
 + b*Sqrt[d]*Sqrt[f] + a*f] - (2*a^2*f^(3/2)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sq
rt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]))/(8*a^(5/2)*d^
2)
```

Maple [A] time = 0.279, size = 519, normalized size = 1.4

$$-\frac{f}{d^2}\ln\left(\frac{1}{x}\left(2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{f}{2d^2}\ln\left(\left(\left(2\frac{-b\sqrt{df}+af+cd}{f}+\frac{1}{f}\left(-2c\sqrt{df}+bf\right)\left(x+\frac{1}{f}\sqrt{df}\right)+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x)
```

```
[Out] -f/d^2/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/2*f/d^2/(1/f
*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*
((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b
*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-1/2*(c*x^2+b*x+a)^(1/2)/a/
d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2)/a^2/d/x-3/8/d*b^2/a^(5/2)*ln((2*a+b*x+2*a^(
1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/2/d*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b
*x+a)^(1/2))/x)+1/2*f/d^2/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^
(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2
)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{1}{\sqrt{cx^2+bx+a}\left(fx^2-d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-dx^3\sqrt{a+bx+cx^2}+fx^5\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-d*x**3*sqrt(a + b*x + c*x**2) + f*x**5*sqrt(a + b*x + c*x**2)
), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.102 $\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=466

$$-\frac{2d^2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2d(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2b\sqrt{a+bx+cx^2}}{cf\left(b^2-4ac\right)}-\frac{}{f\left(b^2-\right.}$$

```
[Out] (-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*d*(b + 2*c*
x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) - (2*d^2*(b*(b^2*f - c*(c*d +
 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*f^2*(b^2*d*f -
(c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (2*b*Sqrt[a + b*x + c*x^2])/(c*(b^2
 - 4*a*c)*f) - ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(c^(3
/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[
f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f
*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a
*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a
*f]*Sqrt[a + b*x + c*x^2])])/(2*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 1.34656, antiderivative size = 466, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6725, 613, 738, 640, 621, 206, 975, 1033, 724}

$$-\frac{2d^2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2d(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2b\sqrt{a+bx+cx^2}}{cf\left(b^2-4ac\right)}-\frac{}{f\left(b^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```


```
[Out] (-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*d*(b + 2*c*
x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) - (2*d^2*(b*(b^2*f - c*(c*d +
 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*f^2*(b^2*d*f -
(c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (2*b*Sqrt[a + b*x + c*x^2])/(c*(b^2
 - 4*a*c)*f) - ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(c^(3
/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[
f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f
```

```
*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a
*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a
*f]*Sqrt[a + b*x + c*x^2])])/(2*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int \left(-\frac{d}{f^2\left(a+bx+cx^2\right)^{3/2}} - \frac{x^2}{f\left(a+bx+cx^2\right)^{3/2}} + \frac{d^2}{f^2\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\right) dx$$

$$= -\frac{d\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f^2} + \frac{d^2\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx}{f^2} - \frac{\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f}$$

$$= -\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2\left(b\left(b^2f-c(cd+3a\right.\right.}{\left(b^2-4ac\right)f^2\left(b^2df\right.}$$

$$= -\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2\left(b\left(b^2f-c(cd+3a\right.\right.}{\left(b^2-4ac\right)f^2\left(b^2df\right.}$$

$$= -\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2\left(b\left(b^2f-c(cd+3a\right.\right.}{\left(b^2-4ac\right)f^2\left(b^2df\right.}$$

$$= -\frac{2x(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} - \frac{2d^2\left(b\left(b^2f-c(cd+3a\right.\right.}{\left(b^2-4ac\right)f^2\left(b^2df\right.}$$

Mathematica [A] time = 1.64, size = 562, normalized size = 1.21

$$\frac{-\frac{2d^2\left(-bc(3af+cd)-2c^2x(af+cd)+b^2cfx+b^3f\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(b^2df-(af+cd)^2\right)} + \frac{f\left(a\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}\left(a(b-2cx)+bcx^2\right)\sqrt{a+x(b+cx)}\right)}{ac^{3/2}\left(4ac-b^2\right)} + \frac{d^{3/2}f\left(\frac{\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tan}{\sqrt{af}}\right.}{}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((2*d*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*f*x^3*(b^2 -
2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*d^2*(b^3*f - b
*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x))/((b^2 - 4*a*c)*(b^2*d*
f - (c*d + a*f)^2)*Sqrt[a + x*(b + c*x)]) + (f*(-2*Sqrt[c]*(b*c*x^2 + a*(b
- 2*c*x))*Sqrt[a + x*(b + c*x)] + a*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])]))/(a*c^(3/2)*(-b^2 + 4*a*c)) + (d^(3/2)*f*(((
```

```
b^2 - 4*a*c)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sq
rt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*S
qrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ((-b^2 + 4*a*
c)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) -
b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*
(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(2*(b^2 - 4*a*c)*(-(b^2
*d*f) + (c*d + a*f)^2)))/f^2
```

Maple [B] time = 0.283, size = 1648, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/f*x/c/(c*x^2+b*x+a)^(1/2)-1/2/f*b/c^2/(c*x^2+b*x+a)^(1/2)-1/f*b^2/c/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)*x-1/2/f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-
1/f/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/f^2*d/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*x*c-2/f^2*d/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b+1/2/f*d^
2/(d*f)^(1/2)/(-b*(d*f)^(1/2)+a*f+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)+2/f^2*d
^2/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d
*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c^2-
1/f*d^2/(d*f)^(1/2)/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)
^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c
*d))^(1/2)*x*b*c+1/f^2*d^2/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+a*f+c*d))^(1/2)*b*c-1/2/f*d^2/(d*f)^(1/2)/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c
-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1
/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2-1/2/f*d^2/(d*f)^(1/2)/(-b*(d*f)^(1/2
)+a*f+c*d)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f
+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a
*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(
1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-1/2/f*d^2/(
d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+
b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)+2/f^2*d^2/(b*(d*f
)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*c^2+1/f*d^2/(d*f)^(1/
2)/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b*c+1/f^2*d
^2/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1
```

```
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b*c+1/2/f*d^2
/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2
+1/2/f*d^2/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/((b*(d*f)^(1/2)+a*f+c*d)/f)^
(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(
1/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.103 $\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=341

$$-\frac{2d\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}-$$

```
[Out] (-2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) - (2*d*(a*(2*c^2*d
 - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*f*(b^2*d*f - (c*d
+ a*f)^2)*Sqrt[a + b*x + c*x^2]) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])])/(2*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*ArcT
anh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*(c*d + b*Sqrt[d
]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 1.04115, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6725, 636, 1018, 1033, 724, 206}

$$-\frac{2d\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{d\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) - (2*d*(a*(2*c^2*d
 - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*f*(b^2*d*f - (c*d
+ a*f)^2)*Sqrt[a + b*x + c*x^2]) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])])/(2*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*ArcT
anh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*(c*d + b*Sqrt[d
]*Sqrt[f] + a*f)^(3/2))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}dx = \int\left(-\frac{x}{f\left(a+bx+cx^2\right)^{3/2}}+\frac{dx}{f\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\right)dx$$

$$= -\frac{\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}dx}{f}+\frac{d\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}dx}{f}$$

$$= -\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}+\frac{(2d}{}$$

$$= -\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}+\frac{d\int}{2}$$

$$= -\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}-\frac{d\,\mathrm{S}}{}$$

$$= -\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}-\frac{d\,\mathrm{ta}}{}$$

Mathematica [A] time = 1.37873, size = 414, normalized size = 1.21

$$\frac{1}{2}\left(\frac{4a^2(bfx+2cd)+8a^3f-4abd(b-3cx)-4b^3dx}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(f\left(b^2d-a^2f\right)-2acdf-c^2d^2\right)}-\frac{d\log\left(\sqrt{d}\sqrt{f}-fx\right)}{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{d\log\left(\sqrt{d}\sqrt{f}+fx\right)}{\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((8*a^3*f - 4*b^3*d*x - 4*a*b*d*(b - 3*c*x) + 4*a^2*(2*c*d + b*f*x))/((b^2
- 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)]
) - (d*Log[Sqrt[d]*Sqrt[f] - f*x])/(Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(3/2)) - (d*Log[Sqrt[d]*Sqrt[f] + f*x])/(Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f]
+ a*f)^(3/2)) + (d*Log[Sqrt[d]*(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x
+ b*Sqrt[f]*x + 2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]
)])/(Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*Log[Sqrt[d]*(b*(Sq
rt[d] + Sqrt[f]*x) + 2*(a*Sqrt[f] + c*Sqrt[d]*x + Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + x*(b + c*x)]))])/(Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*
f)^(3/2)))/2
```

Maple [B] time = 0.287, size = 1480, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/f/c/(c*x^2+b*x+a)^(1/2)+2/f*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/f*b^2/c
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/2/f*d/(-b*(d*f)^(1/2)+a*f+c*d)/((x+(d*f)
^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+a*f+c*d))^(1/2)-2/f^2*d/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^
(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/
2)+a*f+c*d))^(1/2)*(d*f)^(1/2)*x*c^2+1/f*d/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-
b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*b*c-1/f^2*d/(-b*(d*f)^(1/2)+a*f+c*d)/(4
*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*(d*f)^(1/2)*b*c+1/2/f*d/(-b*(d*f)^(1
/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2+1/2/f*d/(-b*(d*f)
^(1/2)+a*f+c*d)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2
)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1
/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d
*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-1/2/f*
d/(b*(d*f)^(1/2)+a*f+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)+2/f^2*d/(b*(d*f)^(1/2)+a*f
+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/f*d/(b*(d*f)^(
1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b*c+1/f^2*d/(b*(d*f)^(1/
2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
```

```
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/f*d/(b*(
d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2+1/2/f*d/(b*(d*f)
^(1/2)+a*f+c*d)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+
c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)
/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.104 $\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=297

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

```
[Out] (2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d
*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] -
2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) +
 (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d + b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 0.453871, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1065, 1033, 724, 206}

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d
*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] -
2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) +
 (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d + b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 1065

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2
)^(q + 1)*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*f - c*(2*a
*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x)
)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*(-(b*(c*d + a*f))) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{-\frac{1}{2}\left(b^2-4ac\right)d(cd+af)+\frac{1}{2}b\left(b^2-4ac\right)dfx}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)}$$

$$= \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{\left(\sqrt{d}\sqrt{f}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)} +$$

$$= \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{d}\sqrt{f}\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2 \cdots}\right.}{cd-b\sqrt{d} \cdots}$$

$$= \frac{2\left(ab(cd-af)+c\left(b^2d-2a(cd+af)\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

Mathematica [A] time = 0.468073, size = 352, normalized size = 1.19

$$\frac{2\left(\frac{a^2f(b+2cx)+acd(2cx-b)-b^2cdx}{\sqrt{a+x(b+cx)}} + \frac{\sqrt{d}\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{\sqrt{d}\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2\left(a\sqrt{\cdots}\right.}{2\sqrt{a+x\cdots}}\right)}{4\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((-(b^2*c*d*x) + a*c*d*(-b + 2*c*x) + a^2*f*(b + 2*c*x))/Sqrt[a + x*(b +
 c*x)] + ((b^2 - 4*a*c)*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]])])/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]) + ((-b^2 + 4*a*c)*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(
-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]])])/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f
] + a*f])))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

Maple [B] time = 0.309, size = 1427, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*d/(d*f)^(1/2)/(-b*(d*f)^
(1/2)+a*f+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(
1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)+2*d/f/(-b*(d*f)^(1/2)+a*f+c*d)/
(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c^2-d/(d*f)^(1/2)/(-b*(d*f)^(1/2
)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*b*c+d/f/(-b*(d*f)^(1/
2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(
x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b*c-1/2*d/(d*f)^(1/2)/
(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2-1/2*d
/(d*f)^(1/2)/(-b*(d*f)^(1/2)+a*f+c*d)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*
ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+
(d*f)^(1/2)/f))-1/2*d/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^
(1/2)+2*d/f/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c
*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*c^
2+d/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+
(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*
x*b*c+d/f/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(
d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b*c+1/
2*d/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+
(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*
b^2+1/2*d/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(
1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1
/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.105 $\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=299

$$-\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2\cdots\right.}{2\sqrt{a+bx+cx^2}\sqrt{af\cdots}}\right.}{2\left(af+b\sqrt{d}\sqrt{f}+c\cdots\right.}$$

```
[Out] (-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^
2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[f]*ArcTanh[(b*Sqrt[d]
 - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x
)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d +
 b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 0.400357, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1018, 1033, 724, 206}

$$-\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{\sqrt{f}\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2\cdots\right.}{2\sqrt{a+bx+cx^2}\sqrt{af\cdots}}\right.}{2\left(af+b\sqrt{d}\sqrt{f}+c\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^
2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[f]*ArcTanh[(b*Sqrt[d]
 - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x
)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*(c*d +
 b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
```

```
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{\frac{1}{2}b\left(b^2-4ac\right)df-\frac{1}{2}\left(b^2-4ac\right)f(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)}$$

$$= -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{f\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}dx}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)} + \frac{f\int}{2}$$

$$= -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{f\,\mathrm{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\right.}{cd-b\sqrt{d}}$$

$$= -\frac{2\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+}}\right.}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

Mathematica [A] time = 0.415264, size = 356, normalized size = 1.19

$$\frac{2\left(\frac{2a^2cf+a\left(b^2(-f)-bcfx+2c^2d\right)+bc^2dx}{\sqrt{a+x(b+cx)}} - \frac{\sqrt{f}\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{\sqrt{f}\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}}{4\sqrt{af+b\sqrt{d}\sqrt{f}}}\right.}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x))/Sqrt[a + x*(b +
 c*x)] - ((b^2 - 4*a*c)*Sqrt[f]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/(4*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]) + ((-b^2 + 4*a*c)*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(
-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])]])/(4*Sqrt[c*d + b*Sqrt[d]*Sqrt[f
] + a*f]))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

Maple [B] time = 0.298, size = 1360, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] -1/2/(-b*(d*f)^(1/2)+a*f+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)-2/f/(-b*(d*f)^(1
/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*
(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*(d*f)^(1/2)*x*c^2+1/(
-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*b*c-1/f/
(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*(d*f)^(1/
2)*b*c+1/2/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2
)*b^2+1/2/(-b*(d*f)^(1/2)+a*f+c*d)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln(
(2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*
f)^(1/2)/f))-1/2/(b*(d*f)^(1/2)+a*f+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)+2/f/(b*(d*f
)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/(b*
(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*
f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b*c+1/f/(b*(d*f)^
(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/(b*(d
*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2+1/2/(b*(d*f)^(1/2
)+a*f+c*d)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/
f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(
1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.106 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal. Leaf size=310

$$-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\cdots}{2\sqrt{a+bx+cx^2}\cdots}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\cdots\right.}$$

```
[Out] (-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2
- 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b*S
qrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]
*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sq
rt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(
2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 0.41061, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {975, 1033, 724, 206}

$$-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{f\tanh^{-1}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f\tanh^{-1}\left(\frac{2a\sqrt{f}+x(b\cdots}{2\sqrt{a+bx+cx^2}\cdots}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2
- 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b*S
qrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]
*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sq
rt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(
2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}\left(b^2-4ac\right)f(cd+af)-\frac{1}{2}b\left(b^2-4ac\right.}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}}{\left(b^2-4ac\right)\left(b^2df-(cd+a\right.}$$

$$= -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{f^{3/2}\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)}$$

$$= -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{f^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4}\right.}{\sqrt{d}\left(cd\right.}$$

$$= -\frac{2\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)x\right)}{\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{f\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\right.}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+b}}\right.}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)}$$

Mathematica [A] time = 0.446705, size = 360, normalized size = 1.16

$$\frac{2\left(\frac{-bc(3af+cd)-2c^2x(af+cd)+b^2cfx+b^3f}{\sqrt{a+x(b+cx)}} + \frac{f\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{-2a\sqrt{f}+b\left(\sqrt{d}-\sqrt{f}x\right)+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{4\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{f\left(4ac-b^2\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\tanh^{-1}\left(\ldots\right.}{4\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+\ldots}}\right.}{\left(b^2-4ac\right)\left((af+cd)^2-b^2df\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*((b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x)/Sqrt[a +
x*(b + c*x)] + ((b^2 - 4*a*c)*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(-2
*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]) + ((-b^2 + 4*a*c)*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh
[(-2*(a*Sqrt[f] + c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/(4*Sqrt[d]*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f])))/((b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

Maple [B] time = 0.292, size = 1376, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2/(d*f)^(1/2)*f/(-b*(d*f)^(1/2)+a*f+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)+2/(
-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*c^2-1/(d
*f)^(1/2)/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)
*x*b*c*f+1/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2
)*b*c-1/2/(d*f)^(1/2)/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f
+c*d))^(1/2)*b^2*f-1/2/(d*f)^(1/2)*f/(-b*(d*f)^(1/2)+a*f+c*d)/(1/f*(-b*(d*f
)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*(d*f)^(1
/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*((x+(d*f)
^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-1/2/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+
c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+a*f+c*d)/f)^(1/2)+2/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*
f+c*d)/f)^(1/2)*x*c^2+1/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x
-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2
)+a*f+c*d)/f)^(1/2)*x*b*c*f+1/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)
^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+
c*d)/f)^(1/2)*b*c+1/2/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(
d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
a*f+c*d)/f)^(1/2)*b^2*f+1/2/(d*f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)*f/((b*(d*f)
^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b
*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)
/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f
)^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.107 $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=394

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{f^{3/2}\tanh^{-}}{2d\left(a\right.}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (2*f*
(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*d*(b^2*
d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*S
qrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2])])/(2*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f
^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*S
qrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*(c*d + b*S
qrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 1.18181, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6725, 740, 12, 724, 206, 1018, 1033}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{f^{3/2}\tanh^{-}}{2d\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```


```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (2*f*
(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*d*(b^2*
d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*S
qrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2])])/(2*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f
^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*S
qrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*(c*d + b*S
```

```
qrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1018

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^
(q + 1)*((g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f
)) + c*(g*(2*c^2*d + b^2*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x))/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
```

```
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*(-(b*f)))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*(-(b*(c*d + a*f))) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx=\int\left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}-\frac{fx}{d\left(a+bx+cx^2\right)^{3/2}\left(-d+fx^2\right)}\right)dx$$

$$=\frac{\int\frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx}{d}-\frac{f\int\frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(-d+fx^2\right)}\,dx}{d}$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}-\frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}-$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}-\frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}+$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}-\frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}-$$

$$=\frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}-\frac{2f\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}-$$

Mathematica [A] time = 0.999934, size = 436, normalized size = 1.11

$$\frac{-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(b^2df-(af+cd)^2\right)}+\frac{f^{3/2}\left(\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\tanh^{-1}\left(\frac{2a\sqrt{f}+b\sqrt{d}+b\sqrt{f}x+2c\sqrt{d}x}{2\sqrt{a+x(b+cx)}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)+\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\right.}{2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\left((af+cd)^2-\right.}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (2*f*(
a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2)*Sqrt[a + x*(b + c*x)]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])]/a^(3/2) + (f^(3/2)*((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^
(3/2)*ArcTanh[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2
*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])] + (c*d - b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x
+ b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)]
)]))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
a*f]*(-(b^2*d*f) + (c*d + a*f)^2)))/d
```

Maple [B] time = 0.26, size = 1518, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/d/a/(c*x^2+b*x+a)^(1/2)-2/d*b/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*c-1/d*b
^2/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/d/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x
^2+b*x+a)^(1/2))/x)-1/2/d*f/(-b*(d*f)^(1/2)+a*f+c*d)/((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))
^(1/2)-2/d/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2
)*(d*f)^(1/2)*x*c^2+1/d/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2
)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a
*f+c*d))^(1/2)*x*b*c*f-1/d/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+a*f+c*d))^(1/2)*(d*f)^(1/2)*b*c+1/2/d/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2
```

```
)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2*f+1/2/d*f/(-b*(d*f)^(1/2)+a*f+c*d)/(1/f*
(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d)+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-1/2/d/(b*(d*f)^(1/2)+a*f+c*
d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d
*f)^(1/2)+a*f+c*d)/f)^(1/2)+2/d/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*
f+c*d)/f)^(1/2)*(d*f)^(1/2)*x*c^2+1/d/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/(
(x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1
/2)+a*f+c*d)/f)^(1/2)*x*b*c*f+1/d/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(
d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
a*f+c*d)/f)^(1/2)*(d*f)^(1/2)*b*c+1/2/d/(b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)
/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^
(1/2)+a*f+c*d)/f)^(1/2)*b^2*f+1/2/d/(b*(d*f)^(1/2)+a*f+c*d)*f/((b*(d*f)^(1/
2)+a*f+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/
f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^
2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1
/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2-d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.108 $\int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal. Leaf size=454

$$-\frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2dx(b^2-4ac)}+\frac{3b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{2f\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{d(b^2-4ac)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\cdots$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x^2]) - (2*
f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 -
4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((3*b^2 - 8*a*c
)*Sqrt[a + b*x + c*x^2])/(a^2*(b^2 - 4*a*c)*d*x) + (3*b*ArcTanh[(2*a + b*x)
/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(5/2)*d) + (f^2*ArcTanh[(b*Sqrt[d
] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f] + a
*f)^(3/2)) + (f^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[
f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d
^(3/2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rubi [A] time = 1.19335, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6725, 740, 806, 724, 206, 975, 1033}

$$-\frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2dx(b^2-4ac)}+\frac{3b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{2f\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{d(b^2-4ac)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```


```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x^2]) - (2*
f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 -
4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((3*b^2 - 8*a*c
)*Sqrt[a + b*x + c*x^2])/(a^2*(b^2 - 4*a*c)*d*x) + (3*b*ArcTanh[(2*a + b*x)
/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(5/2)*d) + (f^2*ArcTanh[(b*Sqrt[d
] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f] + a
*f)^(3/2)) + (f^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[
```

```
f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]))/(2*d
^(3/2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 975

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*
f))*x)*(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^(q + 1))/((b^2 - 4*a*c)*(b^2*d
*f + (c*d - a*f)^2)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int\left(\frac{1}{dx^2\left(a+bx+cx^2\right)^{3/2}} + \frac{f}{d\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx}{d} + \frac{f\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx}{d}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)\cdots\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)\cdots\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)\cdots\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}\cdots}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)dx\sqrt{a+bx+cx^2}} - \frac{2f\left(b\left(b^2f-c(cd+3af)\right)-c\left(2c^2d-b^2f+2acf\right)\cdots\right.}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}\cdots}$$

<u>Mathematica</u> [A] time = 1.34166, size = 488, normalized size = 1.07

$$\frac{\frac{2\left(8ac-3b^2\right)\sqrt{a+x(b+cx)}}{a^2x} + \frac{3b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{5/2}} - \frac{4f\left(-bc(3af+cd)-2c^2x(af+cd)+b^2cfx+b^3f\right)}{\sqrt{a+x(b+cx)}\left(b^2df-(af+cd)^2\right)} - f^2\left(\frac{\left(b^2-4ac\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\tanh^{-1}\left(\frac{2a\sqrt{f}-\cdots}{2\sqrt{a+x(b+c\cdots}}\right)}{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\cdots\right.}{2d\left(b^2-4ac\right)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] ((4*(b^2 - 2*a*c + b*c*x))/(a*x*Sqrt[a + x*(b + c*x)]) - (4*f*(b^3*f - b*c*
(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*d + a*f)*x))/((b^2*d*f - (c*d + a*f)^2
)*Sqrt[a + x*(b + c*x)]) + (2*(-3*b^2 + 8*a*c)*Sqrt[a + x*(b + c*x)])/(a^2*
x) + (3*b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)
])])/a^(5/2) - (f^2*(((b^2 - 4*a*c)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh
[(-(b*Sqrt[d]) + 2*a*Sqrt[f] - 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d - b
```

```
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f] + ((-b^2 + 4*a*c)*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(b*Sqrt
[d] + 2*a*Sqrt[f] + 2*c*Sqrt[d]*x + b*Sqrt[f]*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))
/(Sqrt[d]*(-(b^2*d*f) + (c*d + a*f)^2)))/(2*(b^2 - 4*a*c)*d)
```

Maple [B] time = 0.25, size = 1656, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x)
```

```
[Out] 1/2*f^2/d/(d*f)^(1/2)/(-b*(d*f)^(1/2)+a*f+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)
+2*f/d/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*x*
c^2-f^2/d/(d*f)^(1/2)/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*f
+c*d))^(1/2)*x*b*c+f/d/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x+(d*f)^(1/2)
/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+a*
f+c*d))^(1/2)*b*c-1/2*f^2/d/(d*f)^(1/2)/(-b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2
)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*b^2-1/2*f^2/d/(d*f)^(1/2)/(-b*(d*f)^(1/2)+a*
f+c*d)/(1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+a*f+c*d
)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+a*f+c
*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+1/f*(-b*(d*f)^(1/2)+a*f+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-1/d/a/x/(c*x^2+
b*x+a)^(1/2)-3/2/d*b/a^2/(c*x^2+b*x+a)^(1/2)+3/d*b^2/a^2/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)*x*c+3/2/d*b^3/a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+3/2/d*b/a^(
5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-8/d*c^2/a/(4*a*c-b^2)/(c
*x^2+b*x+a)^(1/2)*x-4/d*c/a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*b-1/2*f^2/d/(d*
f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*
f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)+2*f/d/(b*(d*f)^(1/2
)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*c^2+f^2/d/(d*f)^(1/2)/(b*(d
*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*x*b*c+f/d/(b*(d*f)^(1
/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-
(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b*c+1/2*f^2/d/(d*f)^(1/2)/(
b*(d*f)^(1/2)+a*f+c*d)/(4*a*c-b^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+
```

```
b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*b^2+1/2*f^2/d/(d*
f)^(1/2)/(b*(d*f)^(1/2)+a*f+c*d)/((b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2)*ln((2*(b
*(d*f)^(1/2)+a*f+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*
f)^(1/2)+a*f+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+a*f+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2-d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] sage2
```

3.109 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=761

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3}-\frac{\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e^2-4df}+a\right.\right.\right.}{}$$

```
[Out] -((4*c*e - b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*c*f^2) - ((b^2*f^2 + 4*
c*f*(b*e - a*f) - 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(8*c^(3/2)*f^3) - ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*
Sqrt[e^2 - 4*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2
- 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2
 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sq
rt[e^2 - 4*d*f]))]) + ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d
*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f
]) - b*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])))*Ar
cTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f
]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt
[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt
[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f
]))])
```

Rubi [A] time = 3.13535, antiderivative size = 761, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1067, 1076, 621, 206, 1032, 724}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3}-\frac{\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e^2-4df}+a\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -((4*c*e - b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*c*f^2) - ((b^2*f^2 + 4*
c*f*(b*e - a*f) - 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(8*c^(3/2)*f^3) - ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*
Sqrt[e^2 - 4*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2
- 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2
 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sq
rt[e^2 - 4*d*f]))]) + ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d
*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f
]) - b*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])))*Ar
cTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f
]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt
[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt
[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f
]))])
```

<u>Rule 1067</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((C*(b*f*p - c*e*(2*p +
q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), x] - Dist[1/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[
p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q +
1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(-2*A*f)*(2*p + 2*
q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3
)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2
) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c
^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

<u>Rule 1076</u>

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}d\left(4bce-b^2f-4acf\right)-\frac{1}{4}\left(8c^2de-b^2ef-4acef+4bc\left(e^2-2df\right)\right)x+\frac{1}{4}\left(b^2f^2+\right.}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}}{2cf^2}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}df\left(4bce-b^2f-4acf\right)-\frac{1}{4}d\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)+\left(\frac{1}{4}f\left(-8c^2\right.\right.}{\sqrt{a+bx+cx^2}(}}{2cf}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,\right.}{4cf^3}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+}}\right.}{8c^{3/2}f^3}$$

$$= -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2} - \frac{\left(b^2f^2+4cf(be-af)-8c^2\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+}}\right.}{8c^{3/2}f^3}$$

Mathematica [A] time = 2.39381, size = 552, normalized size = 0.73

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(4cf(af-be)-b^2f^2+8c^2\left(e^2-df\right)\right)}{8c^{3/2}f^3} + \frac{f\sqrt{e^2-4df}\sqrt{a+x(b+cx)}(bf-4ce+2cfx)+\sqrt{2}c\left(e\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] ((-(b^2*f^2) + 4*c*f*(-(b*e) + a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(8*c^(3/2)*f^3) + (f*Sqrt[e^2 - 4*d*f
]*(-4*c*e + b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + Sqrt[2]*c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f
 - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])
*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b +
 c*x)])] + Sqrt[2]*c*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*ArcTanh
[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*
x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-
```

```
e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(4*c*f^3*Sqrt[e^2 - 4*d*f
])
```

Maple [B] time = 0.312, size = 14815, normalized size = 19.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.110 $\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=549

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+ \ldots$$

```
[Out] Sqrt[a + b*x + c*x^2]/f - ((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - ((2*d*f*(c*e - b*f) + (e - Sqrt[e^2
 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*
a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*f*(c*e - b*f) + (e + Sqrt[e
^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[
e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^
2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x
 + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 7.02783, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1019, 1076, 621, 206, 1032, 724}

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+ \ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/f - ((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - ((2*d*f*(c*e - b*f) + (e - Sqrt[e^2
 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*
```

```
a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*f*(c*e - b*f) + (e + Sqrt[e
^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[
e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^
2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x
 + c*x^2])]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
```

```
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bd}{2}+\frac{1}{2}(2cd+be-2af)x+\frac{1}{2}(2ce-bf)x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)+\left(\frac{1}{2}f(2cd+be-2af)-\frac{1}{2}e(2ce-bf)\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{(2ce-bf)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2f^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)\right)-\left(\frac{1}{2}f(2\right.\right.}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} + \frac{\left(2\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)\right)-\left(\frac{1}{2}f(2cd+b\right.\right.\right.}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{\left(2f(cde-bdf)+\left(e-\sqrt{e^2-4df}\right)\left(f(be-\right.\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{}}$$

Mathematica [A] time = 1.94204, size = 496, normalized size = 0.9

$$\frac{4f\sqrt{e^2-4df}\sqrt{a+x(b+cx)}-\sqrt{2}\left(\sqrt{e^2-4df}+e\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(2*
Sqrt[c]*f^2) + (4*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] - Sqrt[2]*(e +
Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f -
b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x
- b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*
x)])] - Sqrt[2]*(-e + Sqrt[e^2 - 4*d*f])*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^
2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c*(
-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*
Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2
 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(4*f^2*Sqrt[e^2 - 4*d*f])
```

Maple [B] time = 0.321, size = 10138, normalized size = 18.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.111 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal. Leaf size=431

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+$$

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
 + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

Rubi [A] time = 0.649644, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {989, 621, 206, 1032, 724}

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
 + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
```

```
- 4*d*f])
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{\cdots}{\left(e-\sqrt{e^2-4df}+2\cdots\right.}\cdots}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(2\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\right)\mathrm{Subst}\left(\int \frac{\cdots}{16af^2-8bf(e\cdots}\cdots\right.}{f\sqrt{e^2\cdots}}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\tanh^{-1}\cdots}{\sqrt{2}f\sqrt{e^2-4df}}$$

Mathematica [A] time = 0.764498, size = 417, normalized size = 0.97

$$\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)\cdots}{\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*
f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a
*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcT
anh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2
*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

Maple [B] time = 0.32, size = 6019, normalized size = 14.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.112 $\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=523

$$\frac{\left(cd\left(e-\sqrt{e^2-4df}\right)-f\left(2bd-a\left(\sqrt{e^2-4df}+e\right)\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(cd\left(\sqrt{e^2-4d\ldots\right.\right.}{\ldots}$$

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) + ((c
*d*(e - Sqrt[e^2 - 4*d*f]) - f*(2*b*d - a*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh
[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x
)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2
- 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) -
 ((c*d*(e + Sqrt[e^2 - 4*d*f]) - f*(2*b*d - a*(e - Sqrt[e^2 - 4*d*f])))*Arc
Tanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]
))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))
])
```

Rubi [A] time = 3.70144, antiderivative size = 521, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {6728, 734, 843, 621, 206, 724, 1019, 1076, 1032}

$$-\frac{\left(-af\left(\sqrt{e^2-4df}+e\right)+2bdf-cd\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(-af\left(e-\ldots\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - ((2
*b*d*f - c*d*(e - Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh
[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x
)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2
- 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2
```

```
 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) +
 ((2*b*d*f - a*f*(e - Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f]))*Arc
Tanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]
))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))
])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d} + \frac{\int \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx}{d}$$

$$= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} - \frac{\int \frac{-\frac{1}{2}(bd-2ae)f-\frac{1}{2}f(2cd-be-2af)x+\frac{1}{2}bf^2x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{df}$$

$$= \frac{a\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{\int \frac{-\frac{1}{2}bdf^2-\frac{1}{2}(bd-2ae)f^2+\left(-\frac{1}{2}bef^2-\frac{1}{2}f^2(2cd-be-2af)\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{df^2}$$

$$= -\frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(2bdf-af\left(e-\sqrt{e^2-4df}\right)-cd\left(e+\sqrt{e^2-4df}\right)\right)\int}{d\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(2\left(2bdf-af\left(e-\sqrt{e^2-4df}\right)-cd\left(e+\sqrt{e^2-4df}\right)\right)\right)\mathrm{Subst}\Big(}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(2bdf-cd\left(e-\sqrt{e^2-4df}\right)-af\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\Big(\frac{}{2\sqrt{2}}}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2}}$$

<u>Mathematica</u> [A] time = 1.38877, size = 454, normalized size = 0.87

$$\frac{\left(\sqrt{e^2-4df}-e\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2}\right.}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(\right.}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/d) + ((-
e + Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*
f - b*(e + Sqrt[e^2 - 4*d*f]))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f]
)*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b
+ c*x)])] + (e + Sqrt[e^2 - 4*d*f])*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4
```

```
*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(4*a*f + 2*c*(-e +
Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[
f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*
d*f])]*Sqrt[a + x*(b + c*x)])])/(2*Sqrt[2]*d*f*Sqrt[e^2 - 4*d*f])
```

Maple [B] time = 0.303, size = 6460, normalized size = 12.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 + ex + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.113 $\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=736

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f(\cdots}{}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/d - (b*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(2*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2) - (f*(2*c*d^2 - b*d*(e + Sqrt[e^2
- 4*d*f]) + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e -
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*(2*c*d^2 -
b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]
)
```

Rubi [A] time = 3.47554, antiderivative size = 736, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6728, 732, 843, 621, 206, 724, 734, 1019, 1076, 1032}

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/d - (b*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(2*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2) - (f*(2*c*d^2 - b*d*(e + Sqrt[e^2
- 4*d*f]) + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e -
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*(2*c*d^2 -
b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]
)
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
```

```
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1019

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d
+ e*x + f*x^2)^(q + 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2} - \frac{e\sqrt{a+bx+cx^2}}{d^2x} + \frac{\left(e^2-df+efx\right)\sqrt{a+bx+cx^2}}{d^2\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{\left(e^2-df+efx\right)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx}{d^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx}{d} - \frac{e\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{e\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d^2} - \frac{\int \frac{\frac{1}{2}f\left(bde-2ae^2+2adf\right)+\frac{1}{2}f\left(2cde-be^2+2bdf-\right.}{\sqrt{a+bx+cx^2}\left(d+ex+f\right.}}{d^2f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{(ae)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2} - \frac{(be)\int \frac{1}{\sqrt{a+b}}}{2d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{d}$$

Mathematica [A] time = 1.83877, size = 520, normalized size = 0.71

$$\frac{(2ae-bd)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{2\sqrt{a}d^2} - \frac{4df\sqrt{e^2-4df}\sqrt{a+x(b+cx)} + \sqrt{2}x\left(e\sqrt{e^2-4df}+2df-e^2\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-\right.\right.}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] ((-(b*d) + 2*a*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/(
2*Sqrt[a]*d^2) - (4*d*f*Sqrt[e^2 - 4*d*f]*Sqrt[a + x*(b + c*x)] + Sqrt[2]*(
-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*x*ArcTanh[(4*a*f - 2*c*(e + Sq
rt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]
*Sqrt[a + x*(b + c*x)])] + Sqrt[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*Sqrt
[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*
f]))]*x*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(4*d^2*f*S
qrt[e^2 - 4*d*f]*x)
```

Maple [B] time = 0.313, size = 6765, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 + ex + d\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(x**2*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.114 $\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=545

$$-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+$$

```
[Out] Sqrt[a + b*x + c*x^2]/(c*f) - (e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[c]*f^2) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*c^(3/2)*f) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f
]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
+ ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e +
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 3.72213, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6728, 621, 206, 640, 1032, 724}

$$-\frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```


```
[Out] Sqrt[a + b*x + c*x^2]/(c*f) - (e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[c]*f^2) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*c^(3/2)*f) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f
]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
```

```
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
+ ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e +
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int\left(-\frac{e}{f^2\sqrt{a+bx+cx^2}} + \frac{x}{f\sqrt{a+bx+cx^2}} + \frac{de+\left(e^2-df\right)x}{f^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{de+\left(e^2-df\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{e\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2} + \frac{\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{(2e)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2cf} + \frac{(2def}{}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf} - $$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{\left(2def-(e\right.}{}$$

Mathematica [A] time = 2.47293, size = 550, normalized size = 1.01

$$-\frac{\frac{bf\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{\sqrt{2}\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}\left(-\frac{e\left(e^2-3df\right)}{\sqrt{e^2-4df}}-\right.}{}}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((-2*f*Sqrt[a + x*(b + c*x)])/c + (2*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + x*(b + c*x)])])/Sqrt[c] + (b*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 x*(b + c*x)])])/c^(3/2) + (Sqrt[2]*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f]
```

```
- d*f*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b
*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]
)])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*
f - b*(e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(e^2 - d*f - (e*(e^2 - 3*d*f))/
Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e
+ Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])
/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 -
 4*d*f]))])/(2*f^2)
```

Maple [B] time = 0.321, size = 3131, normalized size = 5.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] (c*x^2+b*x+a)^(1/2)/c/f-1/2/f*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))-1/f^2*e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/2/f^
2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(
1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d-1/2/f^3*2^(1/2
)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e
^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e
^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2-3/2/f^2/(-4*d*f+e^2)
^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f
```

```
+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d*e+1/2/f^3
/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
)*e^3+1/2/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*
f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b
*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*
d-1/2/f^3*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*
e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^2+
3/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b
*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f
+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f))*d*e-1/2/f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b
*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4
*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2
^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*
f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/
2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f))*e^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.115 $\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=463

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\right.}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{}}}$$

```
[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f) - ((e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*
a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2
*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*
c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 3.43564, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1077, 621, 206, 1032, 724}

$$-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\tanh^{-1}\left(\right.}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f) - ((e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*
a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2
```

```
*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2]])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*
c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 1077

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
 + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a}\cdots}}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{\left(2\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\mathrm{Subst}\left(\int \frac{\cdots}{16af^2-8bf\left(e-\sqrt{e^2-4\cdots}\right.}\right.}{f\cdots}$$

$$= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+\cdots}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2\cdots}}\right.}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce\cdots}}$$

<u>Mathematica</u> [A] time = 1.21055, size = 468, normalized size = 1.01

$$\frac{\frac{\sqrt{2}\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}\left(\frac{2df-e^2}{\sqrt{e^2-4df}}+e\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{\cdots}\right.}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\cdots}}\right.}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+\cdots\right.}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + (Sqrt
[2]*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2
- 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a
 + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(e + (-e^2 + 2*d
*f)/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*
(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)
])])/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e
^2 - 4*d*f]))])/(2*f)
```

Maple [B] time = 0.342, size = 2321, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] 1/f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/2/f^2*2^(1/2)/(((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-
2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*
c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e+1/f/(-4*d*f+e^2)^(1/2)*2^(1
/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c
*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d
*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c
*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*d-1/2/f^2/(-4*d*f+e^2)
^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e^2+1/2/f^2
*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*e-1/f/(-4*d*f
+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
```

```
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*d+
1/2/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b
*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f
+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f))*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + bx + cx^2}\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.116 $\int \frac{x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=402

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4\cdots\right.\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e\cdots}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)\cdots}}$$

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*
f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]
*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 0.962864, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1032, 724, 206}

$$\frac{\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4\cdots\right.\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e\cdots}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*
f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]
*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
```

```
2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = -\left(\left(-1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx\right)+\left(1-\frac{e}{\sqrt{e^2-4}}\right.$$

$$= -\left(\left(2\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4d}\right.}\right.\right.$$

$$= -\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} - \frac{\left(1+\frac{e}{\sqrt{e^2-4}}\right.}{\sqrt{2}}$$

Mathematica [A] time = 1.01437, size = 407, normalized size = 1.01

$$\frac{-\frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-\right.}}\right.}{\sqrt{f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (-(((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x
- b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*
x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2
*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])) - ((1 - e/Sqrt[e^2 - 4*d*f])*ArcTanh[(
4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/Sqrt[2]
```

Maple [B] time = 0.326, size = 1516, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -1/2/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+1/2/(-4*d*f
+e^2)^(1/2)/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/
```

```
f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-
4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*e-1/
2/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)
^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2
)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f))*e-1/2/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2
)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 48.0673, size = 23019, normalized size = 57.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2
*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
 + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 + sqrt(2
)*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e
^2)*f - (2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^
3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*
(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)
*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c
^3*d^5 - 12*b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 -
10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e
 + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6
- 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a
^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)
*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2
*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^
2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2
 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2
 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c
*x^2 + b*x + a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*
c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^
2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt(
(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*
d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*
d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*
d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4
- 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^
2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2
)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e
 + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
- (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*
x - (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*
(4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4
```

```
- 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e
^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b
*d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*
d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)
/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f
^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*
a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a
*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*
c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*
e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4
*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c -
 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*
sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^
4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*
d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b
*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*
e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 -
8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*
c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*
(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 2
2*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)
*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3
*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c
^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(
b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*
d*e^2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 - sqrt(2)*(b^2*d^2*e
^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f - (2*c
^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 +
(b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a
^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^
3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*
b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^
2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/
(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^
5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a
^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*
b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c
^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e
^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*
d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c -
5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x +
 a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*
c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2
*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*
```

c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^
2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*
a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 -
(8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c
+ 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*
e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)
*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f))
)/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 -
4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a
*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x - (2*a*c^2
*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2
*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^
3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d
^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(
b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3
- 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^
2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^
2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*
f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b
*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8
*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c
^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*
d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4*sqrt(2)*sqrt((2*c*d^
2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*
f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*
d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e
^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*
d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 -
2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4
*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2
*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d
^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*
c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*
c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 -
b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)
*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*l
og(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 + sqrt(2)*(b^2*d^2*e^2 - 2*a*b*d
*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f + (2*c^3*d^4*e^2 -
3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a
*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f
^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)
*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4*e
+ a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*(a
*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2

- 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f ^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b* c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8* (b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^ 3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^ 3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d ^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((2* c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^ 2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4* b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a ^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4* a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b ^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^ 2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2 *(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e ^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2* a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d^3*e^2 - 2 *a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e ^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2*b* d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a*b *c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b* c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 - 6*a*b*c)* d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3* d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3) *d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3 *b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a* b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b *c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a ^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2* (a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b *d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b* e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4 *e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2 *c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^ 2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^ 2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4* b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log(-(2*b^2*d^3 - 4*a*b*d^2*e + 2*a^2*d*e^2 - sqrt(2)*(b^2*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4 - 4*(b^2*d^3 - 2*a*b*d^2*e + a^2*d*e^2)*f + (2*c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 - 2*a*b*c*d*e^5 + a^2*c*e^6 + 8*a^3*d^2*f^4 + (b^2*c + 3*a*c^2)*d^2*e^4 - 2*(2*a^2*b*d^2*e + 3*a^3*d*e^2 - 4*(a*b^2 - 3*a^2*c)*d^3)*f^3 + (5*a^2*b*d*e^3 + a^3*e^4 - 8*(b^2*c - 3*a*c^2)*d^4 + 4*(b^3 - 2*a*b*c)*d^3*e - 2*(5*a*b^2 - 11*a^2*c)*d^2*e^2)*f^2 - (8*c^3*d^5 - 12*b*c^2*d^4*e + a^2*b*e^5 + 2*(b^2*c + 9*a*c^2)*d^3*e^2 + (b^3 - 10*a*b*c)*d^2*e^3 - 2*(a*b^2 - 4*a^2*c)*d*e^4)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((2*c*d^2 - b*d*e + a*e^2 - 2*a*d*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) + (4*b*c*d^3 + a*b*d*e^2 - (b^2 + 4*a*c)*d^2*e)*x + (2*a*c^2*d^3*e^2 - 2*a*b*c*d^2*e^3 + 2*a^2*c*d*e^4 - 8*a^3*d^2*f^3 + 2*(4*a^2*b*d^2*e + a^3*d*e^2 - 4*(a*b^2 - 2*a^2*c)*d^3)*f^2 - 2*(4*a*c^2*d^4 - 4*a*b*c*d^3*e + a^2*b*d*e^3 - (a*b^2 - 6*a^2*c)*d^2*e^2)*f + (b*c^2*d^3*e^2 - b^2*c*d^2*e^3 + a*b*c*d*e^4 - 4*a^2*b*d^2*f^3 + (4*a*b^2*d^2*e + a^2*b*d*e^2 - 4*(b^3 - 2*a*b*c)*d^3)*f^2 - (4*b*c^2*d^4 - 4*b^2*c*d^3*e + a*b^2*d*e^3 - (b^3 - 6*a*b*c)*d^2*e^2)*f)*x)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 -

```
(a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 +
 (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2
*a^2*c^2)*d*e^4)*f)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.117 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=374

$$\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 0.313594, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {983, 724, 206}

$$\frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2} f \tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 983

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((
b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q
 + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b
^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(4f)\,\mathrm{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e-\sqrt{e^2-4df}\right)-\left(-2bf+2c \cdots\right.}{\sqrt{a+bx+cx^2}} \cdots\right.}{\sqrt{e^2-4df}} \cdots$$

$$= -\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}f\tanh^{-1}\left(\frac{\cdots}{2\sqrt{2}\cdots}\right.}{\sqrt{e^2-4df}\sqrt{ce^2-\cdots}}$$

Mathematica [A] time = 0.805188, size = 376, normalized size = 1.01

$$\frac{\sqrt{2} f \left(\frac{\tanh^{-1}\left(\frac{4af - b\left(\sqrt{e^2-4df}+e-2fx\right) - 2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*f*(ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^
2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] -
 ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f
] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])]/Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sq
rt[e^2 - 4*d*f]
```

Maple [B] time = 0.319, size = 761, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f))+1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
```

```
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e
^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*
e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 51.6038, size = 22873, normalized size = 61.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
```

^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*
a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^
2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^
2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*
a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*
d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^
3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)
*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^
3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d
^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
e^3 + 2(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a
^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2
*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a
*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2
 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*
a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*
d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (
b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)
*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 -
 a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^
3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d
*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*
a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*
a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a
*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e +
 a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*
b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c
*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3
*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^
2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*
e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c
^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)
*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*
b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*
b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*
c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*

```
e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^
2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^
2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*
f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b
*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8
*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c
^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*
d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*
c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d -
a*b*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*
b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d
^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3
*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^
4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3
*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e
^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f
+ b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^
2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*
c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2
*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2
- 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2
+ (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*
x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c
*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2
)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((
c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d
*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d
*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d
*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 -
 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
+ 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2
 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)
*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e
+ a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
 (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^
2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 -
4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 -
 (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)
*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3
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+ (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2 *b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d ^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c ^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2* c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4 *sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^ 2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a ^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)* f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a* b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^ 3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a *c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2* e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b *d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b ^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3 *e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d ^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^ 2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e ^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2* e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^ 3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2* b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d *e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b* c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^ 2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b *d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b* e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4 *e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2 *c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^ 2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^

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2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4
- 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(
a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*
b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^
2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 +
 a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 -
(4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e
- a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3
*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c
^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^
2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^
2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2
*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^
4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^
3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e
^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3
 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b
^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)
*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*
e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c
*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2
*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d
 + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e +
 a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
(b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 -
 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 +
 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4
 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)
*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3
- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*
d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2
*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e
^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2
*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b
*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e
^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*
e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^
3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 -
 a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*
b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4
- a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b
^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*
a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b
^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
```

```
 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2
*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (
b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2
 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 -
 (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(
a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a
*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f
+ (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 +
(4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*
c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^
2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^
2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2
)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*
(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b
^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx + cx^2}\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.118 $\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=451

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{\right.\right.}}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-}}$$

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) + (f*
(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
 - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*
d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sq
rt[e^2 - 4*d*f]]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt
[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 2.63164, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6728, 724, 206, 1032}

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{\right.\right.}}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) + (f*
(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
 - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*
d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sq
rt[e^2 - 4*d*f]]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt
[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*
```

```
x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} + \frac{-e-fx}{d\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\right) dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{d}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+}}}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{\left(2f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c(}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c(\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)}}\right.}{\sqrt{2}d\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)}}$$

Mathematica [A] time = 2.61047, size = 450, normalized size = 1.

$$\frac{\sqrt{2}f\left(\frac{\left(\sqrt{e^2-4df}-e\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\left(\sqrt{e^2-4df}+e\right)\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2d\right.}}\right)}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\frac{\sqrt{e^2-4df}}{2d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/Sqrt[a] + (Sqr
t[2]*f*(((-e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*
f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(
b + c*x)])])/Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))] + ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f + 2*c*(-e +
 Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt
[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + x*(b + c*x)])])/Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*
```

```
f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sqrt[e^2 - 4*d*f])/(2*d)
```

Maple [B] time = 0.374, size = 859, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2
^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e
^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(
1/2))/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-2*f/(e+(-4*d*f+
e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/
2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*
d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4
*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+
b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.119 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=543

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2\cdots\right.}{\sqrt{\cdots}}$$

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*A
rcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*
f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqr
t[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 4.59176, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6728, 730, 724, 206, 1032}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2\cdots\right.}{\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*A
rcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*
f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqr
```

```
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqr
t[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}} - \frac{e}{d^2x\sqrt{a+bx+cx^2}} + \frac{e^2-df+efx}{d^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\right)$$

$$= \frac{\int \frac{e^2-df+efx}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{d^2} + \frac{\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} - \frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad} + \frac{(2e)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d^2} - \cdots$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} + \frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{f\left(e^2 \cdots\right.}{\cdots}$$

Mathematica [A] time = 1.59828, size = 533, normalized size = 0.98

$$-\frac{-\frac{bd\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} + \frac{\sqrt{2}f\left(e\sqrt{e^2-4df}+2df-e^2\right)\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\sqrt{2}f\left(\frac{e^2-2df}{\sqrt{e^2-4df}}+e\right)\tanh^{-1}\left(\cdots\right)}{\sqrt{f\left(2af+b\left(\cdots\right.\right.}}\cdots}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((2*d*Sqrt[a + x*(b + c*x)])/(a*x) - (b*d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*S
qrt[a + x*(b + c*x)])])/a^(3/2) - (2*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[
a + x*(b + c*x)])])/Sqrt[a] + (Sqrt[2]*f*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f
])*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f
] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*
f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]
*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 -
4*d*f]))]) + (Sqrt[2]*f*(e + (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*
f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2
```

```
*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sq
rt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]])])/Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/(2*d^2)
```

Maple [B] time = 0.344, size = 983, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln
((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1
/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))+16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4
*d*f+e^2)^(1/2))^2/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+4*
f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/
2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)
/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1
/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-
4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)
^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^
2)^(1/2))/a/x*(c*x^2+b*x+a)^(1/2)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^
2)^(1/2))*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\left(fx^2 + ex + d\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.120 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=679

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\cdots\right.}{\sqrt{2}d^3\sqrt{e^2-4d\cdots}}$$

```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x
) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (b*e*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d^2) - ((e^2 - d*f)*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3) + (f*(2
*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e
 - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*e^3 - 4*d*e*f - (
e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
(c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 11.2259, antiderivative size = 679, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6728, 744, 806, 724, 206, 730, 1032}

$$-\frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}+\frac{f\left(-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\cdots\right.}{\sqrt{2}d^3\sqrt{e^2-4d\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```



```
[Out] -Sqrt[a + b*x + c*x^2]/(2*a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d*x
) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (b*e*ArcTanh[(2*a +
```

```
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d^2) - ((e^2 - d*f)*Ar
cTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3) + (f*(2
*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e
 - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*e^3 - 4*d*e*f - (
e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
(c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \int\left(\frac{1}{dx^3\sqrt{a+bx+cx^2}} - \frac{e}{d^2x^2\sqrt{a+bx+cx^2}} + \frac{e^2-df}{d^3x\sqrt{a+bx+cx^2}} + \frac{-e\left(e^2-2\right.}{d^3\sqrt{a+bx}}\right.$$

$$= \frac{\int \frac{-e\left(e^2-2df\right)-f\left(e^2-df\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{d^3} + \frac{\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d^2} + \frac{\left(e^2-df\right)\int}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx}{2ad} + \frac{(be)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad^2} -$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\left(e^2-df\right)\tanh^{-1}\left(\frac{}{2\sqrt{a}}\right.}{\sqrt{a}d^3}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+c}}\right.}{2a^{3/2}d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{}{2}\right.}{8a^{5/2}d}$$

<u>Mathematica</u> [A] time = 2.53449, size = 669, normalized size = 0.99

$$\frac{\frac{d^2\left(\left(4acx-3b^2x\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)+6\sqrt{a}b\sqrt{a+x(b+cx)}\right)}{a^{5/2}x} - \frac{4bde\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{a^{3/2}} - \frac{4d^2\sqrt{a+x(b+cx)}}{ax^2} - \frac{8\left(e^2-df\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a}} - \frac{4\sqrt{2}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((-4*d^2*Sqrt[a + x*(b + c*x)])/(a*x^2) + (8*d*e*Sqrt[a + x*(b + c*x)])/(a*
x) - (4*b*d*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/a^(3/
2) - (8*(e^2 - d*f)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])
/Sqrt[a] + (d^2*(6*Sqrt[a]*b*Sqrt[a + x*(b + c*x)] + (-3*b^2*x + 4*a*c*x)*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + x*(b + c*x)])]))/(a^(5/2)*x) - (4*Sq
rt[2]*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f])*Arc
```

```
Tanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*
f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*
(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]) + (4*Sqrt[2]*f*(e^2 - d*f + (e*(e^2 - 3*d*f))/Sqrt[e^2 - 4*d*f])*ArcTa
nh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*
f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*
(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*(e^2 - 2*d*f - e
*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))])/(8*d^3)
```

Maple [B] time = 0.342, size = 1296, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -8*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln
((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1
/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(
x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-64*f^4/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d
*f+e^2)^(1/2))^3/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*d+64
*f^3/(-e+(-4*d*f+e^2)^(1/2))^3/(e+(-4*d*f+e^2)^(1/2))^3/a^(1/2)*ln((2*a+b*x
+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*e^2-8*f^3/(e+(-4*d*f+e^2)^(1/2))^3/(-4*d
*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*
f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b
*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+
2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/x^2*(c*x^2+b*x+a)^(1/2
)-3*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a^2/x*(c*x^2+b*x+a)^
(1/2)+3/2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b^2/a^(5/2)*ln((
2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-2*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-
```

```
4*d*f+e^2)^(1/2))*c/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1
6*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2/a/x*(c*x^2+b*x+a
)^(1/2)-8*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e^2)^(1/2))^2*b/a^(3/2
)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.121 $\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal. Leaf size=779

$$\frac{2\left(cx\left(\left(e^2-df\right)(abf-2ace+bcd)-de\left(-c(2af+be)+b^2f+2c^2d\right)\right)-\left(adf-ae^2+bde\right)\left(-c(2af+be)+b^2f+2c^2d\right)+\right.}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

```
[Out] (2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*e*(b + 2*c*x))
/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) + (2*(c*d*e*(b*c*d - 2*a*c*e + a
*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*((b
*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f) - d*e*(2*c^2*d + b^2*f - c*(b*e + 2*a*f
)))*x))/((b^2 - 4*a*c)*f^2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a
 + b*x + c*x^2]) + ((2*d*(b*d - a*e)*f + (e - Sqrt[e^2 - 4*d*f])*(c*d^2 - b
*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
- c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[2]*S
qrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*(b*d - a*e
)*f + (e + Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a
*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*
Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d
*f]]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 14.1698, antiderivative size = 779, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6728, 613, 636, 1016, 1032, 724, 206}

$$\frac{2\left(cx\left(\left(e^2-df\right)(abf-2ace+bcd)-de\left(-c(2af+be)+b^2f+2c^2d\right)\right)-\left(adf-ae^2+bde\right)\left(-c(2af+be)+b^2f+2c^2d\right)+\right.}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*e*(b + 2*c*x))
/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) + (2*(c*d*e*(b*c*d - 2*a*c*e + a
*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*((b
*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f) - d*e*(2*c^2*d + b^2*f - c*(b*e + 2*a*f
)))*x))/((b^2 - 4*a*c)*f^2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a
 + b*x + c*x^2]) + ((2*d*(b*d - a*e)*f + (e - Sqrt[e^2 - 4*d*f])*(c*d^2 - b
*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
- c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*S
qrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*(b*d - a*e
)*f + (e + Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a
*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*
Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d
*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
```

```
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \left(-\frac{e}{f^2\left(a+bx+cx^2\right)^{3/2}} + \frac{x}{f\left(a+bx+cx^2\right)^{3/2}} + \frac{de+\left(e^2-df\right)x}{f^2\left(a+bx+cx^2\right)^{3/2}\left(d+ex\right.}\right.$$

$$= \frac{\int \frac{de+\left(e^2-df\right)x}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{f^2} - \frac{e\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f^2} + \frac{\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f}$$

$$= \frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} + \frac{2\left(cde(bcd-2ace\right.}{}$$

$$= \frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} + \frac{2\left(cde(bcd-2ace\right.}{}$$

$$= \frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} + \frac{2\left(cde(bcd-2ace\right.}{}$$

$$= \frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}} + \frac{2\left(cde(bcd-2ace\right.}{}$$

Mathematica [A] time = 2.91359, size = 1066, normalized size = 1.37

$$\frac{\frac{4\left(2fa^3+(-2cd-be+2cex+bfx)a^2+b(b(d-ex)-3cdx)a+b^3dx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}} + \frac{\sqrt{2}\left(c\left(\sqrt{e^2-4df}-e\right)d^2+b\left(e^2-\sqrt{e^2-4df}e-2df\right)d+a\left(-e^3+\sqrt{e^2-4df}e^2+3dfe-df\sqrt{e^2-4df}\right)\right)\log\left(-e-\right.}{\sqrt{e^2-4df}\sqrt{c\left(e^2-\sqrt{e^2-4df}e-2df\right)+f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((4*(2*a^3*f + b^3*d*x + a^2*(-2*c*d - b*e + 2*c*e*x + b*f*x) + a*b*(-3*c*d
*x + b*(d - e*x))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*(c*d^2
*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e
^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-e + Sqr
t[e^2 - 4*d*f] - 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) + (Sqrt[2]*(c*d^2*(e
 + Sqrt[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 -
```

```
3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[e + Sqrt[e^2
- 4*d*f] + 2*f*x])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(c*d^2*(e + Sqrt
[e^2 - 4*d*f]) - b*d*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f
 + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[-4*a*f + 2*c*e*x + 2
*c*Sqrt[e^2 - 4*d*f]*x + b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*Sqrt[2]*Sqrt
[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f
]))]*Sqrt[a + x*(b + c*x)]])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqr
t[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]) - (Sqrt[2]*(c*d^2
*(-e + Sqrt[e^2 - 4*d*f]) + b*d*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a*(-e
^3 + 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f]))*Log[b*(-e +
Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*(2*a*f - c*e*x + c*Sqrt[e^2 - 4*d*f]*x + Sqr
t[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]))/(2
*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Maple [B] time = 0.359, size = 14651, normalized size = 18.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**3/((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.122 $\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=609

$$-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\tanh^{-1}\left(\frac{4\cdots}{2\sqrt{2}\sqrt{a\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\cdots}}$$

```
[Out] (-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/
((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x
^2]) - (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(
4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
- (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e + Sqrt[e^2
- 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*
d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 5.84168, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1061, 1032, 724, 206}

$$-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\tanh^{-1}\left(\frac{4\cdots}{2\sqrt{2}\sqrt{a\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/
((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x
^2]) - (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(
4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/
```

```
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
- (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e + Sqrt[e^2
- 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*
d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 1061

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)
*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a
*f)) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)
^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d -
a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1) + (b^2*(C*d + A*f) - b*((Plus[A])*c*e + a*C*e) + 2*(A*c*(c*
d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)
*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(A*c*e + a*C*e)
 + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4
)))*x - c*f*(b^2*(C*d + A*f) - b*(A*c*e + a*C*e) + 2*(A*c*(c*d - a*f) - a*(
c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1]
&& NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && I
LtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{-\frac{1}{2}\left(b^2-4ac\right)d(c}{\sqrt{a+b}}}{\left(b^2-4ac\right)\left((cd-\right.}$$

$$= -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{\left(f\left(2d(cd-af)-\right.\right.}{\sqrt{}}$$

$$= -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{\left(2f\left(2d(cd-af)\right.\right.}{}$$

$$= -\frac{2\left(a(bcd-2ace+abf)+c\left(b^2d-abe-2a(cd-af)\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{f\left(2d(cd-af)-\right.}{\sqrt{2}\sqrt{e^2-4df}\left(\right.}$$

Mathematica [A] time = 6.61074, size = 1097, normalized size = 1.8

$$\frac{16\sqrt{2}f\sqrt{ce^2-bfe-c\sqrt{e^2-4df}e+2af^2-2cdf+bf\sqrt{e^2-4df}}\left(e+\frac{2df-e^2}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)-\left(2c\left(e-\sqrt{e^2-4}\right.\right.}{2\sqrt{2}\sqrt{ce^2-bfe-c\sqrt{e^2-4df}e+2af^2-2cdf+bf}}\right.}{\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)\left(16af^2-8b\left(e-\sqrt{e^2-4df}\right)f+4c\left(e-\sqrt{e^2-4df}\right)^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(e - (e^2 - 2*d*f)/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e - Sqr
t[e^2 - 4*d*f]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2
))/((b^2 - 4*a*c)*f*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[
```

```
e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (2*(e + (e^2 - 2*d*f)/Sqrt[e^2
- 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(
e + Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*a*c)*f*(4*a*f^2 - 2
*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x
))^(3/2)) + (4*(b + 2*c*x)*(-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)))^(3/2))/
(c*f*(a + x*(b + c*x))^(3/2)*Sqrt[1 - (b + 2*c*x)^2/(b^2 - 4*a*c)]) + (16*S
qrt[2]*f*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b
*f*Sqrt[e^2 - 4*d*f]]*(e + (-e^2 + 2*d*f)/Sqrt[e^2 - 4*d*f])*(a + b*x + c*x
^2)^(3/2)*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c
*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((4*
a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(16*a*
f^2 - 8*b*f*(e - Sqrt[e^2 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x
*(b + c*x))^(3/2)) + (16*Sqrt[2]*f*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
 c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(e - (-e^2 + 2*d*f)/Sqrt[e^
2 - 4*d*f])*(a + b*x + c*x^2)^(3/2)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*
f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + b*x + c*x^2])])/((4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sq
rt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sq
rt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2))
```

Maple [B] time = 0.348, size = 11341, normalized size = 18.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2/((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.123 $\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=609

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{\cdots}{2\sqrt{2}\sqrt{\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\cdots}}$$

```
[Out] (2*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) + c*(b*c*d - 2*a*c*e + a*b*f)*x))
/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*
x^2]) + (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[
(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)
/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4
*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 5.64259, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1016, 1032, 724, 206}

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\tanh^{-1}\left(\frac{\cdots}{2\sqrt{2}\sqrt{\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```


```
[Out] (2*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) + c*(b*c*d - 2*a*c*e + a*b*f)*x))
/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*
x^2]) + (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[
(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)
/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
```

```
4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4
*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx = \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{\frac{1}{2}\left(b^2-4ac\right)d(c\cdots}{\sqrt{a+\cdots}}}{\left(b^2-4ac\right)\left((cd-\cdots\right.}$$

$$= \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{\left(f\left(2d(ce-bf)-\cdots\right.\right.}{\sqrt{\cdots}}$$

$$= \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{\left(2f\left(2d(ce-bf)\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(a\left(2c^2d-bce+b^2f-2acf\right)+c(bcd-2ace+abf)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{f\left(2d(ce-bf)-\cdots\right.}{\sqrt{2}\sqrt{e^2-4df}\left(\left(\cdots\right.\right.}$$

Mathematica [A] time = 6.33336, size = 983, normalized size = 1.61

$$-\frac{16\sqrt{2}\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\sqrt{ce^2-bfe-c\sqrt{e^2-4df}e+2af^2-2cdf+bf\sqrt{e^2-4df}}\left(cx^2+bx+a\right)^{3/2}\tanh^{-1}\left(\frac{4af-b(e-\cdots}{2\sqrt{2}\sqrt{ce^2-bfe-c\sqrt{\cdots}}}\right.}{\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)\left(16af^2-8b\left(e-\sqrt{e^2-4df}\right)f+4c\left(e-\sqrt{e^2-4d\cdots}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(1 - e/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e - Sqrt[e^2 - 4*d*f
]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*
a*c)*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2
)*(a + x*(b + c*x))^(3/2)) + (2*(1 + e/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*
f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)*
```

```
(a + b*x + c*x^2))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f])
+ c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (16*Sqrt[2]*f^2*(
1 - e/Sqrt[e^2 - 4*d*f])*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[
e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2)*ArcTanh[(4*a*
f - b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - Sqrt[e^2 - 4*d*f]))*x)/(
2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] +
b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((4*a*f^2 - 2*b*f*(e - Sqrt
[e^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e - Sqrt[e
^2 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (1
6*Sqrt[2]*f^2*(1 + e/Sqrt[e^2 - 4*d*f])*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*
f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2
)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2
- 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[
e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((4*a*f^2 -
2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*
b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*
x))^(3/2))
```

Maple [B] time = 0.352, size = 7163, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + bx + cx^2\right)^{\frac{3}{2}}\left(d + ex + fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x/((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.124 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=666

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f(f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2-(bd-ae)(c}$$

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rubi [A] time = 1.74594, antiderivative size = 666, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f(f(2af-b(\sqrt{e^2-4df}+e))+c(e\sqrt{e^2-}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2-(bd-ae)(c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
```

```
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{\frac{1}{2}(b}{}}{\left(b^2-\right.}$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} + \frac{\left(f\left(2f\right.\right.}{}$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{\left(2f\left(2f\right.\right.}{}$$

$$= \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} - \frac{f\left(c\left(e^2\right.\right.}{}$$

Mathematica [A] time = 6.57271, size = 700, normalized size = 1.05

$$2f\left(-\frac{2\left(-2c\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)+2b^2f-bc\left(\sqrt{e^2-4df}+e-2fx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(4af^2-2bf\left(\sqrt{e^2-4df}+e\right)+c\left(\sqrt{e^2-4df}+e\right)^2\right)} + \frac{2c\left(cx\left(\sqrt{e^2-4df}-e\right)-2af\right)+2b^2f+bc\left(\sqrt{e^2-4df}-e+2fx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)} + \frac{\sqrt{2}f^2\,\mathrm{ta}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e
 + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
```

```
*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Sqrt[a + x*(b + c*x)]) - (2*
(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e
^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) +
 c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f^2*ArcTanh
[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2) - (Sqrt[2]*f^2*Sq
rt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(-e^2 +
 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f])))^2))
/Sqrt[e^2 - 4*d*f]
```

Maple [B] time = 0.333, size = 4099, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 2/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-b*e*f-2*c*d*f+c*e^2)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-4*
f/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*x*c^2-4/(-4*d*f+e^2)^(1/2)*f^2/((-4*d*f+e^2
)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^
2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/f^2)^(1/2)*x*b*c+4/(-4*d*f+e^2)^(1/2)*f/((-4*d*f+e^2)^(1/2)*b*f-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^
2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*x*c^2*e-2*f/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-
```

```
2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2
*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*b*c-2/(-4*d*f+e^2)^(1/2)*f^2/((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(
4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2)*b^2+2/(-4*d*f+e^2)^(1/2)*f/((-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^
2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*
c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2)^(1/2)*b*c*e-2/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((((-4*d*f+e^2)^(1/2)*b*f-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f
+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)
*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f))-2/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2)^(1/2)-4*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e
^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/
2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*x*c^2+4/(-4*d
*f+e^2)^(1/2)*f^2/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2
)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*x*b*c-4/(-4*d*f+e^2)^(1/
2)*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*x*c^2*e-2*f/(-(-4*d*f+e^2)^(1/2)*b*f+
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^
2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f
*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4
*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
```

```
)^(1/2)*b*c+2/(-4*d*f+e^2)^(1/2)*f^2/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(
-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/
2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*b^2-2
/(-4*d*f+e^2)^(1/2)*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2
-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*b*c*e+2/(-4*d*f+e^2)
^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)*f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*
f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.125 $\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal. Leaf size=816

$$\frac{2\left(b^2+cxb-2ac\right)}{a\left(b^2-4ac\right)d\sqrt{cx^2+bx+a}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d}+\frac{f\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)-2\left(f\left(be^2-afe\cdots\right.\right.\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(c\cdots\right.}$$

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) + (2*(c
*e*(2*a*c*e - b*(c*d + a*f)) + (b*e - a*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*
f)) + c*(2*c^2*d*e + b*f*(b*e - a*f) - b*c*(e^2 + d*f))*x))/((b^2 - 4*a*c)*
d*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) + (f*((e - Sqr
t[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)) - 2*(f*(b*e^2 - b*d*f - a*e
*f) - c*(e^3 - 2*d*e*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b
*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2
]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*((e + Sq
rt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)) - 2*(f*(b*e^2 - b*d*f - a*
e*f) - c*(e^3 - 2*d*e*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[
2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rubi [A] time = 15.9158, antiderivative size = 814, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6728, 740, 12, 724, 206, 1016, 1032}

$$\frac{2\left(b^2+cxb-2ac\right)}{a\left(b^2-4ac\right)d\sqrt{cx^2+bx+a}}-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d}-\frac{f\left(2f\left(be^2-afe-bdf\right)-2c\left(e^3-2def\right)-\left(e-\sqrt{e^2-4df}\right)\left(f\cdots\right.\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) + (2*(c
*e*(2*a*c*e - b*(c*d + a*f)) + (b*e - a*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*
f)) + c*(2*c^2*d*e + b*f*(b*e - a*f) - b*c*(e^2 + d*f))*x))/((b^2 - 4*a*c)*
d*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f*(2*f*(b*e
^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e - Sqrt[e^2 - 4*d*f])*(f*(b*e
 - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b
*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2
]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*f*(b*
e^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e + Sqrt[e^2 - 4*d*f])*(f*(b*
e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[
2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}} + \frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$= \frac{\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx}{d} + \frac{\int \frac{-e-fx}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{d}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} + \frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2\right.\right.}{\left(b^2-4ac\right)d\left((cd-af)^2\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} + \frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2\right.\right.}{\left(b^2-4ac\right)d\left((cd-af)^2\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} + \frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2\right.\right.}{\left(b^2-4ac\right)d\left((cd-af)^2\right.}$$

$$= \frac{2\left(b^2-2ac+bcx\right)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} + \frac{2\left(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2\right.\right.}{\left(b^2-4ac\right)d\left((cd-af)^2\right.}$$

Mathematica [A] time = 6.64223, size = 1121, normalized size = 1.37

$$\frac{16\sqrt{2}\left(\frac{ef}{\sqrt{e^2-4df}}+f\right)\sqrt{ce^2-bfe-c\sqrt{e^2-4df}\,e+2af^2-2cdf+bf\sqrt{e^2-4df}}\left(cx^2+bx+a\right)^{3/2}\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e\right.}}{2\sqrt{2}\sqrt{ce^2-bfe-c\sqrt{e^2-}}}\right.}{d\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)\left(16af^2-8b\left(e-\sqrt{e^2-4df}\right)f+4c\left(e-\sqrt{e^2-4df}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x)*(a + b*x + c*x^2))/(a*(b^2 - 4*a*c)*d*(a + x*(b +
c*x))^(3/2)) - (2*f*(1 + e/Sqrt[e^2 - 4*d*f])*(2*b^2*f - 4*a*c*f - b*c*(e -
 Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*(a + b*x + c
*x^2))/((b^2 - 4*a*c)*d*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 - 4*d*f]) + c*(e - S
qrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (2*f*(1 - e/Sqrt[e^2 - 4*d*
f])*(2*b^2*f - 4*a*c*f - b*c*(e + Sqrt[e^2 - 4*d*f]) + 2*c*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)*(a + b*x + c*x^2))/((b^2 - 4*a*c)*d*(4*a*f^2 - 2*b*f*(
e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/
```

```
2)) - ((a + b*x + c*x^2)^(3/2)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x
+ c*x^2])])/(a^(3/2)*d*(a + x*(b + c*x))^(3/2)) + (16*Sqrt[2]*f^2*(f + (e*f
)/Sqrt[e^2 - 4*d*f])*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2
- 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(3/2)*ArcTanh[(4*a*f -
b*(e - Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(d*(4*a*f^2 - 2*b*f*(e - Sqrt[e
^2 - 4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e - Sqrt[e^2
 - 4*d*f]) + 4*c*(e - Sqrt[e^2 - 4*d*f])^2)*(a + x*(b + c*x))^(3/2)) - (16*
Sqrt[2]*f^2*(-f + (e*f)/Sqrt[e^2 - 4*d*f])*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(a + b*x + c*x^2)^(
3/2)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e
^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sq
rt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(d*(4*a*f
^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2
 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)*(a + x*(b
 + c*x))^(3/2))
```

Maple [B] time = 0.325, size = 4594, normalized size = 5.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/((x-1/2*(-e+(-4*d*f+e^2)^
(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2)^(1/2)-8*f^2/(-e+(-4*d*f+e^2)^(1/2))/((-4*d*f+e^2)^(1/2)*b*f
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f
^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(
c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*
d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)
^(1/2)*x*c^2-8*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)
^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2
/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)
+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c
*e^2)/f^2)^(1/2)*x*b*c+8*f^2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((-
4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4
```

```
*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)
^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+c*e^2)/f^2)^(1/2)*x*c^2*e-4*f^2/(-e+(-4*d*f+e^2)^(1/2))/((-4*d*f+e
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*
c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1
/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))
/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)*b*c-4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(
4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2)*b^2+4*f^2/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^
(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*b*c*e-4*f^3/(-e+(-4*d*f+e^2)^(1/2))/(-4
*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*
e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(
-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e
+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))-4
*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a/(c*x^2+b*x+a)^(1/2)+8*f
/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b/a/(4*a*c-b^2)/(c*x^2+b*x+
a)^(1/2)*x*c+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*b^2/a/(4*a*
c-b^2)/(c*x^2+b*x+a)^(1/2)+4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2
))/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+4*f^3/(e+(-4*d*f+e
^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(
-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d
*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^
(1/2)+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4
*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)
*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*x*c^2-8
*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e
^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-
```

```
c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*
f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(
1/2)*x*b*c+8*f^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(
1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f
*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c
*e^2)/f^2)^(1/2)*x*c^2*e+4*f^2/(e+(-4*d*f+e^2)^(1/2))/(-(-4*d*f+e^2)^(1/2)*
b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^
2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c
+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)
/f^2)^(1/2)*b*c-4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e
^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*
c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2)^(1/2)*b^2+4*f^2/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(
-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)
/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*b*c*e-4*f^3/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+
e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*
f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2+ex+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

3.126 $\int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=140

$$-\frac{1}{4}\sqrt{-x^2-4x-3}x+\frac{5}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{11}{2}$$

```
[Out] (5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*ArcSin[2 + x]
)/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcT
an[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - (5*ArcTanh[x/S
qrt[-3 - 4*x - x^2]])/4
```

Rubi [A] time = 0.500627, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {6728, 619, 216, 640, 742, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{4}\sqrt{-x^2-4x-3}x+\frac{5}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{11}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```


```
[Out] (5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*ArcSin[2 + x]
)/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcT
an[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - (5*ArcTanh[x/S
qrt[-3 - 4*x - x^2]])/4
```


Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
```

```
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int\left(\frac{5}{4\sqrt{-3-4x-x^2}} - \frac{x}{\sqrt{-3-4x-x^2}} + \frac{x^2}{2\sqrt{-3-4x-x^2}} - \frac{15+8x}{4\sqrt{-3-4x-x^2}\left(3+\right.}\right.$$

$$= -\left(\frac{1}{4}\int \frac{15+8x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right) + \frac{1}{2}\int \frac{x^2}{\sqrt{-3-4x-x^2}}\,dx + \frac{5}{4}\int \frac{}{\sqrt{-3}}$$

$$= \sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} - \frac{1}{4}\int \frac{3+6x}{\sqrt{-3-4x-x^2}}\,dx + \frac{1}{2}\int \frac{-}{\sqrt{-3-4x-}}$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{5}{4}\sin^{-1}(2+x) + \frac{1}{8}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+\right.}$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{13}{4}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-}}\right.$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\mathrm{ta}}{}$$

Mathematica [C] time = 0.549183, size = 210, normalized size = 1.5

$$\frac{1}{24}\left(-6\sqrt{-x^2-4x-3}\,x + 60\sqrt{-x^2-4x-3} - \sqrt{1-2i\sqrt{2}}\left(4\sqrt{2}+7i\right)\tanh^{-1}\left(\frac{-i\sqrt{2}x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - \sqrt{1+2i\sqrt{2}}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (60*Sqrt[-3 - 4*x - x^2] - 6*x*Sqrt[-3 - 4*x - x^2] + 132*ArcSin[2 + x] - S
qrt[1 - (2*I)*Sqrt[2]]*(7*I + 4*Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + 2*x -
 I*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 + (2
*I)*Sqrt[2]]*(-7*I + 4*Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2]
)*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/24
```

Maple [A] time = 0.1, size = 159, normalized size = 1.1

$$-\frac{x}{4}\sqrt{-x^2-4x-3}+\frac{5}{2}\sqrt{-x^2-4x-3}+\frac{11\arcsin(2+x)}{2}+\frac{\sqrt{4}\sqrt{3}}{24}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/4*x*(-x^2-4*x-3)^(1/2)+5/2*(-x^2-4*x-3)^(1/2)+11/2*arcsin(2+x)+1/24*3^(1
/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)
^2-12)^(1/2)*2^(1/2))+5*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/
((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(2x^2+4x+3)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 2.19684, size = 494, normalized size = 3.53

$$-\frac{1}{4}\sqrt{-x^2-4x-3}(x-10)+\frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*
sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)
*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 11/2*arctan(sqrt(-x^2 - 4
*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 5/16*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*
x + 3)/x^2) - 5/16*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Giac [A] time = 1.27305, size = 254, normalized size = 1.81

$$-\frac{1}{4}\sqrt{-x^2-4x-3}(x-10)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-}{x+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqr
t(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqr
```

```
t(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 11/2*arcsin(x + 2) - 5/8*log(2*(sqrt
(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 +
1) + 5/8*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) -
 1)^2/(x + 2)^2 + 3)
```

3.127 $\int \frac{x^3}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

Optimal. Leaf size=115

$$-\frac{1}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-2\sin^{-1}(x+2)$$

```
[Out] -Sqrt[-3 - 4*x - x^2]/2 - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4
*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])
/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rubi [A] time = 0.420103, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {6728, 619, 216, 640, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{2}\sqrt{-x^2-4x-3}+\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}+\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-2\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -Sqrt[-3 - 4*x - x^2]/2 - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4
*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])
/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int\left(-\frac{1}{\sqrt{-3-4x-x^2}}+\frac{x}{2\sqrt{-3-4x-x^2}}+\frac{6+5x}{2\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\right)dx$$

$$= \frac{1}{2}\int \frac{x}{\sqrt{-3-4x-x^2}}\,dx + \frac{1}{2}\int \frac{6+5x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \int \frac{1}{\sqrt{-3-4x}}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right) - \frac{5}{8}\int \frac{-6}{\sqrt{-3-4x-x^2}}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - \sin^{-1}(2+x) + \frac{1}{8}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \frac{1}{8}\int$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{2}\int \frac{}{\sqrt{-3-}}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 4\,\mathrm{Subst}\left(\int \frac{}{-4}\right.$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{1}{6}\,\mathrm{Subst}\left(\int \frac{}{\frac{1}{3}}\right.$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{}{-}\right.$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.436546, size = 192, normalized size = 1.67

$$\frac{1}{8}\left(-4\left(\sqrt{-x^2-4x-3}+4\sin^{-1}(x+2)\right)+\frac{\left(5\sqrt{2}-2i\right)\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)}{\sqrt{1-2i\sqrt{2}}}+\frac{\left(5\sqrt{2}+2i\right)\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x}}\right)}{\sqrt{1+2i\sqrt{2}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-4*(Sqrt[-3 - 4*x - x^2] + 4*ArcSin[2 + x]) + ((-2*I + 5*Sqrt[2])*ArcTanh[
(2 + (2*I)*Sqrt[2] + 2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 -
```

```
4*x - x^2])])/Sqrt[1 - (2*I)*Sqrt[2]] + ((2*I + 5*Sqrt[2])*ArcTanh[(2 - (2*
I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^
2])])/Sqrt[1 + (2*I)*Sqrt[2]])/8
```

Maple [A] time = 0.135, size = 144, normalized size = 1.3

$$-\frac{1}{2}\sqrt{-x^2-4x-3}-2\arcsin(2+x)+\frac{\sqrt{4}\sqrt{3}}{24}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)-4\,\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/2*(-x^2-4*x-3)^(1/2)-2*arcsin(2+x)+1/24*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^
2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-4*arct
anh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3
/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(2x^2+4x+3)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 2.20015, size = 474, normalized size = 4.12

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{2}\sqrt{-x^2-4x-3}+2\,\mathrm{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) - 1/2*sqrt(-x^2 - 4*x - 3) + 2*arctan(sqrt(-x^2 - 4*x - 3)*(x +
2)/(x^2 + 4*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) +
1/4*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Giac [A] time = 1.29332, size = 250, normalized size = 2.17

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{-x^2-4x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
- 1/2*sqrt(-x^2 - 4*x - 3) - 2*arcsin(x + 2) + 1/2*log(2*(sqrt(-x^2 - 4*x -
 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*log(
2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)
^2 + 3)
```

3.128 $\int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}\sin^{-1}(x+2)$$

```
[Out] ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2
] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/
Sqrt[-3 - 4*x - x^2]]/2
```

Rubi [A] time = 0.198155, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1077, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{1}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2
] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/
Sqrt[-3 - 4*x - x^2]]/2
```

Rule 1077

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
 + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
```

```
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx + \frac{1}{2}\int \frac{-3-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

$$= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right)\right) + \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,d$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{4}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx + \frac{1}{4}\int -\frac{4}{\sqrt{-3-4x-x^2}}$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-}}\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - 8\operatorname{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx,\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, -\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1\right.\right.$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\tanh^{-1}\left(\frac{}{\sqrt{-3-}}\right.$$

Mathematica [C] time = 0.199376, size = 159, normalized size = 1.62

$$\frac{1}{4}\left(-i\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + i\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + 2\sin^{-1}(x+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (2*ArcSin[2 + x] - I*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + 2
*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + I*Sqrt[
1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2
+ (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/4
```

Maple [A] time = 0.1, size = 130, normalized size = 1.3

$$\frac{\arcsin(2+x)}{2} - \frac{\sqrt{4}\sqrt{3}}{12}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right) - \operatorname{Artanh}\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-\ldots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/2*arcsin(2+x)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/
(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3
/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(2x^2+4x+3\right)\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 1.85723, size = 441, normalized size = 4.5

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}}{x^2+4x+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
```

```
/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/
8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4
*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Giac [B] time = 1.24748, size = 231, normalized size = 2.36

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\arcsin(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.129 $\int \frac{x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=68

$$\frac{\tan^{-1}\left(\frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]) + ArcTan[(1 +
 (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0615909, antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1026, 1161, 618, 204}

$$\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTan[(1 + (3
 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
```

```
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx &= 8\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)\\
&= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)\right)-\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,d\right.\\
&= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx\right.\\
&= \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.18009, size = 174, normalized size = 2.56

$$\frac{\left(1-i\sqrt{2}\right)\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)+\left(1+i\sqrt{2}\right)\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ((1 - I*Sqrt[2])*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 -
I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + (1 + I*Sqrt
[2])*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x
)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/(6*Sqrt[2])
```

Maple [A] time = 0.098, size = 92, normalized size = 1.4

$$\frac{\sqrt{4}\sqrt{3}\sqrt{2}}{12}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)\frac{1}{\sqrt{\left(x^2\left(-\frac{3}{2}-x\right)^{-2}-4\right)\left(1+x\left(-\frac{3}{2}-x\right)^{-1}\right)^{-2}}}\left(1+x\left(-\frac{3}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/12*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2
-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(
1/2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 1.5533, size = 138, normalized size = 2.03

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(6\,x^2+20\,x+15\right)\sqrt{-x^2-4\,x-3}}{4\left(2\,x^3+11\,x^2+18\,x+9\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*
x^3 + 11*x^2 + 18*x + 9))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Giac [A] time = 1.21604, size = 92, normalized size = 1.35

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
```

3.130 $\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=95

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] -(Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + (Sqrt[2]*
ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + ArcTanh[x/Sqrt[-3 -
 4*x - x^2]]/3
```

Rubi [A] time = 0.111173, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -(Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + (Sqrt[2]*
ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + ArcTanh[x/Sqrt[-3 -
 4*x - x^2]]/3
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = -\left(\frac{1}{6}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right)+\frac{1}{6}\int -\frac{4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}$$
$$= -\left(\frac{2}{3}\int \frac{x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx\right)+\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-}}\right.$$
$$= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)-\frac{16}{3}\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right.$$
$$= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)+\frac{2}{9}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)+\frac{2}{9}$$
$$= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)-\frac{4}{9}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right.\right.$$
$$= -\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)+\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)+\frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-}}\right.$$

Mathematica [C] time = 0.109496, size = 150, normalized size = 1.58

$$\frac{1}{6}i\left(\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)-\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2+i\sqrt{2}\right)x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (I/6)*(Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])
*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - Sqrt[1 + (2*I)*Sqrt[2
]]*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]
*Sqrt[-3 - 4*x - x^2])])
```

Maple [A] time = 0.1, size = 121, normalized size = 1.3

$$-\frac{\sqrt{4}\sqrt{3}}{18}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)+\mathrm{Artanh}\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}}\right)\right)\frac{1}{\sqrt{\left(x^2\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -1/18*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^
2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^
(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [A] time = 1.60765, size = 365, normalized size = 3.84

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)-\frac{1}{6}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)-\frac{1}{12}\log\left(-\frac{2\sqrt{-x^2-4\,x-}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) - 1/12*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/12*lo
```

```
g((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Giac [B] time = 1.25614, size = 223, normalized size = 2.35

$$-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/3*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) -
 1)^2/(x + 2)^2 + 1) - 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt
(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.131 $\int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal. Leaf size=130

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)-\frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[Out] -ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/(3*Sqrt[3]) + (Sqrt[2]*Ar
cTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (Sqrt[2]*ArcTan[(1 +
(3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (4*ArcTanh[x/Sqrt[-3 - 4*x - x^
2]])/9
```

Rubi [A] time = 0.417422, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {6728, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)-\frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/(3*Sqrt[3]) + (Sqrt[2]*Ar
cTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (Sqrt[2]*ArcTan[(1 +
(3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (4*ArcTanh[x/Sqrt[-3 - 4*x - x^
2]])/9
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 724

```
Int[1/((((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
```

```
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int \left(\frac{1}{3x\sqrt{-3-4x-x^2}} - \frac{2(2+x)}{3\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\right) dx$$

$$= \frac{1}{3}\int \frac{1}{x\sqrt{-3-4x-x^2}}\,dx - \frac{2}{3}\int \frac{2+x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

$$= \frac{1}{6}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \frac{1}{3}\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{18}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx - \frac{1}{18}\int - \cdots$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{2}{9}\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{16}{9}\,\mathrm{Subst}\left(\int \frac{1+3x\cdots}{-4-8x^2-\cdots}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{27}\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^{\cdots}}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{27}\,\mathrm{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx\cdots\right.$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-\cdots}}}{\sqrt{2}}\right.$$

Mathematica [C] time = 0.453093, size = 200, normalized size = 1.54

$$\frac{1}{54}\left(-6\sqrt{3}\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1-2i\sqrt{2}}\left(\sqrt{2}+2i\right)\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - 3\sqrt{1+2i\sqrt{2}}\left(\sqrt{2}-\cdots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-6*Sqrt[3]*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])] - 3*Sqrt[1 - (
2*I)*Sqrt[2]]*(2*I + Sqrt[2])*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*
```

```
x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] - 3*Sqrt[1 + (2*I)*Sqrt[
2]]*(-2*I + Sqrt[2])*ArcTanh[(2 + (2*I)*Sqrt[2] + (2 + I*Sqrt[2])*x)/(Sqrt[
2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/54
```

Maple [A] time = 0.102, size = 152, normalized size = 1.2

$$\frac{\sqrt{3}}{9}\arctan\left(\frac{(-6-4x)\sqrt{3}}{6}\frac{1}{\sqrt{-x^2-4x-3}}\right)+\frac{\sqrt{4}\sqrt{3}}{54}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)+4\,\mathrm{A}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] 1/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))+1/54*3^(1/2)*4^
(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)
^(1/2)*2^(1/2))+4*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/
(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2+4x+3)\sqrt{-x^2-4x-3}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x), x)
```

Fricas [A] time = 1.63283, size = 473, normalized size = 3.64

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}(2x+3)}{3(x^2+4x+3)}\right)+\frac{1}{18}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{18}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2 + 4*x +
3)) + 1/18*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) + 1/18*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*
x - 3))/(2*x + 3)) + 1/9*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1
/9*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Giac [A] time = 1.30737, size = 269, normalized size = 2.07

$$\frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) + 1/9*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3)
- 1)^2/(x + 2)^2 + 1) + 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqr
t(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.132 $\int \frac{1}{x^2\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{2\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{10}{27}\tanh^{-1}\left(\right.$$

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x
^2])])/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/S
qrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]
)/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/27
```

Rubi [A] time = 0.453625, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {6728, 730, 724, 204, 1028, 986, 12, 1026, 1161, 618, 1027, 206}

$$\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{2\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{10}{27}\tanh^{-1}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x
^2])])/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/S
qrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]
)/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/27
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
```

```
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int \left(\frac{1}{3x^2\sqrt{-3-4x-x^2}} - \frac{4}{9x\sqrt{-3-4x-x^2}} + \frac{2(5+4x)}{9\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\right)$$

$$= \frac{2}{9}\int \frac{5+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx + \frac{1}{3}\int \frac{1}{x^2\sqrt{-3-4x-x^2}}\,dx - \frac{4}{9}\int$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} - \frac{2}{9}\int \frac{1}{x\sqrt{-3-4x-x^2}}\,dx - \frac{2}{9}\int \frac{1}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{4\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{9\sqrt{3}} + \frac{1}{27}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{27}\int$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{32}{27}\,\mathrm{S}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{4}{81}\,\mathrm{S}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{8}{81}\,\mathrm{S}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{2}{27}$$

Mathematica [C] time = 0.455593, size = 225, normalized size = 1.49

$$\frac{3\left(\sqrt{-x^2-4x-3}+2\sqrt{3}x\tan^{-1}\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)\right)+\sqrt{1-2i\sqrt{2}}\left(2\sqrt{2}+i\right)x\tanh^{-1}\left(\frac{-i\sqrt{2}x+2x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right)+\sqrt{1+2i\sqrt{2}}\left(2\sqrt{2}\right.}{27x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (3*(Sqrt[-3 - 4*x - x^2] + 2*Sqrt[3]*x*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 -
4*x - x^2])]) + Sqrt[1 - (2*I)*Sqrt[2]]*(I + 2*Sqrt[2])*x*ArcTanh[(2 - (2*I
```

```
)*Sqrt[2] + 2*x - I*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2
])] + Sqrt[1 + (2*I)*Sqrt[2]]*(-I + 2*Sqrt[2])*x*ArcTanh[(2 + (2*I)*Sqrt[2]
 + (2 + I*Sqrt[2])*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/(27*
x)
```

Maple [A] time = 0.111, size = 169, normalized size = 1.1

$$-\frac{2\sqrt{3}}{9}\arctan\left(\frac{(-6-4x)\sqrt{3}}{6}\frac{1}{\sqrt{-x^2-4x-3}}\right)+\frac{\sqrt{4}\sqrt{3}}{81}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right)-5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x)
```

```
[Out] -2/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))+1/81*3^(1/2)*4
^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12
)^(1/2)*2^(1/2))-5*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2
/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))+1/9*(-x^2-4*x-3)^(1/2
)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x^2+4x+3\right)\sqrt{-x^2-4x-3}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2), x)
```

Fricas [A] time = 1.63239, size = 518, normalized size = 3.43

$$-\frac{12\sqrt{3}x\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}(2x+3)}{3\left(x^2+4x+3\right)}\right)-2\sqrt{2}x\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-2\sqrt{2}x\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+5x\,l}{54x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/54*(12*sqrt(3)*x*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2
+ 4*x + 3)) - 2*sqrt(2)*x*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x
 - 3))/(2*x + 3)) - 2*sqrt(2)*x*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^
2 - 4*x - 3))/(2*x + 3)) + 5*x*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^
2) - 5*x*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) - 6*sqrt(-x^2 - 4*x
- 3))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{-(x+1)(x+3)}\left(2x^2+4x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Giac [B] time = 1.2789, size = 363, normalized size = 2.4

$$\frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{2}{27}\sqrt{2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1
)) + 2/27*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) +
1)) - 1/18*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 2)/((sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/27*log(2*(sq
```

```
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2
+ 1) - 5/27*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3
) - 1)^2/(x + 2)^2 + 3)
```

3.133 $\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$

Optimal. Leaf size=149

$$-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}-\frac{873\left(12x^2+17x+6\right)^{7/2}}{1792}+\frac{25091(24x+17)\left(12x^2+17x+6\right)^{5/2}}{24576}-\frac{125455(24x+17)}{471}$$

```
[Out] (125455*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/150994944 - (125455*(17 + 24*x
)*(6 + 17*x + 12*x^2)^(3/2))/4718592 + (25091*(17 + 24*x)*(6 + 17*x + 12*x^
2)^(5/2))/24576 - (873*(6 + 17*x + 12*x^2)^(7/2))/1792 - ((10 - 3*x)*(6 + 1
7*x + 12*x^2)^(7/2))/32 - (125455*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 1
7*x + 12*x^2])])/(603979776*Sqrt[3])
```

Rubi [A] time = 0.0933228, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1002, 742, 640, 612, 621, 206}

$$-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}-\frac{873\left(12x^2+17x+6\right)^{7/2}}{1792}+\frac{25091(24x+17)\left(12x^2+17x+6\right)^{5/2}}{24576}-\frac{125455(24x+17)}{471}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (125455*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/150994944 - (125455*(17 + 24*x
)*(6 + 17*x + 12*x^2)^(3/2))/4718592 + (25091*(17 + 24*x)*(6 + 17*x + 12*x^
2)^(5/2))/24576 - (873*(6 + 17*x + 12*x^2)^(7/2))/1792 - ((10 - 3*x)*(6 + 1
7*x + 12*x^2)^(7/2))/32 - (125455*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 1
7*x + 12*x^2])])/(603979776*Sqrt[3])
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx = \int (10-3x)^2 \left(6+17x+12x^2\right)^{5/2} dx$$
$$= -\frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2} + \frac{1}{96}\int \left(11331-\frac{7857x}{2}\right)\left(6+1\right.$$
$$= -\frac{873\left(6+17x+12x^2\right)^{7/2}}{1792} - \frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2} + \frac{752}{51}$$
$$= \frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576} - \frac{873\left(6+17x+12x^2\right)^{7/2}}{1792} -$$
$$= -\frac{125455(17+24x)\left(6+17x+12x^2\right)^{3/2}}{4718592} + \frac{25091(17+24x)\left(6+17\right.}{24576}$$
$$= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24x)\left(6+17x\right.}{4718592}$$
$$= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24x)\left(6+17x\right.}{4718592}$$
$$= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24x)\left(6+17x\right.}{4718592}$$

Mathematica [A] time = 0.225207, size = 87, normalized size = 0.58

$$\frac{12\sqrt{12x^2+17x+6}\left(171228266496x^7-732816211968x^6-1190083166208x^5+3438453030912x^4+8974844476416x^3\right.}{12683575296}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(474999091769 + 3132157281976*x + 7899203409792
*x^2 + 8974844476416*x^3 + 3438453030912*x^4 - 1190083166208*x^5 - 73281621
1968*x^6 + 171228266496*x^7) - 878185*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[1
8 + 51*x + 36*x^2])])/12683575296
```

Maple [A] time = 0.057, size = 147, normalized size = 1.

$$\frac{129220757\,x}{458752}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}} - \frac{125455\,\sqrt{12}}{3623878656}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,x^2+17\,x+6}\right) + \frac{2132735+3010920\,x}{150994944}\sqrt{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x)
```

```
[Out] 129220757/458752*x*(12*x^2+17*x+6)^(3/2)-125455/3623878656*ln(1/12*(17/2+12
*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))*12^(1/2)+125455/150994944*(17+24*x)*(12
*x^2+17*x+6)^(1/2)+27/2*x^5*(12*x^2+17*x+6)^(3/2)-8613/112*x^4*(12*x^2+17*x
+6)^(3/2)+14991/1792*x^3*(12*x^2+17*x+6)^(3/2)+4267751/14336*x^2*(12*x^2+17
*x+6)^(3/2)+2473875847/33030144*(12*x^2+17*x+6)^(3/2)
```

Maxima [A] time = 1.5339, size = 209, normalized size = 1.4

$$\frac{27}{2}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^5-\frac{8613}{112}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^4+\frac{14991}{1792}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^3+\frac{4267751}{14336}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"maxima")
```

```
[Out] 27/2*(12*x^2 + 17*x + 6)^(3/2)*x^5 - 8613/112*(12*x^2 + 17*x + 6)^(3/2)*x^4
 + 14991/1792*(12*x^2 + 17*x + 6)^(3/2)*x^3 + 4267751/14336*(12*x^2 + 17*x
+ 6)^(3/2)*x^2 + 129220757/458752*(12*x^2 + 17*x + 6)^(3/2)*x + 2473875847/
33030144*(12*x^2 + 17*x + 6)^(3/2) + 125455/6291456*sqrt(12*x^2 + 17*x + 6)
*x - 125455/1811939328*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x
 + 17) + 2132735/150994944*sqrt(12*x^2 + 17*x + 6)
```

Fricas [A] time = 1.57645, size = 398, normalized size = 2.67

$$\frac{1}{1056964608}\left(171228266496\,x^7-732816211968\,x^6-1190083166208\,x^5+3438453030912\,x^4+8974844476416\,x^3+7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 1/1056964608*(171228266496*x^7 - 732816211968*x^6 - 1190083166208*x^5 + 343
8453030912*x^4 + 8974844476416*x^3 + 7899203409792*x^2 + 3132157281976*x +
474999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/3623878656*sqrt(3)*log(-8*sq
```

```
rt(3)*sqrt(12*x^2 + 17*x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{(3x+2)(4x+3)}\,(3x-10)^2\,(3x+2)^2\,(4x+3)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**2*(-12*x**2+31*x+30)**2*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))*(3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2,
 x)
```

Giac [A] time = 1.16059, size = 115, normalized size = 0.77

$$\frac{1}{1056964608}\,(8\,(48\,(24\,(96\,(24\,(48\,(168\,x-719)x-56047)x+3886417)x+973832951)x+20570842213)x+391519660$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/1056964608*(8*(48*(24*(96*(24*(48*(168*x - 719)*x - 56047)*x + 3886417)*x
 + 973832951)*x + 20570842213)*x + 391519660247)*x + 474999091769)*sqrt(12*
x^2 + 17*x + 6) + 125455/1811939328*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x
 - sqrt(12*x^2 + 17*x + 6)) - 17))
```

3.134 $\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$

Optimal. Leaf size=103

$$-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2}+\frac{97}{768}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}+\frac{97\tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{98304\sqrt{3}}$$

```
[Out] (-97*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/24576 + (97*(17 + 24*x)*(6 + 17*x
 + 12*x^2)^(3/2))/768 - (6 + 17*x + 12*x^2)^(5/2)/20 + (97*ArcTanh[(17 + 24
*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(98304*Sqrt[3])
```

Rubi [A] time = 0.0417328, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1002, 640, 612, 621, 206}

$$-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2}+\frac{97}{768}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}+\frac{97\tanh^{-1}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{98304\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```


```
[Out] (-97*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/24576 + (97*(17 + 24*x)*(6 + 17*x
 + 12*x^2)^(3/2))/768 - (6 + 17*x + 12*x^2)^(5/2)/20 + (97*ArcTanh[(17 + 24
*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(98304*Sqrt[3])
```


Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx &= \int (10-3x)\left(6+17x+12x^2\right)^{3/2}\,dx\\
&= -\frac{1}{20}\left(6+17x+12x^2\right)^{5/2}+\frac{97}{8}\int \left(6+17x+12x^2\right)^{3/2}\,dx\\
&= \frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/2}-\frac{1}{20}\left(6+17x+12x^2\right)^{5/2}-\frac{97}{512}\int\\
&= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/}\\
&= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/}\\
&= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/}\end{aligned}$$

Mathematica [A] time = 0.0345904, size = 72, normalized size = 0.7

$$\frac{12\sqrt{12x^2+17x+6}\left(-884736x^4+1963008x^3+6837888x^2+5455144x+1353611\right)+485\sqrt{3}\tanh^{-1}\left(\frac{24x+17}{4\sqrt{36x^2+51x+18}}\right)}{1474560}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (12*Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 + 1963008*x^
3 - 884736*x^4) + 485*Sqrt[3]*ArcTanh[(17 + 24*x)/(4*Sqrt[18 + 51*x + 36*x^
2])])/1474560
```

Maple [A] time = 0.05, size = 96, normalized size = 0.9

$$-\frac{3x^2}{5}\left(12x^2+17x+6\right)^{\frac{3}{2}}+\frac{349x}{160}\left(12x^2+17x+6\right)^{\frac{3}{2}}+\frac{7093}{3840}\left(12x^2+17x+6\right)^{\frac{3}{2}}-\frac{1649+2328x}{24576}\sqrt{12x^2+17x+6}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x)
```

```
[Out] -3/5*x^2*(12*x^2+17*x+6)^(3/2)+349/160*x*(12*x^2+17*x+6)^(3/2)+7093/3840*(1
2*x^2+17*x+6)^(3/2)-97/24576*(17+24*x)*(12*x^2+17*x+6)^(1/2)+97/589824*ln(1
/12*(17/2+12*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))*12^(1/2)
```

Maxima [A] time = 1.47097, size = 140, normalized size = 1.36

$$-\frac{3}{5}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^2+\frac{349}{160}\left(12x^2+17x+6\right)^{\frac{3}{2}}x+\frac{7093}{3840}\left(12x^2+17x+6\right)^{\frac{3}{2}}-\frac{97}{1024}\sqrt{12x^2+17x+6}x+\frac{97}{294912}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="max
ima")
```

```
[Out] -3/5*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 349/160*(12*x^2 + 17*x + 6)^(3/2)*x +
7093/3840*(12*x^2 + 17*x + 6)^(3/2) - 97/1024*sqrt(12*x^2 + 17*x + 6)*x + 9
7/294912*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x + 17) - 1649/
24576*sqrt(12*x^2 + 17*x + 6)
```

Fricas [A] time = 1.55974, size = 263, normalized size = 2.55

$$-\frac{1}{122880}\left(884736\,x^4-1963008\,x^3-6837888\,x^2-5455144\,x-1353611\right)\sqrt{12\,x^2+17\,x+6}+\frac{97}{589824}\,\sqrt{3}\log\left(8\,\sqrt{3}\sqrt{12\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/122880*(884736*x^4 - 1963008*x^3 - 6837888*x^2 - 5455144*x - 1353611)*sq
rt(12*x^2 + 17*x + 6) + 97/589824*sqrt(3)*log(8*sqrt(3)*sqrt(12*x^2 + 17*x
+ 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -152x\sqrt{12x^2+17x+6}\,dx-\int -69x^2\sqrt{12x^2+17x+6}\,dx-\int 36x^3\sqrt{12x^2+17x+6}\,dx-\int -60\sqrt{12x^2+17x+6}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x**2+31*x+30)*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] -Integral(-152*x*sqrt(12*x**2 + 17*x + 6), x) - Integral(-69*x**2*sqrt(12*x
**2 + 17*x + 6), x) - Integral(36*x**3*sqrt(12*x**2 + 17*x + 6), x) - Integ
ral(-60*sqrt(12*x**2 + 17*x + 6), x)
```

Giac [A] time = 1.15629, size = 95, normalized size = 0.92

$$-\frac{1}{122880}\,(8\,(48\,(72\,(32\,x-71)x-17807)x-681893)x-1353611)\sqrt{12\,x^2+17\,x+6}-\frac{97}{294912}\,\sqrt{3}\log\left(\left|-4\,\sqrt{3}\left(2\,\sqrt{3}x-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/122880*(8*(48*(72*(32*x - 71)*x - 17807)*x - 681893)*x - 1353611)*sqrt(1
2*x^2 + 17*x + 6) - 97/294912*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x - sqr
t(12*x^2 + 17*x + 6)) - 17))
```

3.135 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)}\,dx$

Optimal. Leaf size=28

$$\frac{1}{42}\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

```
[Out] ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42
```

Rubi [A] time = 0.0482939, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {1002, 724, 206}

$$\frac{1}{42}\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

```
[Out] ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)}\,dx &= \int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}}\,dx\\ &= -\left(2\,\text{Subst}\left(\int \frac{1}{7056-x^2}\,dx, x, \frac{-206-291x}{\sqrt{6+17x+12x^2}}\right)\right)\\ &= \frac{1}{42}\tanh^{-1}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.109895, size = 37, normalized size = 1.32

$$\frac{1}{42}\log\left(84\sqrt{12x^2+17x+6}+291x+206\right)-\frac{1}{42}\log(10-3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

```
[Out] -Log[10 - 3*x]/42 + Log[206 + 291*x + 84*Sqrt[6 + 17*x + 12*x^2]]/42
```

Maple [B] time = 0.058, size = 163, normalized size = 5.8

$$-\frac{4}{49}\sqrt{12\,(x+3/4)^2-x-\frac{3}{4}}+\frac{\sqrt{12}}{294}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,(x+3/4)^2-x-\frac{3}{4}}\right)-\frac{1}{588}\sqrt{12\,(x-10/3)^2+97\,x-\frac{382}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x)
```

```
[Out] -4/49*(12*(x+3/4)^2-x-3/4)^(1/2)+1/294*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x+
3/4)^2-x-3/4)^(1/2))*12^(1/2)-1/588*(12*(x-10/3)^2+97*x-382/3)^(1/2)-97/141
12*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x-10/3)^2+97*x-382/3)^(1/2))*12^(1/2)+
1/42*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^(1/2))+1/12*(12*(
x+2/3)^2+x+2/3)^(1/2)+1/288*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x+2/3)^2+x+2/
```

```
3)^(1/2))*12^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(12\,x^2-31\,x-30\right)(3\,x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)), x)
```

Fricas [B] time = 1.54378, size = 153, normalized size = 5.46

$$\frac{1}{84}\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-\frac{1}{84}\log\left(\frac{291\,x-84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="fri
cas")
```

```
[Out] 1/84*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 1/84*log((291*x -
84*sqrt(12*x^2 + 17*x + 6) + 206)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{12x^2+17x+6}}{36x^3-69x^2-152x-60}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)/(-12*x**2+31*x+30),x)
```

```
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(36*x**3 - 69*x**2 - 152*x - 60), x)
```

Giac [B] time = 1.1727, size = 85, normalized size = 3.04

$$\frac{1}{42}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12x^2+17x+6}+42\right|\right)-\frac{1}{42}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12x^2+17x+6}-42\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="gia
c")
```

```
[Out] 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) -
 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))
```

3.136 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}+\frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)}+\frac{97\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

```
[Out] -(275 + 388*x)/(98*(10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + (3137*Sqrt[6 + 17*
x + 12*x^2])/(38416*(10 - 3*x)) + (97*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17
*x + 12*x^2])])/3226944
```

Rubi [A] time = 0.0777517, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1002, 740, 806, 724, 206}

$$-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}+\frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)}+\frac{97\tanh^{-1}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

```
[Out] -(275 + 388*x)/(98*(10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + (3137*Sqrt[6 + 17*
x + 12*x^2])/(38416*(10 - 3*x)) + (97*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17
*x + 12*x^2])])/3226944
```

Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = \int \frac{1}{(10-3x)^2\left(6+17x+12x^2\right)^{3/2}}\,dx$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} - \frac{1}{882}\int \frac{-\frac{14859}{2}-10476x}{(10-3x)^2\sqrt{6+17x+12x^2}}\,dx$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97\int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}}}{76832}$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} - \frac{97\,\mathrm{Subst}\left(\int \frac{1}{7056-x^2}\,dx\right.}{38416}$$

$$= -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97\tanh^{-1}\left(\frac{206+291x}{84\sqrt{6+17x+1}}\right.}{3226944}$$

Mathematica [A] time = 0.243785, size = 114, normalized size = 1.36

$$\frac{\sqrt{12x^2+17x+6}\left(97\left(36x^3-69x^2-152x-60\right)\tanh^{-1}\left(\frac{7\sqrt{3x+2}}{6\sqrt{4x+3}}\right)-42\sqrt{3x+2}\sqrt{4x+3}\left(37644x^2-98767x-88978\right)\right)}{1613472(3x-10)(3x+2)^{3/2}(4x+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(-42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(-88978 - 98767*x
 + 37644*x^2) + 97*(-60 - 152*x - 69*x^2 + 36*x^3)*ArcTanh[(7*Sqrt[2 + 3*x]
)/(6*Sqrt[3 + 4*x])]))/(1613472*(-10 + 3*x)*(2 + 3*x)^(3/2)*(3 + 4*x)^(3/2)
)
```

Maple [B] time = 0.064, size = 245, normalized size = 2.9

$$\frac{32}{2401}\left(12\,(x+3/4)^2-x-\frac{3}{4}\right)^{\frac{3}{2}}\left(x+\frac{3}{4}\right)^{-2}+\frac{384}{117649}\sqrt{12\,(x+3/4)^2-x-\frac{3}{4}}-\frac{16\sqrt{12}}{117649}\ln\left(\frac{\sqrt{12}}{12}\left(\frac{17}{2}+12\,x\right)+\sqrt{12\,(x+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x)
```

```
[Out] 32/2401/(x+3/4)^2*(12*(x+3/4)^2-x-3/4)^(3/2)+384/117649*(12*(x+3/4)^2-x-3/4
)^(1/2)-16/117649*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x+3/4)^2-x-3/4)^(1/2))*
12^(1/2)-1/72/(x+2/3)^2*(12*(x+2/3)^2+x+2/3)^(3/2)+1/288*(12*(x+2/3)^2+x+2/
3)^(1/2)+1/6912*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x+2/3)^2+x+2/3)^(1/2))*12
^(1/2)-97/45177216*(12*(x-10/3)^2+97*x-382/3)^(1/2)-7057/813189888*ln(1/12*
(17/2+12*x)*12^(1/2)+(12*(x-10/3)^2+97*x-382/3)^(1/2))*12^(1/2)+97/3226944*
arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^(1/2))-1/67765824/(x-1
0/3)*(12*(x-10/3)^2+97*x-382/3)^(3/2)+1/135531648*(17+24*x)*(12*(x-10/3)^2+
97*x-382/3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(12\,x^2-31\,x-30\right)^2(3\,x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2), x)
```

Fricas [A] time = 1.58619, size = 370, normalized size = 4.4

$$\frac{97\left(36\,x^3-69\,x^2-152\,x-60\right)\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-97\left(36\,x^3-69\,x^2-152\,x-60\right)\log\left(\frac{291\,x-84\,\sqrt{12\,x^2+17\,x+6}+2}{x}\right.}{6453888\left(36\,x^3-69\,x^2-152\,x-60\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"fricas")
```

```
[Out] 1/6453888*(97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x + 84*sqrt(12*x^2 +
17*x + 6) + 206)/x) - 97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x - 84*sqr
t(12*x^2 + 17*x + 6) + 206)/x) - 168*(37644*x^2 - 98767*x - 88978)*sqrt(12*
x^2 + 17*x + 6))/(36*x^3 - 69*x^2 - 152*x - 60)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(3x+2)(4x+3)}}{(3x-10)^2(3x+2)^2(4x+3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**2/(-12*x**2+31*x+30)**2,x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))/((3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2
), x)
```

Giac [B] time = 1.22682, size = 215, normalized size = 2.56

$$\frac{1}{9680832}\sqrt{3}\left(\sqrt{3}\left(175672\sqrt{3}+97\log\left(\frac{7\sqrt{3}-12}{7\sqrt{3}+12}\right)\right)\mathrm{sgn}\left(\frac{1}{3x+2}\right)-\left(97\sqrt{3}\log\left(\frac{\left|-28\sqrt{3}+24\sqrt{\frac{1}{3x+2}+4}\right|}{4\left(7\sqrt{3}+6\sqrt{\frac{1}{3x+2}+4}\right)}\right)+134456\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"giac")
```

```
[Out] 1/9680832*sqrt(3)*(sqrt(3)*(175672*sqrt(3) + 97*log((7*sqrt(3) - 12)/(7*sqr
t(3) + 12)))*sgn(1/(3*x + 2)) - (97*sqrt(3)*log(1/4*abs(-28*sqrt(3) + 24*sq
rt(1/(3*x + 2) + 4))/(7*sqrt(3) + 6*sqrt(1/(3*x + 2) + 4))) + 134456*sqrt(1
/(3*x + 2) + 4) + 28*(221183/(3*x + 2) - 18436)/(12*(1/(3*x + 2) + 4)^(3/2)
 - 49*sqrt(1/(3*x + 2) + 4)))*sgn(1/(3*x + 2)))
```

3.137 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx$

Optimal. Leaf size=139

$$-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}-\frac{1634466587\sqrt{12x^2+17x+6}}{7589772288(10-3x)}-\frac{50555899\sqrt{12x^2+17x+6}}{19361664(10-3x)^2}+\frac{1042556x+7}{8232(10-3x)^2\sqrt{12}}$$

```
[Out] -(275 + 388*x)/(294*(10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + (738029 + 104
2556*x)/(8232*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) - (50555899*Sqrt[6 + 17
*x + 12*x^2])/(19361664*(10 - 3*x)^2) - (1634466587*Sqrt[6 + 17*x + 12*x^2]
)/(7589772288*(10 - 3*x)) + (40325*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x
+ 12*x^2])])/637540872192
```

Rubi [A] time = 0.117485, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {1002, 740, 822, 834, 806, 724, 206}

$$-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}-\frac{1634466587\sqrt{12x^2+17x+6}}{7589772288(10-3x)}-\frac{50555899\sqrt{12x^2+17x+6}}{19361664(10-3x)^2}+\frac{1042556x+7}{8232(10-3x)^2\sqrt{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```


```
[Out] -(275 + 388*x)/(294*(10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + (738029 + 104
2556*x)/(8232*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) - (50555899*Sqrt[6 + 17
*x + 12*x^2])/(19361664*(10 - 3*x)^2) - (1634466587*Sqrt[6 + 17*x + 12*x^2]
)/(7589772288*(10 - 3*x)) + (40325*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x
+ 12*x^2])])/637540872192
```


Rule 1002

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[((d*g)/a + (f*h*x)/c
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx = \int \frac{1}{(10-3x)^3\left(6+17x+12x^2\right)^{5/2}}\,dx$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} - \frac{\int \frac{\frac{109953}{2}-41904x}{(10-3x)^3\left(6+17x+12x^2\right)^{3/2}}\,dx}{2646}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} + \frac{\int \frac{-\frac{5020024\cdots}{4}\cdots}{(10-3x\cdots}}{\cdots}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{5055589\cdots}{19361\cdots}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{5055589\cdots}{19361\cdots}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{5055589\cdots}{19361\cdots}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{5055589\cdots}{19361\cdots}$$

Mathematica [A] time = 0.38004, size = 131, normalized size = 0.94

$$\frac{\sqrt{12x^2+17x+6}\left(42\sqrt{3x+2}\sqrt{4x+3}\left(706089565584x^5-3206824169544x^4-1096520427663x^3+9848047480070x^2\right.\right.}{318770436096(10-3x)^2(3x+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(42*Sqrt[2 + 3*x]*Sqrt[3 + 4*x]*(2773753482408 + 1
0124325497244*x + 9848047480070*x^2 - 1096520427663*x^3 - 3206824169544*x^4
 + 706089565584*x^5) + 40325*(60 + 152*x + 69*x^2 - 36*x^3)^2*ArcTanh[(7*Sq
rt[2 + 3*x])/(6*Sqrt[3 + 4*x])]))/(318770436096*(10 - 3*x)^2*(2 + 3*x)^(5/2
)*(3 + 4*x)^(5/2))
```

Maple [B] time = 0.071, size = 306, normalized size = 2.2

$$\frac{1}{79692609024}\left(12\,(x-10/3)^2+97\,x-\frac{382}{3}\right)^{\frac{3}{2}}\left(x-\frac{10}{3}\right)^{-2}+\frac{47}{1152}\left(12\,(x+2/3)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}\left(x+\frac{2}{3}\right)^{-2}-\frac{230400}{5764801}\left(12\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x)
```

```
[Out] 1/79692609024/(x-10/3)^2*(12*(x-10/3)^2+97*x-382/3)^(3/2)+47/1152/(x+2/3)^2
*(12*(x+2/3)^2+x+2/3)^(3/2)-230400/5764801/(x+3/4)^2*(12*(x+3/4)^2-x-3/4)^(
3/2)-23/110592*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x+2/3)^2+x+2/3)^(1/2))*12^
(1/2)-570457/31239502737408*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(x-10/3)^2+97*
x-382/3)^(1/2))*12^(1/2)+58752/282475249*ln(1/12*(17/2+12*x)*12^(1/2)+(12*(
x+3/4)^2-x-3/4)^(1/2))*12^(1/2)-23/4608*(12*(x+2/3)^2+x+2/3)^(1/2)+40325/63
7540872192*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^(1/2))-4032
5/8925572210688*(12*(x-10/3)^2+97*x-382/3)^(1/2)-1410048/282475249*(12*(x+3
/4)^2-x-3/4)^(1/2)+1261/62479005474816*(17+24*x)*(12*(x-10/3)^2+97*x-382/3)
^(1/2)-128/352947/(x+3/4)^3*(12*(x+3/4)^2-x-3/4)^(3/2)-1261/31239502737408/
(x-10/3)*(12*(x-10/3)^2+97*x-382/3)^(3/2)-1/2592/(x+2/3)^3*(12*(x+2/3)^2+x+
2/3)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(12\,x^2-31\,x-30\right)^3(3\,x+2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"maxima")
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3), x)
```

Fricas [A] time = 1.6898, size = 674, normalized size = 4.85

$$\frac{40325\left(1296\,x^6-4968\,x^5-6183\,x^4+16656\,x^3+31384\,x^2+18240\,x+3600\right)\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-40325\,(129}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"fricas")
```

```
[Out] 1/1275081744384*(40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*
x^2 + 18240*x + 3600)*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 4
0325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 36
00)*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) + 168*(706089565584*x
^5 - 3206824169544*x^4 - 1096520427663*x^3 + 9848047480070*x^2 + 1012432549
7244*x + 2773753482408)*sqrt(12*x^2 + 17*x + 6))/(1296*x^6 - 4968*x^5 - 618
3*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**3/(-12*x**2+31*x+30)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21618, size = 313, normalized size = 2.25

$$\frac{\sqrt{3}\left(282273\,\sqrt{3}\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^3-11460924\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^2-37551180\,\sqrt{3}\left(2\,\sqrt{3}x-\sqrt{12\,x^2\dots}\right.\right.}{159385218048\left(3\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^2-40\,\sqrt{3}\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)-188\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"giac")
```

```
[Out] 1/159385218048*sqrt(3)*(282273*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x +
6))^3 - 11460924*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 37551180*sqrt(
3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 83365264)/(3*(2*sqrt(3)*x - sq
rt(12*x^2 + 17*x + 6))^2 - 40*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6
)) - 188)^2 + 1/2213683584*((8*(2860316794*x + 6078171227)*x + 34383350229)
*x + 8090114146)/(12*x^2 + 17*x + 6)^(3/2) + 40325/637540872192*log(abs(-6*
sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 40325/637540872
192*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))
```

3.138 $\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rubi [A] time = 0.0039147, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {629}

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

Mathematica [A] time = 0.0067621, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] time = 0.046, size = 16, normalized size = 1.1

$$\frac{(-9+3x)x}{5}\left(x^2-3x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)*(x^2-3*x)^(2/3),x)
```

```
[Out] 3/5*(-3+x)*x*(x^2-3*x)^(2/3)
```

Maxima [A] time = 0.988348, size = 15, normalized size = 1.

$$\frac{3}{5}\left(x^2-3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Fricas [A] time = 1.49361, size = 31, normalized size = 2.07

$$\frac{3}{5}\left(x^2-3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [B] time = 0.448597, size = 31, normalized size = 2.07

$$\frac{3x^2\left(x^2-3x\right)^{\frac{2}{3}}}{5}-\frac{9x\left(x^2-3x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x**2-3*x)**(2/3),x)
```

```
[Out] 3*x**2*(x**2 - 3*x)**(2/3)/5 - 9*x*(x**2 - 3*x)**(2/3)/5
```

Giac [A] time = 1.15488, size = 15, normalized size = 1.

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="giac")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

3.139 $\int ((-3+x)x)^{2/3}(-3+2x)\,dx$

Optimal. Leaf size=16

$$\frac{3}{5}(-(3-x)x)^{5/3}$$

```
[Out] (3*(-((3 - x)*x))^(5/3))/5
```

Rubi [A] time = 0.006378, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1588}

$$\frac{3}{5}(-(3-x)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
```

```
[Out] (3*(-((3 - x)*x))^(5/3))/5
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3}{5}(-(3-x)x)^{5/3}$$

Mathematica [A] time = 0.0031555, size = 13, normalized size = 0.81

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] time = 0.045, size = 14, normalized size = 0.9

$$\frac{(-9+3x)x}{5}((-3+x)x)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-3+x)*x)^(2/3)*(-3+2*x),x)
```

```
[Out] 3/5*(-3+x)*x*((-3+x)*x)^(2/3)
```

Maxima [A] time = 1.01458, size = 12, normalized size = 0.75

$$\frac{3}{5}((x-3)x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="maxima")
```

```
[Out] 3/5*((x - 3)*x)^(5/3)
```

Fricas [A] time = 1.66149, size = 31, normalized size = 1.94

$$\frac{3}{5}\left(x^2-3x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [A] time = 10.0711, size = 10, normalized size = 0.62

$$\frac{3\left(x\left(x-3\right)\right)^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)**(2/3)*(-3+2*x),x)
```

```
[Out] 3*(x*(x - 3))**(5/3)/5
```

Giac [A] time = 1.18262, size = 15, normalized size = 0.94

$$\frac{3}{5}\left(x^{2}-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="giac")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

3.140 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} \, dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rubi [A] time = 0.0282344, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1631, 629}

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx$$
$$= \frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

Mathematica [A] time = 0.0057753, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] time = 0.046, size = 20, normalized size = 1.3

$$\frac{3(-3+x)^2x^2}{5}\frac{1}{\sqrt[3]{x^2-3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x)
```

```
[Out] 3/5*(-3+x)^2*x^2/(x^2-3*x)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(2x^2-9x+9)x}{(x^2-3x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)
```

Fricas [A] time = 1.47822, size = 31, normalized size = 2.07

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\left(x-3\right)\left(2x-3\right)}{\sqrt[3]{x\left(x-3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**2-9*x+9)/(x**2-3*x)**(1/3),x)
```

```
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(2\,x^2-9\,x+9\right)x}{\left(x^2-3\,x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)
```

3.141 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx$

Optimal. Leaf size=15

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rubi [A] time = 0.0575034, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1985, 1631, 629}

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 1985

```
Int[(u_)^(p_.)*(v_)^(q_.)*(z_)^(m_.), x_Symbol] :> Int[ExpandToSum[z, x]^m*
ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{m, p, q}, x] && Linea
rQ[z, x] && QuadraticQ[{u, v}, x] &&  !(LinearMatchQ[z, x] && QuadraticMatc
hQ[{u, v}, x])
```

Rule 1631

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{(-3+x)x}}\,dx &= \int \frac{x\left(9-9x+2x^2\right)}{\sqrt[3]{-3x+x^2}}\,dx\\ &= \int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx\\ &= \frac{3}{5}\left(-3x+x^2\right)^{5/3}\end{aligned}$$

Mathematica [A] time = 0.0039718, size = 13, normalized size = 0.87

$$\frac{3}{5}((x-3)x)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
```

```
[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] time = 0.046, size = 18, normalized size = 1.2

$$\frac{3\,(-3+x)^2\,x^2}{5}\frac{1}{\sqrt[3]{(-3+x)\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x)
```

```
[Out] 3/5*(-3+x)^2*x^2/((-3+x)*x)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^2-9\,x+9\right)x}{((x-3)x)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)
```

Fricas [A] time = 1.45791, size = 31, normalized size = 2.07

$$\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x\,(x-3)\,(2x-3)}{\sqrt[3]{x\,(x-3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x**2-9*x+9)/((-3+x)*x)**(1/3),x)
```

```
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(2\,x^2-9\,x+9\right)x}{\left((x-3)x\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)
```

3.142 $\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\left(g^2+3h^2x^2\right)}\,dx$

Optimal. Leaf size=242

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}-\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}+\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

```
[Out] ((1 - (9*h^2*x^2)/g^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^(2
/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*Sqrt[3]*h*(-((c*g^2)/h^2) +
 9*c*x^2)^(1/3)) + ((1 - (9*h^2*x^2)/g^2)^(1/3)*Log[g^2 + 3*h^2*x^2])/(6*2^
(2/3)*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)) - ((1 - (9*h^2*x^2)/g^2)^(1/3)*Lo
g[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2*2^(2/3)*h*(-((
c*g^2)/h^2) + 9*c*x^2)^(1/3))
```

Rubi [A] time = 0.0927643, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1009, 1008}

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}-\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}+\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```



```
[Out] ((1 - (9*h^2*x^2)/g^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^(2
/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*Sqrt[3]*h*(-((c*g^2)/h^2) +
 9*c*x^2)^(1/3)) + ((1 - (9*h^2*x^2)/g^2)^(1/3)*Log[g^2 + 3*h^2*x^2])/(6*2^
(2/3)*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)) - ((1 - (9*h^2*x^2)/g^2)^(1/3)*Lo
g[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2*2^(2/3)*h*(-((
c*g^2)/h^2) + 9*c*x^2)^(1/3))
```


Rule 1009

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Dist[(1 + (c*x^2)/a)^(1/3)/(a + c*x^2)^(1/3), Int[(g + h*x)/
((1 + (c*x^2)/a)^(1/3)*(d + f*x^2)), x], x] /; FreeQ[{a, c, d, f, g, h}, x]
 && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] &&  !GtQ[a, 0]
```

Rule 1008

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(Sqrt[3]*h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*a^(1/3)*f), x] + (-Simp[(
3*h*Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2^(5/3)*a^
(1/3)*f), x] + Simp[(h*Log[d + f*x^2])/(2^(5/3)*a^(1/3)*f), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\left(g^2+3h^2x^2\right)}\,dx = \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\int \frac{g+hx}{\left(g^2+3h^2x^2\right)\sqrt[3]{1-\frac{9h^2x^2}{g^2}}}\,dx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}$$

$$= \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3hx}{g}}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\ 2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} - \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}}{}$$

Mathematica [C] time = 0.581694, size = 268, normalized size = 1.11

$$\frac{h^2x\left(-\frac{2g^5F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)}{\left(g^2+3h^2x^2\right)\left(g^2F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)+2h^2x^2\left(F_1\left(\frac{3}{2};\frac{4}{3},1;\frac{5}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)-F_1\left(\frac{3}{2};\frac{1}{3},2;\frac{5}{2};\frac{9h^2x^2}{g^2},-\frac{3h^2x^2}{g^2}\right)\right)\right)}-hx\sqrt[3]{1-\frac{9h^2x^2}{g^2}}F_1\left(1;\frac{1}{3},1;2;\right.}{2cg^2\left(g^2-9h^2x^2\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```

```
[Out] (h^2*x*(c*(-(g^2/h^2) + 9*x^2))^(2/3)*(-(h*x*(1 - (9*h^2*x^2)/g^2)^(1/3)*Ap
pellF1[1, 1/3, 1, 2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2]) - (2*g^5*AppellF1[
```

```
1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2])/((g^2 + 3*h^2*x^2)*(g
^2*AppellF1[1/2, 1/3, 1, 3/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + 2*h^2*x^
2*(-AppellF1[3/2, 1/3, 2, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2] + AppellF
1[3/2, 4/3, 1, 5/2, (9*h^2*x^2)/g^2, (-3*h^2*x^2)/g^2])))))/(2*c*g^2*(g^2 -
 9*h^2*x^2))
```

Maple [F] time = 0.803, size = 0, normalized size = 0.

$$\int \frac{hx+g}{3\,h^{2}x^{2}+g^{2}}\frac{1}{\sqrt[3]{-\frac{cg^{2}}{h^{2}}+9\,cx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)

[Out] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{hx+g}{\left(3\,h^{2}x^{2}+g^{2}\right)\left(9\,cx^{2}-\frac{cg^{2}}{h^{2}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
maxima")

[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{g+hx}{\sqrt[3]{c\left(-\frac{g}{h}+3x\right)\left(\frac{g}{h}+3x\right)}\left(g^2+3h^2x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g**2/h**2+9*c*x**2)**(1/3)/(3*h**2*x**2+g**2),x)
```

```
[Out] Integral((g + h*x)/((c*(-g/h + 3*x)*(g/h + 3*x))**(1/3)*(g**2 + 3*h**2*x**2
)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{hx+g}{\left(3\,h^2x^2+g^2\right)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
giac")
```

```
[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

3.143 $\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$

Optimal. Leaf size=488

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)\right.}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9c}}$$

```
[Out] (3*3^(1/6)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2
))/(2*c*g - b*h)^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b + 2*c*x)
)/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3
))])/(f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3)) +
 (3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2)
)/(2*c*g - b*h)^2)^(1/3)*Log[(f*(c^2*g^2 - b*c*g*h + b^2*h^2))/(3*c^2*h^2)
+ (b*f*x)/c + f*x^2])/(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x
+ 9*c*x^2)^(1/3)) - (3*3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^
2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*Log[(1 - (3*h*(b + 2*c*x))/(2
*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)])/
(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3))
```

Rubi [A] time = 0.360419, antiderivative size = 488, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 104, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1041, 1040}

$$\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)\right.}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9c}}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)
^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x
)/c + f*x^2)),x]
```

```
[Out] (3*3^(1/6)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2
))/(2*c*g - b*h)^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b + 2*c*x)
)/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3
))])/(f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3)) +
 (3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2)
)/(2*c*g - b*h)^2)^(1/3)*Log[(f*(c^2*g^2 - b*c*g*h + b^2*h^2))/(3*c^2*h^2)
+ (b*f*x)/c + f*x^2])/(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x
+ 9*c*x^2)^(1/3)) - (3*3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^
2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*Log[(1 - (3*h*(b + 2*c*x))/(2
*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)])/
(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3))
```

Rule 1041

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = -(c/(b^2 - 4*a*c))},
Dist[(q*(a + b*x + c*x^2))^(1/3)/(a + b*x + c*x^2)^(1/3), Int[(g + h*x)/((q
*a + b*q*x + c*q*x^2)^(1/3)*(d + e*x + f*x^2)), x], x]] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0]
&& EqQ[c^2*g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] &&  !GtQ[4*a - b^2/c,
0]
```

Rule 1040

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = ((-9*c*h^2)/(2*c*g -
b*h)^2)^(1/3)}, Simp[(Sqrt[3]*h*q*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b
+ 2*c*x))/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*
h))^(1/3))])/f, x] + (-Simp[(3*h*q*Log[(1 - (3*h*(b + 2*c*x))/(2*c*g - b*h)
)^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)])/(2*f), x] +
 Simp[(h*q*Log[d + e*x + f*x^2])/(2*f), x])] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] && EqQ[c^2*
g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] && GtQ[(-9*c*h^2)/(2*c*g - b*h)^2
, 0]
```

Rubi steps

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx = \frac{\sqrt[3]{-\frac{c\left(\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2\right)}{b^2-\frac{4\left(-c^2g^2+bcgh+2b^2h^2\right)}{9h^2}}}\int \frac{}{\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}\right.}}{\sqrt[3]{}}$$

$$= \frac{3\sqrt[6]{3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(}{\sqrt{3}}\right.}{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$

Mathematica [F] time = 0.549227, size = 0, normalized size = 0.

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(g + h*x)/((((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x +
c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 +
(b*f*x)/c + f*x^2)),x]
```

```
[Out] Integrate[(g + h*x)/((((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x +
c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 +
(b*f*x)/c + f*x^2)), x]
```

Maple [F] time = 3.165, size = 0, normalized size = 0.

$$\int (hx+g)\frac{1}{\sqrt[3]{\frac{2\,b^2h^2+bcgh-c^2g^2}{9\,ch^2}+bx+cx^2}}\left(\frac{f}{c^2}\left(b^2+\frac{-2\,b^2h^2-bcgh+c^2g^2}{3\,h^2}\right)+\frac{bfx}{c}+fx^2\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2
+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

```
[Out] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2
+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$3\int \frac{hx+g}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="maxima")
```

```
[Out] 3*integrate((h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(
c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^
2)/h^2)*f/c^2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b**2*h**2+b*c*g*h-c**2*g**2)/c/h**2+b*x+c*x**2)**
(1/3)/(f*(b**2+1/3*(-2*b**2*h**2-b*c*g*h+c**2*g**2)/h**2)/c**2+b*f*x/c+f*x*
*2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\left(hx+g\right)}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="giac")
```

```
[Out] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(
c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^
2)/h^2)*f/c^2)), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```